QUICK ARITHMETIC

For Competitive Examinations

QUICK ARITHMETIC

[This Book explains the Short-Cut methods of Solving the Problems]

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PREFACE TO SECOND EDITION

It is a great pleasure to note the appreciation shown by the students for the first edition. The second edition is thoroughly revised and enlarged according to the latest trend of the various competitive exames and feedback from the readers. New and better methods of solving the problems have been provided in the book. Important questions from previous papers have been included.

I am sure that this edition will be of immense help to every student preparing for any competitive exam.

ASHISH AGGARWAL

PREFACE TO FIRST EDITION

I am happy to present this book to the students who wish to appear in the objective type competitive examinations. In such examinations, the students are required to solve questions in a limited time. The conventional method of solving the mathematical questions is not only time-consuming but also prone to mistakes because of the lengthy methods involved. This book explains the short-cut methods of solving the sums in a faster way. For the convenience of the students and easy understanding, the questions are divided into various chapters. The book covers basic terms, short-cut methods of multiplication, ratio and proportion, percentage, profit and loss, time and distance, time and work, simple and compound interest, mensuration, etc. Each chapter contains rules and formulae followed by fully solved exercise. Questions are also taken from previous papers of SSC, Banks and other exams. The important rules are given in the boxes so that one can locate them easily for future reference. In the chapters, logic of formulae is explained so that the students learn the logic of formulae, instead of memorising the formulae directly.

While teaching, I have experienced students facing difficulty in memorising methods, and also making wrong application of rules, leading to wrong answers. I hope that this book will be of immense help to those who appear in objective type mathematical papers. I am very thankful to my family and friends for their encouragement and support in writing this book. Any suggestion for improvement of quality of the book and rectification of errors, is most welcome.

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CONTENTS

Chapter 1

BASIC

TYPE OF NUMBERS

Natural Numbers (N):

1, 2, 3, 4, ...

Natural numbers are also called Positive Integers.

Whole Numbers (W):

0, 1, 2, 3, ...

Integers (I):

.... –3, –2, –1, 0, 1, 2, 3, 4, 5,............

Even Numbers:

Numbers which are exactly divisible by 2.

i.e. 2, 4, 6, 8, ...

Odd Numbers:

Numbers which are not exactly divisible by 2.

i.e. 1, 3, 5, 7, 9, ...

Note: Sum of any two odd numbers is always an even number.

Example:

$3 + 5 = 8$ (an even number)

$7 + 23 = 30$ (an even number)

Note: Product of any two odd numbers is always an odd number.

Example:

$3 \times 7 = 21$ (an odd number)

$5 \times 13 = 65$ (an odd number)

Rational Numbers:

Numbers which can be written in the form of $\frac{p}{q}$, where p and q both are integers, and q is a non-zero integer.

A number expressed in the form of $\frac{p}{q}$ is also called a **fraction,** where 'p' is numerator and 'q' is denominator. A fraction when converted into decimal, gives finite or recurring digits after decimal sign.

Example:

$\frac{1}{3}, \frac{1}{2}, \frac{7}{4}$

Note :

$\frac{x}{1} = x$, and $\frac{0}{x} = 0$

Irrational Numbers:

Numbers which cannot be written in the form of $\frac{p}{q}$ i.e. the numbers which are not rational.

These numbers when converted into decimal, give infinite and non-recurring digits after decimal sign.

Example:

π, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{27}$ etc.

Real numbers:

Real numbers include both rational and irrational numbers.

Mixed Numbers:

Number which consists of a whole number and a fraction.

Example:

$3\frac{1}{3}, 5\frac{2}{3}$ etc.

Prime Numbers:

Numbers which are not divisible by any number other than 1 and itself:

Example:

2, 3, 5, 7, 11, 13, 17, 19, 23

Note: 1 is neither a prime number nor a composite number.

Note: 2 is the only even prime number.

Note: Product of two prime numbers is always a composite number, since the product is divisible by both the prime numbers.

Example:

7 × 11 = 77 (a composite number)

Composite Number:

The number which is not a prime number i.e., the number which is divisible by any number other than 1 and itself.

Example:

4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24

Co-prime Numbers:

Co-prime numbers are the numbers which are prime to each other i.e. they don't have any common factor other than 1.

Since, they do not have any common factor, their HCF is 1 and LCM is equal to products of the numbers.

Note: Co-prime numbers can be prime or composite numbers. Any two prime numbers are always co-prime numbers.

Example:

1. 3 and 5: Both numbers are prime numbers.
2. 8 and 15: Both numbers are composite numbers but they are prime to each other i.e. they don't have any common factor.

Face value and Place value:

Face Value is absolute value of a digit in a number.

Place Value is value of a digit in relation to its position in the number.

Example:

Face value and Place value of 9 in 921 is 9 and 900 respectively.

RULES

1. Product of two positive numbers is always a positive number.

   i.e. $(+a) \times (+b) = +(a \times b)$

Example:

$(4) \times (3) = (+)\ 12 = 12$

2. Product of two negative numbers is always a positive number.

   i.e. $(-a) \times (-b) = +(a \times b)$

Example:

$(-4) \times (-3) = (+)\ 12 = 12$

3. Product of a negative number and a positive number is always a negative number.

   $(-a) \times (+b) = (+a) \times (-b) = -(a \times b)$

Example:

$(-4) \times (3) = (-)\ 12 = -12$

$(4) \times (-3) = (-)\ 12 = -12$

4. Any number multiplied by '0' gives '0'.

   i.e. $a \times 0 = 0 \times a = 0$

Example:

$0 \times 127 = 0$

$92567 \times 0 = 0$

5. If $x + y = c$ (where 'c' is a constant number).

   Then, xy is maximum when $x = y = \frac{c}{2}$

6. How to check whether a given number is prime or not ?

Steps:

1. Find approximate square root of the given number.
2. Divide the given number by every prime number less than the approximate square root.
3. If the given number is divisible by atleast one of the prime numbers, the number is a composite number otherwise a prime number.

Example:

Is 401 a prime number?

Solution:

Approximate square root of 401 is 20.

Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

Since, 401 is not divisible by 2, 3, 5, 7, 11, 13, 17 and 19.

∴ 401 is a prime number.

Hint: Next prime number after 19 is 23 which is greater than 20, so we need not check further.

7. If sum of digits of a natural number is subtracted from the number itself, the resulting number is always divisible by 9.

Example 1:

143: Sum of digits = 1 + 4 + 3 = 8

Now, 143 – 8 = 135 which is divisible by 9.

Example 2:

106: Sum of digits = 1 + 0 + 6 = 7

Now, 106 – 7 = 99 which is divisible by 9.

8. A three-digit number formed by repeating a one-digit number thrice, is always divisible by 111.

Since 111 = 3 × 37, such numbers are divisible by 3 and 37 also.

Example:

222, 777, 888 etc. are divisible by 3, 37 and 111.

9. A four-digit number formed by repeating a two-digit number, is always divisible by 101.

Example:

2525, 3636, 8383, etc. are divisible by 101.

10. The number zero is surrounded by the same two-digit number on both sides. Any such five-digit number is always divisible by 1001.

Since 1001 = 7 × 11 × 13. Such numbers are divisible by 7, 11 and 13 also.

Example:

52052, 68068, 93093, etc. are divisible by 7, 11, 13 and 1001.

11. A six-digit number formed by repeating a three-digit number is always divisible by 1001.

Since 1001 = 7 × 11 × 13. Such numbers are divisible by 7, 11 and 13 also.

Example:

436436, 619619, 851851 etc. are divisible by 7, 11, 13 and 1001.

SHORT-CUT METHODS

12. Product of any 'n' consecutive natural numbers is always divisible by $\underline{|n}$.

Where, $\underline{|n}$ = (n) × (n-1) × (n-2) × .... × 2 × 1.

Example:

Product of any 2, 3 and 4 consecutive numbers are always divisible by $\underline{|2}$, $\underline{|3}$, $\underline{|4}$ i.e. divisible by 2, 6 and 24 respectively.

13. If on adding 9 to a two-digit number, its digits are reversed.
Then unit's digit of the number exceeds its ten's digit by 1.

Proof:

Let the original number is xy.

On adding 9 to it, the number becomes yx.

'xy' can be written as $10x + y$, and

'yx' can be written as $10y + x$

$\therefore (10x + y) + 9 = (10y + x)$

$\therefore (10x - x) + 9 = (10y - y)$

$\therefore 9x + 9 = 9y$

$\therefore x + 1 = y$

Similarly, we can prove that if on adding '9a' to a two-digit number (say xy), digits of the number are reversed.

Then, $x + a = y$

Note: If digits are reversed on subtracting '9a' from it, then $x - a = y$.

SOLVED EXERCISE

1. On subtracting 5 from one-fifth of a number, the result is 10. What is the number?

(a) 15 (b) 25 (c) 50 (d) 75

Solution:

(d) Let the number is x.

Then, $\frac{x}{5} - 5 = 10$

$\therefore \frac{x}{5} = 10+5 = 15$

$\therefore x = 5 \times 15 = 75$

2. In a class of 50 students, 25 students can speak English and 40 students Hindi. If each student can speak atleast one language, how many students can speak both languages?

(a) 10 (b) 15 (c) 20 (d) 25

Solution:

(*b*) Students who can speak both languages

$= 25 + 40 - 50 = 15$

3. In a group of 40 persons, 15 drink tea but not coffee and 18 drink coffee but not tea. If they drink atleast one of the drinks, how many persons in the group drink tea?

(a) 15 (b) 18 (c) 22 (d) 25

Solution:

(c) Persons who drink tea

= Total persons – Persons who don't drink tea

$= 40 - 18 = 22$

4. A sum of Rs. 100 is divided among A, B and C. If A and B together receive Rs. 70, B and C together receive Rs. 50. Find the amount received by B.

(a) Rs. 10 (b) Rs. 20 (c) Rs. 40 (d) Rs. 60

Solution:

(b) (A + B) + (B + C) is Rs.70 + Rs.50 = Rs.120

∴ B's share = Rs. 120 – Rs. 100 = Rs. 20

5. A bucket full of water weighs 50 kg, while half-filled bucket weighs 30 kg. Find the weight of empty bucket.

(a) 10 kg (b) 15 kg (c) 20 kg (d) 25 kg

Solution:

(a) Weight of bucket + full water = 50 kg.

And weight of bucket + half water = 30 kg.

∴ Weight of half water = (50 – 30) kg = 20 kg.

∴ Weight of empty bucket = 30 kg – 20 kg = 10 kg.

6. A number when multiplied by 21 is increased by 420. Find the number.

(a) 19 (b) 20 (c) 21 (d) 22

Solution:

(c) Let 'x' is the required number.

Then, $21x = x + 420$

$\therefore 21x - x = 420$

$\therefore x = \frac{420}{20} = 21$

Trick:

21 times – 1 time = 420

∴ 20 times of a number = 420

∴ The number is $\frac{420}{20} = 21$

7. Find 307th term of the sequence:

1, 2, 5, 8, 1, 2, 5, 8, 1, 2, 5, 8, ...

(a) 1 (b) 2 (c) 5 (d) 8

Solution:

(c) Four terms make one set.
Remainder on dividing 307 by 4 is 3.
307th term is same as third term in the series.
Third term in the series is 5.

8. Find the sum of first 50 terms in the series:

3 + 2 – 5 + 3 + 2 – 5 ....

(a) 0 (b) 2 (c) 3 (d) 5

Solution:

(d) Three terms make one set and total of one set is 3 + 2 – 5 = 0

Since $50 = 3 \times 16 + 2$.

$\therefore$ 50 terms mean 16 sets and two terms.

Total of two terms $= 3 + 2 = 5$

$\therefore$ Total of 50 terms $= 0 + 5 = 5$

9. Find the sum of first 50 terms in the series:

$5 + 3 - 7 + 5 + 3 - 7$ ....

(a) 1 (b) 8 (c) 24 (d) 40

Solution:

(c) Three terms make one set.

Since $50 = 3 \times 16 + 2$.

$\therefore$ 50 terms mean 16 sets and 2 terms.

Now, total of one set $= 5 + 3 - 7 = 1$.

$\therefore$ Total of 50 terms $= 1 \times 16 + (5 + 3) = 24$.

10. The sum of the digits of a two-digit number is 9. If 9 is added to the number, the digits are reversed. What is the number?

(a) 18 (b) 36 (c) 45 (d) 81

Solution:

(c) On adding 9 to the number, its digits are reversed.

$\therefore$ Unit's digit is $\frac{9}{9} = 1$ more than the ten's digit.

Ten's digit is $\frac{9-1}{2} = 4$

$\therefore$ Unit's digit is $4 + 1 = 5$

$\therefore$ The number is 45.

Note: Ten's digit is always smaller than unit's digit if digits are reversed on adding something to the number.

11. The sum of the digits of a two-digit number is 13. If 9 is added to the number, the digits are reversed. What is the number?

(a) 36 (b) 67 (c) 76 (d) 117

Solution:

(b) On adding 9 to the number, its digits are reversed.

$\therefore$ Unit's digit is $\frac{9}{9} = 1$ more than the ten's digit.

$\therefore$ Ten's digit $= \frac{13-1}{2} = 6$

Unit's digit $= 6 + 1 = 7$

$\therefore$ The number is 67.

12. The sum of the digits of a two-digit number is 11. On adding 27 to the given number, its digits are reversed. Find the number.

(a) 38 (b) 47 (c) 74 (d) 83

Solution:

(b) On adding 27 to the number, its digits are reversed.

∴ Unit's digit is $\frac{27}{9} = 3$ more than the ten's digit.

∴ Ten's digit = $\frac{11-3}{2} = 4$

Unit's digit = 4 + 3 = 7

∴ The number is 47.

13. The sum of the digits of a two-digit number is 15. On subtracting 27 from the given number, the digits are reversed. Find the number.

(a) 69 (b) 78 (c) 87 (d) 96

Solution:

(d) On subtracting 27 from the number, its digits are reversed.

∴ Unit's digit is smaller by $\frac{27}{9} = 3$ than the ten's digit.

∴ Unit's digit = $\frac{15-3}{2} = 6$

Ten's digit = 6 + 3 = 9

∴ The number is 96.

14. A number consisting of two digits is four times the sum of its digits. On adding 18 to the number, the digits of the given number are reversed. Find the original number.

(a) 24 (b) 36 (c) 46 (d) 68

Solution:

(a) On adding 18 to the number, its digits are reversed.

∴ Unit's digit is bigger than ten's digit by $\frac{18}{9} = 2$

∴ 36 is not the required number.

Since, the number is four times the sum of its digits, whatever is the sum of digits, the number is divisible by 4.

∴ 46 also is not the required number.

Now, we check 24 and 68.

Sum of the digits should be $\frac{24}{4}$ and $\frac{68}{4}$ i.e. 6 and 17 respectively.

Now, $6 + 8 = 14 \neq 17$

∴ 68 is not the required number.

And, 2 + 4 = 6 = 6

∴ 24 is the required number.

15. A number consists of two digits. If the digits interchange places and the new number is added to the original number, the resulting number will be divisible by :

(*a*) 3 (*b*) 5 (*c*) 9 (*d*) 11

Solution:

(*d*) Let the number is $10x + y$

On interchanging places, the new number is $10y + x$

Sum of two numbers $= 10x + y + 10y + x$

$= 11x + 11y = 11\ (x + y)$

$\therefore$ The sum is always divisible by 11.

16. Three numbers which are co-prime to one another are such that the product of the first two is 475 and that of the last two numbers is 1075. Find the sum of the three numbers.

(*a*) 75 (*b*) 78 (*c*) 87 (*d*) 89

Solution :

(*c*) Let the number are x, y and z.

$x \times y = 475 = 19 \times 25$

and $y \times z = 1075 = 25 \times 43$

Sum of the numbers $= 19 + 25 + 43 = 87$

17. Product of any five consecutive numbers is always divisible by:

(a) 5 (b) 15 (c) 20 (d) 120

Solution:

(d) Product of any five consecutive numbers is always divisible by $\underline{|5}$.

$\underline{|5} = 1 \times 2 \times 3 \times 4 \times 5 = 120$

18. Product of two numbers is 162. If the greater number is 2 times the smaller number, what is the greater number?

(a) 9 (b) 15 (c) 18 (d) 21

Solution:

(c) Let the smaller number is x.

Then the bigger number is 2x.

Product of the two numbers is $(x) \times (2x) = 162$

$$\therefore x^2 = \frac{162}{2} = 81$$

$$\therefore x = \sqrt{81} = 9$$

$\therefore$ The bigger number is $2x = 2 \times 9 = 18$.

19. $\underline{|6} - \underline{|5} - \underline{|4} = ?$

(a) $6 \times 4 \times \underline{|4}$ (b) $6 \times 5 \times \underline{|4}$ (c) $5 \times 4 \times \underline{|5}$ (d) $6 \times 5 \times \underline{|6}$

Solution:

(a) $\underline{|6} - \underline{|5} - \underline{|4}$

$= 6 \times 5 \times \underline{|4} - 5 \times \underline{|4} - \underline{|4}$

$= (6 \times 5 - 5 - 1) \times \underline{|4}$

$= (6 \times 5 - 6) \times \underline{|4}$

$= 6 \times (5 - 1) \times \underline{|4}$

$= 6 \times 4 \times \underline{|4}$

20. $\underline{|9} - \underline{|8} - \underline{|7} = ?$

(a) $9 \times 8 \times \underline{|7}$ (b) $9 \times 7 \times \underline{|7}$ (c) $8 \times 7 \times \underline{|7}$ (d) $8 \times 7 \times \underline{|6}$

Solution:

(b) $\underline{|9} - \underline{|8} - \underline{|7}$

$= 9 \times 8 \times \underline{|7} - 8 \times \underline{|7} - \underline{|7}$

$= (9 \times 8 - 8 - 1) \times \underline{|7}$

$= (9 \times 8 - 9) \times \underline{|7}$

$= 9 \times (8 - 1) \times \underline{|7}$

$= 9 \times 7 \times \underline{|7}$

21. 64 and all the odd numbers up to 100 are multiplied together. The number of zeros at the end of the product will be:

(a) 0 (b) 5 (c) 6 (d) 12

Solution:

(c) There are 10 numbers up to 100 viz. 5, 15, 25, 35,…………95 which have 5 at the unit's place but 25 and 75 are divisible by 5 twice i.e. the product can be divided 10 + 2 = 12 times by 5.

And $64 = 2^6$

We know that 5 × 2 = 10 i.e. product of one pair of 2 and 5 gives one zero.
In the given situation 6 such pairs can be made.

∴ Total 0s at the end of the product are 6.

22. All the even numbers up to 100 i.e. 2, 4, 6,…………..100 are multiplied together. The number of zeros at the end of the product will be:

(a) 10 (b) 11 (c) 12 (d) 13

Solution:

(c) There are 10 numbers up to 100 viz. 10, 20, 30,…………100.
These numbers have '0' at the unit place but 100 has two 0s.
∴ 0s in the product of these numbers = 10 + 1 = 11.

50 = 5 × 10
∴ Product of 5 and an even number is also divisible by 10.
∴ 0s in the product = 11 + 1 = 12.

23. A man engaged a servant on the condition that he would pay him Rs. 3000 and one uniform after one year of service. He served only for 9 months and got Rs. 2200 and a uniform. What is the price of uniform ?

(*a*) 100 (*b*) 200 (*c*) 250 (*d*) 400

Solution :

(*b*) 12 months wages = Rs. 3000 + Uniform
9 months wages = Rs. 2200 + Uniform
∴ 3 months wages = Rs. 800
∴ 12 months wages = 4 × Rs. 800 = Rs. 3200
∴ Cost of uniform = Rs. 3200 – Rs. 3000 = Rs. 200

Chapter 2

FRACTIONS AND DECIMALS

Fractions: Numbers which can be written in the form of $\frac{p}{q}$, where p and q both are integers, and q is a non-zero integer, and 'p' is called numerator and 'q' is called denominator. Like natural numbers, fractions can also be added, subtracted, multiplied and divided.

Proper Fractions : Numerator is less than the denominator.

Example :

$$\frac{2}{3}, \frac{1}{5}, \frac{5}{7}$$

Improper Fractions : Numerator is greater than the denominator.

Example :

$$\frac{4}{3}, \frac{8}{5}, \frac{10}{7}$$

Note : The value of an improper fraction is greater than 1.

Mixed numbers : A number which consists of an Integar and a proper fraction.

Example :

$$3\frac{1}{2}, 5\frac{3}{4}.$$

Mixed numbers can be converted into improper fractions and vice-versa.

How to convert a Mixed number into improper fraction :

Multiply the whole number with the denominator of the fraction and then add it to the numerator of the fraction. The result is the numerator and the denominator will be the denominator of the improper fraction.

Examples :

$$3\frac{1}{2} = \frac{3 \times 2 + 1}{2} = \frac{7}{2}$$

$$5\frac{3}{4} = \frac{5 \times 4 + 3}{4} = \frac{23}{4}$$

How to convert an improper fraction into a Mixed number :

Divide the numerator by the denominator. The dividend is the whole number, the remainder is the numerator of the fraction of the mixed number and the denominator remains the same.

Examples :

$$\frac{5}{3} = 1\frac{2}{3}; \frac{9}{2} = 4\frac{1}{2}$$

Addition/Subtraction of fractions :

Method :

1. Find LCM of the denominators of the fractions.
2. Divide this LCM by each denominator of the fractions and multiply the dividend with the respective numerators.

3. Add the multiplication results. The sum is the numerator and the LCM is the denominator.

Example 1 :

Find the value of $\frac{1}{2}+\frac{2}{3}+\frac{2}{5}$.

Solution :

LCM of denominators 2, 3 and 5 is 30

On dividing 30 by 2, 3 and 5, the dividends are 15, 10 and 6 respectively.

Multiply the numerators 1, 2 and 2 with 15, 10 and 6 respectively.

$$\frac{1}{2}+\frac{2}{3}+\frac{2}{5}=\frac{1\times15+2\times10+2\times6}{30}=\frac{15+20+12}{30}=\frac{47}{30}=1\frac{17}{30}$$

Example 2 :

Find the value of $\frac{3}{2}-\frac{1}{3}-\frac{2}{7}$.

Solution :

LCM of 2, 3 and 7 is 42.

On dividing 42 by 2, 3 and 7, the dividends are 21, 14 and 6 respectively.

$$\frac{3}{2}-\frac{1}{3}-\frac{2}{7}=\frac{3\times21-1\times14-2\times6}{42}=\frac{63-14-12}{42}=\frac{37}{42}$$

Another method of adding/subtracting the fractions :

Multiply the denominator and numerator of each fraction so as to make the denominators of all the fractions equal to their LCM. Then, add or subtract the numerators and divide them by the LCM. Example 1 can be solved by using this method as follows :

L.C.M. of 2, 3 and 5 is 30.

∴ The denominators of these fractions are made equal to 30.

$$\frac{1}{2}=\frac{1}{2}\times\frac{15}{15}=\frac{15}{30}$$

$$\frac{2}{3}=\frac{2}{3}\times\frac{10}{10}=\frac{20}{30}$$

$$\frac{2}{5}=\frac{2}{5}\times\frac{6}{6}=\frac{12}{30}$$

$$\frac{1}{2}+\frac{2}{3}+\frac{2}{5}=\frac{15}{30}+\frac{20}{30}+\frac{12}{30}=\frac{47}{30}=1\frac{17}{30}$$

Multiplication of fractions :

$$\frac{2}{3}\times\frac{5}{7}=\frac{2\times5}{3\times7}=\frac{10}{21}$$

$$1\frac{2}{3}\times\frac{4}{5}=\frac{1\times3+2}{3}\times\frac{4}{5}=\frac{5}{3}\times\frac{4}{5}=\frac{4}{3}$$

Note : Before multiplying a mixed number, convert it into an improper fraction.

COMPARISON OF FRACTIONS

Two or more fractions can be compared by using the following methods:

$\frac{A}{X} > \frac{B}{X}$, if $A > B$

⇒ If denominators of two or more fractions are same.

Higher the numerator, higher the value of fraction is.

Example:

$\frac{13}{7} > \frac{11}{7} > \frac{5}{7}$

$\frac{A}{M} < \frac{A}{N}$, if $M > N$

⇒ If numerators of two or more fractions are same.

Higher the denominator, smaller the value of fraction is.

Example:

$\frac{4}{5} > \frac{4}{7} > \frac{4}{13}$

$\frac{A}{M} < \frac{A+k}{M+k}$, where 'k' is a constant number and $A < M$

⇒ If a constant number is added to both the numerator and the denominator of a proper fraction, the value of fraction with higher numerator is more.

Example:

$\frac{2}{3} < \frac{4}{5} < \frac{17}{18}$

Note: In this case difference between numerator and denominator of the given fractions will remain same.

$\frac{A}{B} > \frac{A+K}{B+K}$, where '$k$' is a constant number and $A > B$

⇒ If a constant number is added to both the numerator and the denominator of a improper fraction, the value of fraction with smaller numerator is more.

Example:

$\frac{5}{4} > \frac{8}{7} > \frac{10}{9}$

Note : Difference between numerator and denominator of each fraction is same.

Cross Multiplication Method:

$\frac{A}{B} > \frac{X}{Y}$, if $AY > BX$

⇒ The fraction with greater numerator after cross-multiplication, is bigger.

Note: In this method we actually make the denominators of the fractions equal.

Example:

Which of the two fractions $\frac{5}{6}$ or $\frac{2}{5}$ is bigger?

Solution:

Applying the rule of cross multiplication, we get the following values:

5×5 and 2×6 or 25 and 12

Clearly 25 is bigger than 12.

$\therefore \frac{5}{6} > \frac{2}{5}$

LCM Method:

Change numerators of the fractions after making denominators of fractions equal to the LCM of all the denominators.

Example:

Compare $\frac{5}{6}, \frac{7}{4}$ and $\frac{9}{10}$.

Solution:

LCM of the denominators i.e. LCM of 6, 4 and 10 is 60.

$\therefore$ Denominators of the given numbers are made equal to 60.

Numerators become $5 \times 10, 7 \times 15, 9 \times 6 = 50, 105$ and 54.

$\because 105 > 54 > 50$

$\therefore \frac{7}{4} > \frac{9}{10} > \frac{5}{6}$

Division Method :-

Fractions are converted into decimals by dividing the numerators of the given fractions by their respective denominators.

Example:

Compare $\frac{14}{3}$ and $\frac{17}{5}$

Solution:

$\frac{14}{3} = 4.67$ and $\frac{17}{5} = 3.4$

We know that $4.67 > 3.4$

$\therefore \frac{14}{3} > \frac{17}{5}$

Fractions can be converted into decimals by dividing the denominators of the fractions by their respective numerators.

Note: This method can be used conveniently when the denominator is greater than numerator.

Example:

Compare $\frac{3}{11}, \frac{2}{9}$ and $\frac{1}{5}$

Solution:

$$\frac{3}{11}=\frac{1}{3.67},\ \frac{2}{9}=\frac{1}{4.5}\ \text{and}\ \frac{1}{5}=\frac{1}{5}$$

$$\because\quad \frac{1}{3.67}>\frac{1}{4.5}>\frac{1}{5}$$

$$\therefore\quad \frac{3}{11}>\frac{2}{9}>\frac{1}{5}$$

CONVERSION OF DECIMALS INTO FRACTIONS

(a) Non-recurring decimals:

Method:

Write the given number as numerator after removing the decimal sign.

Denominator consists of 1 followed by as many 0s as many are the digits after the decimal point in the given number.

Example:

$$0.7=\frac{7}{10},\ 0.23=\frac{23}{100},\ 0.47=\frac{47}{100},\ 5.18=5\frac{18}{100}$$

(b) Recurring Decimals:

(*i*) **Pure recurring decimals**: Where all the digits after decimal point are repeated:

Method:

The given number after removing the decimal sign is written as numerator of the fraction.

Denominator consists of as many 9 as many digits are in the given number after the decimal point.

Example:

$$0.\overline{1}=\frac{1}{9},\ 0.\overline{2}=\frac{2}{9},\ 0.\overline{3}=\frac{3}{9},\ 15.\overline{07}=15\frac{7}{99}$$

$$0.\overline{01}=\frac{1}{99},\ 0.\overline{02}=\frac{2}{99},\ 0.\overline{17}=\frac{17}{99},\ 0.\overline{376}=\frac{376}{999},\ 78.\overline{08}=78\frac{8}{99}$$

(*ii*) **Mixed recurring decimals**: Some digits after decimal point are not repeated while remaining digits are repeated:

Method:

From the number deduct non-recurring digits and write the result as numerator.

Denominator consists of as many 9s as many digits are recurring followed by as many 0s as are non-recurring digits after the decimal point

Examples:

$$0.1\overline{6}=\frac{16-1}{90}=\frac{15}{90};\ 0.3\overline{7}=\frac{37-3}{90}=\frac{34}{90};\ 0.12\overline{43}=\frac{1243-12}{9900}=\frac{1231}{9900}$$

$$2.\bar{3} = \frac{23-2}{9} = \frac{21}{9} ; 5.\bar{1} = \frac{51-5}{9} = \frac{46}{9} ; 12.\overline{52} = \frac{1252-12}{99} = \frac{1240}{99} \text{ or } 12\frac{52}{99}$$

$$2.\bar{3} = 2 + 0.\bar{3} = 2 + \frac{3}{9} = \frac{21}{9}$$

Similarly $5.\bar{1} = 5 + 0.\bar{1} = 5 + \frac{1}{9} = \frac{46}{9}$

$$87.\overline{05} = \frac{8705-87}{99} = \frac{8618}{99} \text{ or } 87\frac{5}{99}$$

$$2.53\bar{6} = 2 + 0.53\bar{6} = 2 + \frac{536-53}{900} = 2\frac{161}{300}$$

$$2.1\bar{8} = 2 + 0.1\bar{8} = 2\frac{18-1}{90} = 2\frac{17}{90} \text{ or } \frac{218-21}{90} = 2\frac{17}{90}$$

$$1.1\bar{6} = 1 + 0.1\bar{6} = 1 + \frac{16-1}{90} = 1\frac{15}{90} = 1\frac{1}{6} \text{ or } \frac{116-11}{90} = \frac{105}{90} = \frac{7}{6}$$

FRACTIONS AND THEIR EQUIVALENT IN DECIMALS

$\frac{1}{2} = 0.5$ $\frac{1}{3} = 0.\bar{3}$ $\frac{1}{4} = 0.25$ $\frac{1}{5} = 0.2$

$\frac{1}{6} = 0.1\bar{6} = 0.167$ (approx.) $\frac{1}{8} = 8.125$ $\frac{2}{3} = 0.\bar{6}$

MULTIPLICATION OF NUMBERS WITH DECIMALS

Steps:

1. The numbers are multiplied as if there is no decimal sign in the given numbers.
2. Put the decimal after 'n' digits of the solution counting from right side where 'n' is equal to total digits to the right side of the decimal point in the numbers multiplied.

Example: 1.41 × 1.1 = ?

Solution: 141 × 11 = 1551

After the decimal point, there are 2 digits in the first number and 1 digit in the second number.

∴ The multiplication will have 2 + 1 = 3 digits after the decimal point.

∴ 1.41 × 1.1 = 1.551

SOLVED EXERCISE

1. $\frac{5}{2} + \frac{3}{2} = ?$

(a) 1 (b) 2 (c) 3 (d) 4

Solution:

(d) $\frac{5}{2} + \frac{3}{2} = \frac{5+3}{2} = \frac{8}{2} = 4$

Note: Denominators of the fractions are same.

$\therefore$ The numerators are added.

2. $\frac{5}{6} + \frac{6}{7} = ?$

(a) $\frac{11}{13}$ (b) $\frac{71}{42}$ (c) $\frac{11}{42}$ (d) $\frac{30}{13}$

Solution:

(b) $\frac{5}{6} + \frac{6}{7} = \frac{5 \times 7 + 6 \times 6}{6 \times 7} = \frac{35 + 36}{42} = \frac{71}{42}$

Note: The denominators are co-prime numbers.

$\therefore$ Cross multiplication method is used.

3. $\frac{3}{5} + \frac{7}{10} = ?$

(a) $\frac{13}{10}$ (b) $\frac{10}{15}$ (c) $\frac{21}{50}$ (d) 1

Solution:

(a) $\frac{3}{5} + \frac{7}{10} = \frac{3 \times 2 + 7 \times 1}{10} = \frac{6 + 7}{10} = \frac{13}{10}$

Note: One denominator is fully divisible by the other denominator.

$\therefore$ Bigger denominator is the LCM of the denominators.

4. $\frac{1}{4} + \frac{5}{6} = ?$

(a) $\frac{3}{5}$ (b) $\frac{5}{24}$ (c) $\frac{13}{12}$ (d) $\frac{15}{6}$

Solution:

(c) $\frac{1}{4} + \frac{5}{6} = \frac{1 \times 3 + 5 \times 2}{12} = \frac{3 + 10}{12} = \frac{13}{12}$

Note: Denominators are neither same nor co-prime numbers.

$\therefore$ LCM of the denominators is taken.

5. $2\frac{1}{7} + 5\frac{3}{7} = ?$

(a) $10\frac{4}{7}$ (b) $10\frac{3}{49}$ (c) $7\frac{4}{7}$ (d) $7\frac{3}{49}$

Solution:

(c) $2\frac{1}{7} + 5\frac{3}{7}$

$= (2 + 5) + \left(\frac{1}{7} + \frac{3}{7}\right) = 7 + \frac{4}{7} = 7\frac{4}{7}$

Note: While adding two or more mixed numbers, we can add whole numbers and fractions separately.

6. $3\frac{4}{5}+2\frac{3}{5}-1\frac{2}{5}=?$

(a) $3\frac{3}{25}$ (b) $3\frac{14}{25}$ (c) $4\frac{3}{5}$ (d) 5

Solution:

(d) $3\frac{4}{5}+2\frac{3}{5}-1\frac{2}{5}$

$=(3+2-1)+\left(\frac{4}{5}+\frac{3}{5}-\frac{2}{5}\right)$

$=4+\frac{5}{5}$

$=4+1=5$

7. $8\frac{1}{4}+7\frac{1}{2}-3\frac{3}{8}=?$

(a) $18\frac{3}{8}$ (b) $14\frac{1}{8}$ (c) $12\frac{1}{8}$ (d) $12\frac{3}{8}$

Solution:

(d) $8\frac{1}{4}+7\frac{1}{2}-3\frac{3}{8}$

$=(8+7-3)+\left(\frac{1}{4}+\frac{1}{2}-\frac{3}{8}\right)$

$=12+\frac{3}{8}=12\frac{3}{8}$

8. $3\frac{1}{6}+8\frac{1}{4}-5\frac{7}{8}=?$

(a) $4\frac{13}{24}$ (b) $5\frac{11}{24}$ (c) $5\frac{13}{24}$ (d) $6\frac{13}{24}$

Solution:

(c) $3\frac{1}{6}+8\frac{1}{4}-5\frac{7}{8}$

$=(3+8-5)+\left(\frac{1}{6}+\frac{1}{4}-\frac{7}{8}\right)$

$=6-\frac{11}{24}=5\frac{13}{24}$

9. $3\frac{1}{3}\times5\frac{2}{5}=?$

(a) 18 (b) $15\frac{2}{15}$ (c) $15\frac{11}{15}$ (d) $8\frac{2}{15}$

Solution:

(a) $3\frac{1}{3} \times 5\frac{2}{5} = \frac{10}{3} \times \frac{27}{5} = 18$

> **Note:** While multiplying two or more mixed numbers, whole numbers and fractions can't be multiplied separately as is done in case of Addition and Subtraction.

10. $0.03 + 0.7 = ?$

(a) 0.10 (b) 0.01 (c) 0.37 (d) 0.73

Solution:

(d) $0.03 + 0.70 = 0.73$

11. $31.2 \times 56.8 = ?$

(a) 177216 (b) 17.7216 (c) 1772.16 (d) 177.216

Solution:

(c) Ignoring decimal point, all the options given are same i.e. 177216.

Decimal is to put after two digits from Right Side as both the numbers have decimal after 1 digit from Right Side and 1 + 1 = 2.

∴ The solution is 1772.16.

12. If $154 \times 18 = 2772$, then $27.72 \div 1.8 = ?$

(a) 1.54 (b) 15.4 (c) 154 (d) 1540

Solution:

(b) $2772 \div 18 = 154$, what we are to find is the place of decimal.

Decimal is to put after 1 digit i.e. 2 digits (of the dividend) – 1 digit (of the divisor).

∴ The solution is 15.4.

13. Find the smallest fraction that should be deducted from the sum of $1\frac{1}{2} + 3\frac{1}{4} + 5\frac{1}{8}$ to make the result a whole number.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{2}$

Solution:

(c) Sum of the fractions $= \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

∴ Deducting $\frac{7}{8}$ from the sum of the numbers will make the result a whole number.

14. Find the smallest fraction that should be added to the sum of $1\frac{1}{2} + 3\frac{3}{4} + 5\frac{5}{8}$ to make the result a whole number.

(a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{1}{2}$

Solution:

(b) Sum of the fractions $= \frac{1}{2} + \frac{3}{4} + \frac{5}{8} = \frac{4+6+5}{8} = \frac{15}{8}$

$\therefore\ \frac{1}{8}$ is the answer as adding $\frac{1}{8}+\frac{15}{8}=\frac{16}{8}=2$, a whole number.

15. $\frac{6}{7}$ of a number exceeds its $\frac{4}{5}$ by 4. The number is:

(a) 28 (b) 35 (c) 70 (d) 140

Solution:

(c) Let, x is the number.

Then, $\frac{6x}{7}-\frac{4x}{5}=\frac{(30-28)x}{35}=\frac{2x}{35}=4$

$\therefore\ x=\frac{4\times 35}{2}=70$

16. The sum of $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{8}$ of a number is 14. The number is:

(a) 12 (b) 14 (c) 16 (d) 18

Solution:

(c) Let, x is the number.

Then, $\frac{x}{2}+\frac{x}{4}+\frac{x}{8}=14$

$\therefore\ \frac{7x}{8}=14$

$\therefore\ x=14\times\frac{8}{7}=16$

17. A student was asked to multiply a number by $\frac{7}{8}$. But he divided the number by $\frac{7}{8}$. If the difference between the two results is 15, the number is:

(a) 64 (b) 56 (c) 49 (d) 15

Solution:

(b) Let, x is the number.

He is to find $\frac{7x}{8}$ instead of $\frac{8x}{7}$.

$\therefore$ Difference between the two results

$=\frac{8x}{7}-\frac{7x}{8}=\frac{15x}{56}=15$

$\therefore\ x=\frac{15\times 56}{15}=56$

Note: $\frac{x}{y}-\frac{y}{x}=\frac{x^2-y^2}{xy}=\frac{(x+y)\times(x-y)}{xy}$

18. $\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\ldots\ldots\ldots\ldots\times\left(1-\frac{1}{n}\right)=?$

(a) 0 (b) 1 (c) $\frac{1}{n}$ (d) $\frac{1}{n-1}$

Solution:

(c) On solving, we get:

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \ldots\ldots\ldots\ldots \times \frac{n-1}{n} = \frac{1}{n}$$

Note: Denominator of a fraction is equal to the numerator of the next fraction. Hence, we are left with numerator of the first fraction and denominator of the last fraction.

19. $\left(1-\frac{1}{1}\right) \times \left(1-\frac{1}{2}\right) \times \left(1-\frac{1}{3}\right) \times \ldots\ldots\ldots\ldots \times \left(1-\frac{1}{n}\right) = ?$

(a) 0 (b) 1 (c) $\frac{1}{n}$ (d) $\frac{1}{n-1}$

Solution:

(a) First number is $1 - 1 = 0$.

We know that, any number multiplied by 0 always gives 0.

$\therefore$ 0 is the answer.

20. $\left(1\frac{1}{2}\right) \times \left(1\frac{1}{3}\right) \times \left(1\frac{1}{4}\right) \times \ldots\ldots\ldots\ldots \times \left(1\frac{1}{120}\right) = ?$

(*a*) $\frac{1}{40}$ (*b*) 121 (*c*) $\frac{121}{120}$ (*d*) $\frac{121}{2}$

Solution :

(*d*) Given Expression $= \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \ldots\ldots\ldots\ldots \times \frac{121}{120} = \frac{121}{2}$

21. $\frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} = ?$

(*a*) 0.1 (*b*) 0.5 (*c*) 0.75 (*d*) 1.0

Solution :

(*a*) Given Expression $= \left(\frac{1}{5}-\frac{1}{6}\right) + \left(\frac{1}{6}-\frac{1}{7}\right) + \left(\frac{1}{7}-\frac{1}{8}\right) + \left(\frac{1}{8}-\frac{1}{9}\right) + \left(\frac{1}{9}-\frac{1}{10}\right)$

$$= \frac{1}{5} - \frac{1}{10} = \frac{1}{10} = 0.1$$

22. If sum of two numbers is 8 and their product is 15, find the sum of their reciprocals?

(a) 120 (b) $\frac{8}{15}$ (c) $\frac{23}{120}$ (d) $\frac{7}{120}$

Solution:

(b) Given, $x + y = 8$ and $xy = 15$

Now, $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{8}{15}$

23. If difference between two numbers is 7 and their product is 78, find the difference between their reciprocals?

(a) $\frac{6}{13}$ (b) $\frac{7}{15}$ (c) $\frac{13}{78}$ (d) $\frac{7}{78}$

Solution:

(d) Given, $x - y = 7$ and $xy = 78$

Now, $\frac{1}{x} - \frac{1}{y} = \frac{x - y}{xy} = \frac{7}{78}$

24. Which of the following numbers is greatest?

(a) $\frac{11}{16}$ (b) $\frac{13}{18}$ (c) $\frac{13}{16}$ (d) $\frac{11}{17}$

Solution:

(c) $\frac{11}{16} > \frac{11}{17}$ (Bigger the denominator, smaller the number)

$\frac{13}{16} > \frac{11}{16}$ (Bigger the numerator, bigger the number)

$\frac{13}{16} > \frac{13}{18}$ (Bigger the denominator, smaller the number)

$\therefore \frac{13}{16}$ is the greatest number.

25. Which of the following fractions is smallest?

(a) $\frac{5}{8}$ (b) $\frac{7}{10}$ (c) $\frac{3}{4}$ (d) $\frac{9}{16}$

Solution:

(d) $\frac{5}{8} < \frac{7}{10}$

Now we compare $\frac{5}{8}, \frac{3}{4}$ and $\frac{9}{16}$.

LCM of denominators i.e. LCM of 8, 4 and 16 is 16.

$\frac{5}{8} = \frac{10}{16}, \frac{3}{4} = \frac{12}{16}, \frac{9}{16} = \frac{9}{16}$

Since $\frac{9}{16} < \frac{10}{16} < \frac{12}{16}$

$\therefore \frac{9}{16} < \frac{5}{8} < \frac{3}{4}$

$\therefore \frac{9}{16}$ is the smallest fraction.

26. Which of the following numbers is greatest?

(a) $\frac{14}{3}$ (b) $\frac{19}{4}$ (c) $\frac{23}{5}$ (d) $\frac{29}{6}$

Solution:

(d) Converting the fractions into decimal, we get:

$\frac{14}{3} = 4.67; \frac{19}{4} = 4.75; \frac{23}{5} = 4.6; \frac{29}{6} = 4.83$

Clearly, 4.83 or $\frac{29}{6}$ is the biggest.

27. Arrange the fractions in ascending order: $\frac{4}{15}, \frac{5}{17}, \frac{10}{33}$ and $\frac{8}{27}$.

(a) $\frac{4}{15}, \frac{5}{17}, \frac{10}{33}, \frac{8}{27}$ (b) $\frac{5}{17}, \frac{10}{33}, \frac{8}{27}, \frac{4}{15}$

(c) $\frac{4}{15}, \frac{5}{17}, \frac{8}{27}, \frac{10}{33}$ (d) $\frac{10}{33}, \frac{8}{27}, \frac{5}{17}, \frac{4}{15}$

Solution:

(c) Dividing the denominators by the numerators:

$$\frac{4}{15} = \frac{1}{3.75}, \frac{5}{17} = \frac{1}{3.4}, \frac{10}{33} = \frac{1}{3.3}, \frac{8}{27} = \frac{1}{3.375}$$

Now $\frac{1}{3.75} < \frac{1}{3.4} < \frac{1}{3.375} < \frac{1}{3.3}$

$\therefore \frac{4}{15} < \frac{5}{17} < \frac{8}{27} < \frac{10}{33}$

28. $0.\overline{6} + 0.\overline{4} = ?$

(a) 1.0 (b) 0.10 (c) 1.1 (d) $1.\overline{1}$

Solution:

(d) $0.\overline{6} + 0.\overline{4} = \frac{6}{9} + \frac{4}{9} = \frac{10}{9} = 1\frac{1}{9} = 1.\overline{1}$

29. $0.\overline{4} + 0.\overline{5} + 0.\overline{9} + 0.\overline{13} = ?$

(a) $0.\overline{31}$ (b) $1.\overline{93}$ (c) $2.\overline{13}$ (d) $2.\overline{31}$

Solution:

(c) **Note :-** $0.\overline{9} = \frac{9}{9} = 1$

$\therefore 0.\overline{4} + 0.\overline{5} + 0.\overline{9} + 0.\overline{13}$

$= 0.\overline{9} + 0.\overline{9} + 0.\overline{13}$

$= 1 + 1 + 0.\overline{13} = 2.\overline{13}$

30. $2.\overline{07} + 3.\overline{19} + 13.\overline{08} = ?$

(a) $18.\overline{34}$ (b) $18.\overline{35}$ (c) $19.\overline{34}$ (d) $19.\overline{35}$

Solution:

(a) $2.\overline{07} + 3.\overline{19} + 13.\overline{08} = ?$

$2 + 3 + 13 + 0.\overline{07} + 0.\overline{19} + 0.\overline{08}$

$= 18 + \frac{7}{99} + \frac{19}{99} + \frac{8}{99}$

$$= 18 + \frac{34}{99} = 18.\overline{34}$$

Direct Method:

$$2+3+13+0.\overline{07}+0.\overline{19}+0.\overline{08} = 18.\overline{34}$$

Note: $2 + 3 + 13 = 18$

And $0.0\overline{7}+0.\overline{19}+0.\overline{08}=0.\overline{34}$

31. $1.\overline{19}+5.\overline{3}+0.\overline{07} = ?$

(a) $6.\overline{56}$ (b) $6.\overline{59}$ (c) $7.\overline{56}$ (d) $7.\overline{59}$

Solution:

(b) $1.\overline{19}+5.\overline{3}+0.\overline{07}$

$$=1.\overline{19}+5.\overline{33}+0.\overline{07}$$

$$=(1+5+0)+\frac{19}{99}+\frac{33}{99}+\frac{7}{99} = 6+\frac{59}{99} = 6\cdot\overline{59}$$

Direct Method:

$$1.\overline{19}+5.\overline{33}+0.\overline{07} = 6.\overline{59}$$

32. $0.11\overline{62} + 0.07\overline{21} = ?$

(a) $0.18\overline{83}$ (b) $0.18\overline{84}$ (c) $0.19\overline{83}$ (d) $0.19\overline{84}$

Solution:

(a) $\frac{1162-11}{9900} + \frac{721-7}{9900}$

$$=\frac{(1162+721)-(11+7)}{9900}$$

$$=\frac{1883-18}{9900} = 0.18\overline{83}$$

Direct Method:

$$0.11\overline{62}+0.07\overline{21}$$

$$= (0.11+0.07) + (0.00\overline{62}+0.00\overline{21})$$

$$= 0.18\overline{83}$$

33. If $5\frac{7}{a}\times b\frac{1}{13} = 12$, find the value of a.

(*a*) 8 (*b*) 9 (*c*) 11 (*d*) 13

Solution :

(*b*) Clearly $b = 2$

$\therefore \quad 5\frac{7}{a} \times 2\frac{1}{13} = 12$

$\therefore \quad 5\frac{7}{a} = 12 \times \frac{13}{27} = \frac{52}{9} = 5\frac{7}{9}$

$\therefore \quad a = 9$

34. What is square root of $0.\bar{4}$?

(*a*) $0.\bar{2}$ (*b*) $0.\bar{4}$ (*c*) $0.\bar{6}$ (*d*) $0.\bar{8}$

Solution :

(*c*) $\sqrt{0.\bar{4}} = \sqrt{\frac{4}{9}} = \frac{2}{3} = \frac{6}{9} = 0.\bar{6}$

Chapter 3

DIVISIBILITY TEST

A number is exactly divisible by a certain divisor if on dividing the number, the remainder is zero. On dividing a number by the divisor, we can tell whether the number is exactly divisible and if not what is the remainder. In this chapter, we are discussing the short-cut methods of finding whether a number is divisible by the given divisor, if not what will be the remainder on dividing the number.

RULES OF DIVISIBILITY TEST

Rules of Divisibility Test for some important numbers are given below:

A number is divisible by:

2: If unit's digit of the number is '0' or a multiple of '2' i.e. unit's digit is '0' or an even number.

Example:

18, 110, 98514 are divisible by 2.

But 23, 147, 1521 etc. are not divisible by 2.

3: If sum of the digits of the number is exactly divisible by '3'.

Note : while totalling the digits, any digit or pairs of digits totalling to 9 may be ignored and the final sum is reduced to one digit.

Example:

132 is divisible by 3 as sum of its digits i.e. $1 + 3 + 2 = 6$ is exactly divisible by 3.

But 275 is not divisible by 3 as sum of the digits i.e. $2 + 7 + 5 = 14$ is not divisible by 3.

In this example, we can ignore $2 + 7 = 9$. Ignoring 9, we get 5 as sum of digits, which is not divisible by 3.

4: If last two digits of the number are either '0s' or multiple of '4'.

Example:

24, 524, 9900, 97632 are divisible by 4.

But 918, 547622, 6853 are not divisible by 4.

Note: A number having odd number at unit's place is never divisible by 2 or by any other even number.

5: If unit's digit of the number is '0' or '5'.

Example:

135, 93215, 75290 are divisible by 5.

But 1019, 50852 are not divisible by 5.

7: Deduct double of unit's digit from the remaining number and check whether the result is divisible by 7.

If the result is divisible by 7, the number is also divisible by 7.

Example:

161 : 16 – 2 × 1 = 14

∴ The number is divisible by 7.

147 : 14 – 2 × 7 = 0

∴ The number is divisible by 7.

8: If its last three digits are '0' or a multiple of '8'.

Example:

721000, 1345144 are divisible by 8.
But 172433 is not divisible by 8.

9: If sum of digits of the number is exactly divisible by '9'.

Note: while totalling the digits, any digit or pair of digits totalling to 9 may be ignored and the final sum is reduced to one digit.

Example:

216 is divisible by 9 as sum of digits i.e. 2 + 1 + 6 = 9 is divisible by 9.
3294 is divisible by 9 as sum of digits i.e. 3 + 2 + 4 = 9 (ignoring 9) is divisible by 9.

Note: If we ignore both the 9's while totalling digits of 3294, then we get 0 as the sum of digits.

⇒ If we ignore all 9's while totalling the digits of a number, sum of digits is always 0 when the number is exactly divisible by 9.

10: If unit's digit of the number is '0'.

Example:

2760, 38610 are divisible by 10.
But 1531, 2376 are not divisible by 10.

11: If difference between sum of digits at the even places and sum of digits at odd places is either '0' or is a multiple of '11'.

Examples:

1. 3256 : Difference between (3 + 5) and (2 + 6) is 0.

   ∴ The number is divisible by 11.
2. 7183 : Difference between (7 + 8) and (1 + 3) is 11.

   ∴ The number is divisible by 11.
3. 291918 : Difference between (2 + 1 + 1) and (9 + 9 + 8) is 22, which is a multiple of 11.

   ∴ The number is divisible by 11.
4. 253 : Difference between (2 + 3) and 5 is 0.

   ∴ The number is divisible by 11.
5. 517 : Difference between (5 + 7) and 1 is 11.

   ∴ The number is divisible by 11.
6. 10065: Difference between (1 + 0 + 5) and (0 + 6) is 0.

   ∴ The number is divisible by 11.

7. 10901 : Difference between (1 + 9 + 1) and (0 + 0) is 11.

∴ The number is divisible by 11.

8. 123 : Difference between (1 + 3) and (2) is 2.

∴ The number is not divisible by 11.

25: If last two digits of the number are 0's or divisible by 25.

∴ Numbers divisible by 25 must end in 00, 25, 50 or 75.

Example:

2125, 4350, 1375, 7800 are divisible by 25.

But 4327, 7681 are not divisible by 25.

99: Starting from right-hand side make pairs of two digits each. Add all the pairs and divide the sum by 99. If the sum is exactly divisible by 99, the number is also divisible by 99.

Examples:

297 : 2 + 97 = 99 (99 is divisible by 99)

∴ 297 is divisible by 99.

8217: 82 + 17 = 99 (99 is divisible by 99)

∴ 8217 is divisible by 99.

73854: 7 + 38 + 54 = 99 (99 is divisible by 99)

∴ 73854 is divisible by 99.

598: 5 + 98 = 103 (103 is not divisible by 99)

∵ 598 is not divisible by 99.

4351: 43 + 51 = 94 (94 is not divisible by 99)

∴ 4351 is not divisible by 99.

999: Starting from right-hand side make pairs of three digits each. Add all the pairs and divide the sum by 999. If the sum is exactly divisible by 999, the number is also divisible by 999.

Examples:

4995: 4 + 995 = 999 (999 is divisible by 999)

∴ 4995 is divisible by 999.

17982: 17 + 982 = 999 (999 is divisible by 999)

∴ 17982 is divisible by 999.

175824: 175 + 824 = 999 (999 is divisible by 999)

∴ 175824 is divisible by 999.

17424: 17 + 424 = 441 (441 is not divisible by 999)

∴ 17424 is not divisible by 999.

The methods explained above can be used to find whether a divisor exactly divides a given number or not provided the divisor can be written as a product of two or more co-prime numbers for which Divisibility rules are explained above.

6: If the number is divisible by 2 and 3 (Co-prime numbers)

Example:

1. Is 204 divisible by 6?

Solution:

204 is divisible by 2.

204 is divisible by 3.

∴ 204 is divisible by 6.

2. Is 1431 divisible by 6?

Solution:

1431 is not divisible by 2.

∴ The number is not divisible by 6.

3. Is 2156 divisible by 6?

Solution:

2156 is divisible by 2.

2156 is not divisible by 3.

∴ The number is not divisible by 6.

In the above examples, we see that a number is divisible by the given divisor, if and only if all the co-prime factors of the given divisor divide the given number exactly.

12: If the number is divisible by 3 and 4 (Co-prime numbers)

Example:

Is 1068 divisible by 12?

Solution:

1068 is divisible by 3.

1068 is divisible by 4.

∴ 1068 is divisible by 12.

14: If the number is divisible by 2 and 7 (Co-prime numbers)

Example:

Is 938 divisible by 14?

Solution:

938 is divisible by 2.

938 is divisible by 7.

Hint: $93 - 2 \times 8 = 77$ is divisible by 7

∴ 938 is divisible by 14.

15: If the number is divisible by 3 and 5 (Co-prime numbers)

Example:

Is 345 divisible by 15?

Solution:

345 is divisible by 3.

345 is divisible by 5.

∴ 345 is divisible by 15.

18: If the number is divisible by 2 and 9 (Co-prime numbers)

Example:

Is 1134 divisible by 18?

Solution:

1134 is divisible by 2.

1134 is divisible by 9.

∴ 1134 is divisible by 18.

REMAINDER RULE

If a number is exactly divisible by a divisor, remainder is '0' and in the other cases remainder may be any number between one and one less than the divisor. Generally, remainders are obtained by actually dividing the given number by the divisor. Using the Rules of Divisibility Test, we can also find the remainder without actually dividing the number, as follows :

2: Remainder is always 1, if the given number is not divisible by 2 i.e. when the number ends in an odd number.

3: Remainder is obtained by dividing the sum of digits of the given number by '3'.

Note: while totalling the digits, any digit or pairs of digits totalling to 9 may be ignored.

Example:

275 is not divisible by 3 as sum of the digits i.e. 2 + 7 + 5 = 5 is not divisible by 3.

On dividing 5 by 3, we get remainder 2.

∴ Remainder on dividing 275 by 3 is 2.

4: Remainder is obtained by dividing the last two digits of the number by '4'.

Example:

Remainders obtained on dividing 918, 547621, 6855 by 4 are what we get remainder on dividing 18, 21, 55 by 4 i.e. 2, 1 and 3 respectively.

5: Remainder is obtained by dividing the unit's digit of the given number by '5'.

Example:

Remainders on dividing 1019, 50852 by 5 are 4 and 2 respectively i.e. remainders obtained on dividing 9 and 2 by 5.

7: Steps for finding remainder on dividing a number by 7:

1. Subtract double of unit's digit from the remaining number.
2. Divide the number so obtained by 7, and find the remainder.
3. (a) If the remainder so obtained is an odd number:
   Subtract the number so obtained from 7.

   (b) If the remainder so obtained is an even number:

   Subtract the number so obtained from 14.
4. Divide the number obtained in step 3) by 2.

Example 1: Find the remainder on dividing 211 by 7.

Solution:

Step 1: $21 - 2 \times 1 = 19$

Step 2: Remainder on dividing 19 by 7 is 5.

Step 3: $7 - 5 = 2$ (Since 5 is an odd number)

Step 4: Actual remainder $= \frac{2}{2} = 1$

Example 2: Find the remainder on dividing 713 by 7.

Solution:

Step 1: $71 - 2 \times 3 = 65$

Step 2: Remainder on dividing 65 by 7 is 2.

Step 3: $14 - 2 = 12$ (Since 2 is an even number)

Step 4: Actual remainder $= \frac{12}{2} = 6$

More Examples:

Number	Step 1	Step 2	Step 3	Step 4
1072 :	$107 - 2 \times 2 = 103$	R = 5	$7 - 5 = 2$	AR $= \frac{2}{2} = 1$
1073 :	$107 - 2 \times 3 = 101$	R = 3	$7 - 3 = 4$	AR $= \frac{4}{2} = 2$
1075 :	$107 - 2 \times 5 = 97$	R = 6	$14 - 6 = 8$	AR $= \frac{8}{2} = 4$
1076 :	$107 - 2 \times 6 = 95$	R = 4	$14 - 4 = 10$	AR $= \frac{10}{2} = 5$

8: Remainder is obtained on dividing the last three digits of the given number by '8'.

Example:

172433: On dividing 433 by 8, we get remainder 1.

∴ Remainder on dividing 172433 by 8 is 1.

9: Remainder is obtained on dividing the sum of digits of the given number by '9'.

Note: while totalling the digits, a digit or pairs of digits totalling to 9 may be ignored.

Example:

2183: Sum of digits = 2 + 1 + 8 + 3 = 2 + 3 = 5 (ignoring 9)

∴ Remainder is 5.

10: Unit's digit of the given number is the remainder.

Example:

Find the remainder on dividing 1276 by 10.

Solution:

Remainder is unit's digit i.e. 6.

11: Remainder rule for 11 can be divided into two parts:

(*i*) When sum of the part that includes unit's digit is more than the sum of other part:

The remainder is what we get as remainder on dividing the difference by 11.

Example:

343 : (3 + 3) – 4 = 6 – 4 = 2 (Remainder)

4326 : (3 + 6) – (4 + 2) = 9 – 6 = 3 (Remainder)

1321 : (3 + 1) – (1 + 2) = 4 – 3 = 1 (Remainder)

281936 : (8 + 9 + 6) – (2 + 1 + 3) = 23 – 6 = 17;

Remainder = 17 – 11 = 6

(*ii*) When sum of part that includes unit's digit is less than that of the other part:

The remainder is what we get as remainder on deducting the difference from the multiple of 11.

Example:

4383 : (3 + 3) – (4 + 8) = 6 – 12 = – 6; Remainder = 11 – 6 = 5

3286 : (2 + 6) – (3 + 8) = 8 – 11 = – 3; Remainder = 11 – 3 = 8

7184 : (1 + 4) – (7 + 8) = 5 – 15 = – 10; Remainder = 11 – 10 = 1

8194 : (1 + 4) – (8 + 9) = 5 – 17 = – 12; Remainder = 22 – 12 = 10

929184 : (2 + 1 + 4) – (9 + 9 + 8) = 7 – 26 = – 19;

Remainder = 22 – 19 = 3

99: Starting from right-hand side make pairs of two digits each. Add all the pairs and divide the sum by 99. Remainder obtained is the answer.

Example 1: What is the remainder when 175394 is divided by 99?

Solution:

Sum of pairs is 94 + 53 + 17 = 164.

Remainder on dividing 164 by 99 is 65.

∴ Remainder on dividing 175394 by 99 is 65.

Example 2: What is the remainder when 25134 is divided by 99?

Solution:

Sum of pairs is 34 + 51 + 2 = 87.

Remainder on dividing 87 by 99 is 87.

∴ Remainder on dividing 25134 by 99 is 87.

999: Starting from right-hand side make pairs of three digits each. Add all the pairs and divide the sum by 999. Remainder obtained is the answer.

Example 1: What is the remainder when 4216 is divided by 999?

Solution:

Sum of pairs is 4 + 216 = 220.

∴ Remainder on dividing 4216 by 999 is 220.

Example 2: What is the remainder when 13528 is divided by 999?

Solution:

Sum of pairs is 13 + 528 = 541.

∴ Remainder on dividing 13528 by 999 is 541.

SOLVED EXERCISE

1. Which of the following numbers is exactly divisible by 45?

(a) 2283 (b) 3920 (c) 4275 (d) 5184

Solution:

(c) A number is divisible by 45 if it is divisible by both 5 and 9.

Here, we can easily rule out 2283 and 5184 for not being divisible by 5 as these numbers don't have 5 or 0 at unit's place.

Both the remaining numbers i.e. 3920, 4275 are divisible by 5.

3920: Sum of digits of 3920 is 3 + 2 = 5.

∴ 3920 is not divisible by 9.

4275: It being the only option left, is the answer.

However, we can check 4275

Sum of digits = 4 + 2 + 7 + 5 = 0 (ignoring both 9's)

∴ 4275 is divisible by 9 also.

2. Which of the following numbers is exactly divisible by 45:

(a) 4145 (b) 4185 (c) 4581 (d) 4158

Solution:

(b) 4581 and 4158 are not divisible by 5 as the numbers don't have '5' or '0' in the unit's place.

Now, sum of digits of 4145 and 4185 are 5 and 9 respectively.

∴ 4145 is not divisible by 9.

∴ 4185 is the only number, which is divisible by 5 and 9 both.

3. Find the greatest four-digit number which is exactly divisible by 5, 6, 8 and 9.

(a) 8640 (b) 9999 (c) 9800 (d) 9720

Solution:

(d) We start from the biggest number to the lowest, applying the Rules of Divisibility Test for 5, 6, 8 and 9.

9999: Not divisible by 5 as its unit's digit is neither 0 nor 5.

9800: Not divisible by 9 as sum of digits i.e. 8 is not divisible by 9.

Now, 8640 and 9720 both are divisible by 5, 6, 8 and 9, but 9720 being the greater number is the answer.

4. The largest number of four digits exactly divisible by 88 is:

(a) 9944 (b) 8888 (c) 9988 (d) 9999

Solution:

(a) A number is divisible by 88 if it is divisible by 8 and 11.

9999: Not divisible by 8 as its unit's digit is an odd number.

9988: Not divisible by 8 as the number formed by its last three digits is not divisible by 8.

> Hint: 1000 is exactly divisible by 8.
>
> 988 is 12 less than 1000.
>
> Since, 12 is not divisible by 8, 9988 is also not divisible by 8.

8888: Divisible by 8 and 11.

9944: Last three digits i.e. 944 is 56 less than 1000 and 56 is divisible by 8.

$\therefore$ 9944 is divisible by 8.

9944 is divisible by 11 as $(9 + 4) - (9 + 4) = 0$.

$\therefore$ 8888 and 9944 both are divisible by 8 and 11.

But, 9944 being the bigger number, is the answer.

5. What value must be given to * so that the number 9781*4 is exactly divisible by 6:

(a) 0 (b) 1 (c) 2 (d) 3

Solution:

(b) A number is divisible by 6 if it is divisible by 2 and 3.

The given number is divisible by 2 as it has an even number in the unit's place.

Sum of the digits of the given number is 11 or 2.

On adding 1 to it, total will become 3 and hence divisible by 3.

$\therefore$ * is to be replaced by 1.

6. What value must be given to * to make 5*8428 exactly divisible by 11?

(a) 0 (b) 2 (c) 3 (d) 8

Solution:

(c) Sum of digits at odd places (from left side) = 5 + 8 + 2 = 15

Sum of digits at even places (from left side) = * + 4 + 8 = 12 + *

Difference between the two sums is '0' when 12 + * = 15

$\therefore$ * = 15 − 12 = 3

7. What value must be given to * to make 8597*65 exactly divisible by 11:

(a) 0 (b) 2 (c) 4 (d) 7

Solution:

(d) Sum of digits at odd places (from left side) = 8 + 9 + * + 5 = 22 + *

Sum of digits at even places (from left side) = 5 + 7 + 6 = 18

Difference between the two sums = 4 + *

We know that any number is divisible by 11, if the difference is '0' or a multiple of 11.

The difference between 22 + * and 18 will be 11 when * = 7.

8. What must be added to 583171 to make it exactly divisible by 11:

(a) 5 (b) 6 (c) 7 (d) 8

Solution:

(a) Sum of digits at odd places (from left side) = 5 + 3 + 7 = 15.

Sum of digits at even places (from left side) = 8 ~~+ 1~~ + 1 = 10.

Whatever we add to the given number, will increase the sum of digits which includes the unit's digit i.e. to 10. If we add 5 to given number difference between the sum of digits becomes 15 − 15 = 0.

$\therefore$ 5 is to be added to the given number.

9. A number when divided by 45 leaves remainder 42. Same number when divided by 5 will leave remainder:

(a) 0 (b) 2 (c) 5 (d) 42

Solution:

(b) Any number which is exactly divisible by a certain divisor is also exactly divisible by factor of that divisor.

∴ Any number which is exactly divisible by 45 will always be divisible by 5 as 5 is a factor of 45.

But, the number is not exactly divisible by 45, it leaves a remainder 42.

∴ We have to divide 42 by 5.

∴ 42 when divided by 5 leaves a remainder 2.

10. Between 100 and 300, how many numbers are divisible by 7?

(a) 14 (b) 21 (c) 28 (d) 35

Solution:

(c) Since, $14 \times 7 + 2 = 100$

∴ Between 1 and 100, 14 numbers are divisible by 7.

Since, $42 \times 7 + 6 = 300$

∴ Between 1 and 300, 42 numbers are divisible by 7.

∴ Between 100 and 300, $42 - 14 = 28$ numbers are divisible by 7.

Hint: $100 = 14 \times 7 + 2$

∴ $300 = 3 \times (14 \times 7 + 2) = 42 \times 7 + 6$

11. An even number, total of whose digits is 36, is always divisible by:

(a) 9 (b) 18 (c) 36 (d) 72

Solution:

(b) An even number is always divisible by 2.

Further any number sum of whose digits is 36 is always divisible by 9.

∴ The number is divisible by both 2 and 9.

∴ The number is divisible by $2 \times 9 = 18$.

12. The smallest number to be added to 1000 so that it is exactly divisible by 45, is:

(a) 8 (b) 20 (c) 35 (d) 80

Solution:

(c) 1000 is exactly divisible by 5 but not by 9, as sum of digits is 1 which is not divisible by 9.

∴ Number to be added must be divisible by 5 and sum of digits should be 8.

35 is the smallest of the given numbers which fulfils both the conditions.

Hint: On adding 8 to 1000, number becomes = 1008.

∴ Sum of digits is $1 + 8 = 9$.

∴ The number is divisible by 9.

But, the number is not divisible by 5 as unit's digit i.e. 8 is not divisible by 5.

13. What should be added to 15762 to make it divisible by 18?

(a) 4 (b) 6 (c) 9 (d) 12

Solution:

(b) A number is divisible by 18, if it is divisible by 2 and 9 both.

A number is divisible by 2, if its unit's digit is an even number.

Unit's digit of the given number is an even number.

∴ The number to be added must be an even number so that sum of two numbers is even number.

∴ 9 is not possible.

Now, a number is divisible by 9, if sum of its digits is divisible by 9.

Sum of digits of 15762 is 3.

3 + 6 = 9 i.e. sum of digits is divisible by 9.

∴ 6 is the answer.

14. What should be subtracted from 331683 to make it divisible by 36?

(a) 6 (b) 11 (c) 15 (d) 24

Solution:

(c) A number is divisible by 36, if it is divisible by 4 and 9 both.

A number is divisible by 4, if its last two digits are divisible by 4.

In 331683, last two digits are 83, and 83 is an odd number.

Any number divisible by 4 must be an even number.

∴ The number to be deducted from 331683 must be odd number.

∴ 6 and 24 are not possible.

11: 83 – 11 = 72 is divisible by 4. (Possible)

15: 83 – 15 = 68 is divisible by 4. (Possible)

Now, we check divisibility rule for 9:

Sum of digits of 331683 is 6.

∴ Sum of digits of the number deducted must be 6 so that the result is divisible by 9.

11: Sum of digits is 1 + 1 = 2 (not possible)

15: Sum of digits is 1 + 5 = 6 (possible)

∴ 15 is the answer.

15. By which least number 325 must be multiplied to get a multiple of 35?

(a) 3 (b) 5 (c) 7 (d) 11

Solution:

(c) 35 = 5 × 7

∴ Any multiple of 35 must be divisible by 5 and 7.

325 = 5 × 5 × 13

∴ 325 is divisible by 5, but not by 7.

∴ 325 must be multiplied by 7 to make it a multiple of 35.

Chapter 4

MULTIPLICATION OF NUMBERS - SHORT-CUT METHODS

Multiplication of numbers is extensively used in solving the numerous problems in the examinations. Conventional method of multiplication of numbers is very lengthy especially when the numbers to be multiplied are larger. As the conventional method involves a lot of calculations, the method is prone to mistakes. By using the short-cut methods explained in this chapter, even the multiplication of large numbers can be done in few seconds by adding and subtracting the numbers. In this chapter we will also learn the method of checking the correctness of product of two or more numbers.

SHORT-CUT METHODS OF MULTIPLICATION

1. Multiplication of numbers – one or more of which have **0's at the end**:

Steps:

1. Multiply the numbers ignoring 0's at the end of the given numbers.
2. At the end of multiplication result put as many 0s as many 0s are at the end of the given numbers.

Example 1: 257 × 100 = ?

Solution:

Second number has two 0s at the end of it.

Ignoring 0s at the end, we get 257 × 1 = 257.

Now we will put two 0s at the end of the result.

∴ 257 × 100 = 25700

Example 2: 34 × 20 = ?

Solution:

Second number has one '0' at the end of it.

Ignoring '0' at the end, we get 34 × 2 = 68.

Now we will put one '0' at the end of the result.

∴ 34 × 20 = 680

Example 3: 125 × 400 = ?

Solution:

Second number has two 0s at the end of it.

Ignoring 0s at the end, we get 125 × 4 = 500.

Now we will put two 0s at the end of the result.

∴ 125 × 400 = 50000

Example 4: 130 × 1200 = ?

Solution:

First number has one '0' and second number has two 0s i.e. total three 0s.

Ignoring 0s at the end, we get 13 × 12 = 156.

Now we will put three 0s at the end of the result.

$\therefore$ 130 × 1200 = 156000

2. Multiplication of a number **by 5, 25, 50 and 125**:

(a) Multiplication of a number **by 5**:

Method:

Divide the given number by 2 and multiply the result by 10.

Note: $\frac{10}{2} = 5$

Example:

$$112 \times 5 = \frac{112}{2} \times 10 = 56 \times 10 = 560$$

$$576 \times 5 = \frac{576}{2} \times 10 = 288 \times 10 = 2880$$

$$175 \times 5 = \frac{175}{2} \times 10 = 87.5 \times 10 = 875$$

(b) Multiplication of a number **by 25**:

Method:

Divide the number by 4 and multiply the result by 100.

Note: $\frac{100}{4} = 25$

Example:

$$16 \times 25 = \frac{16}{4} \times 100 = 4 \times 100 = 400$$

$$128 \times 25 = \frac{128}{4} \times 100 = 32 \times 100 = 3200$$

$$73 \times 25 = \frac{73}{4} \times 100 = 18\frac{1}{4} \times 100 = 1825$$

$$127 \times 25 = \frac{127}{4} \times 100 = 31\frac{3}{4} \times 100 = 3175$$

(c) Multiplication of a number **by 50**:

Method:

Divide the number by 2 and multiply the result by 100.

Note: $\frac{100}{2} = 50$

Example:

$$16 \times 50 = \frac{16}{2} \times 100 = 8 \times 100 = 800$$

$$214 \times 50 = \frac{214}{2} \times 100 = 107 \times 100 = 10700$$

$$123 \times 50 = \frac{123}{2} \times 100 = 61\frac{1}{2} \times 100 = 6150$$

(d) Multiplication of a number **by 125**:

Method:

> Divide the number by 8 and Multiply the result by 1000.
>
> **Note:** $\frac{1000}{8} = 125$

Example:

$$24 \times 125 = \frac{24}{8} \times 1000 = 3 \times 1000 = 3000$$

$$45 \times 125 = \frac{45}{8} \times 1000 = 5\frac{5}{8} \times 1000$$

$$= 5 \times 1000 + \frac{5}{8} \times 1000 = 5000 + 5 \times 125 = 5625$$

3. Multiplication of a number **by 9**:

Steps:

> 1. Put one '0' to the right of the given number.
> 2. From the new number such formed subtract the original number.

Example:

$$\begin{array}{r} 98 \times 9 \\ \hline 980 \\ -98 \\ \hline 882 \\ \hline \end{array} \qquad \begin{array}{r} 238 \times 9 \\ \hline 2380 \\ -238 \\ \hline 2142 \\ \hline \end{array}$$

4. Multiplication by **99, 999, 9999 etc.**:

(*a*) Steps for multiplication **by 99**:

> 1. Put two 0s to the right of the given number.
> 2. From the new number such formed subtract the original number.

Example:

$$\begin{array}{r} 178 \\ \times 99 \\ \hline 17800 \\ -178 \\ \hline 17622 \\ \hline \end{array} \qquad \begin{array}{r} 217 \\ \times 99 \\ \hline 21700 \\ -217 \\ \hline 21483 \\ \hline \end{array} \qquad \begin{array}{r} 5281 \\ \times 99 \\ \hline 528100 \\ -5281 \\ \hline 522819 \\ \hline \end{array}$$

(b) Multiplication **by 999, 9999, 99999, ……..:**

Method:

1. **At the end of the given number put as many 0s as many 9s are in the multiplier.**
2. **From the number such formed subtract the original number.**

Example:

$$\begin{array}{r} 258 \\ \times 999 \\ \hline 258000 \\ -258 \\ \hline \underline{257742} \end{array} \qquad \begin{array}{r} 4381 \\ \times 999 \\ \hline 4381000 \\ -4381 \\ \hline \underline{4376619} \end{array} \qquad \begin{array}{r} 1295 \\ \times 9999 \\ \hline 12950000 \\ -1295 \\ \hline \underline{12948705} \end{array}$$

5. **Multiplication of a number by 11:**

Steps: (Solution is found from right to left)

1. **The unit's digit of the number is written down as the first part of the solution.**
2. **Last two digits are added to get the next part and successively 2 digits are added by shifting to the left side by 1 digit each time to get the next parts.**
3. **First digit of the number becomes last part of the solution.**

Note: One digit (right-hand side) is taken from each part and extra digit, if any, is carried forward and added to the next part.

Example 1: 23 × 11 = ?

Solution:

Part I: 3

Part II: 2 + 3 = 5

Part III: 2

∴ 23 × 11 = 253

Example 2: 124 × 11 = ?

Solution:

Part I: 4

Part II: 2 + 4 = 6

Part III: 1 + 2 = 3

Part IV: 1

∴ 124 × 11 = 1364

Example 3: 2475 × 11 = ?

Solution:

Part I: 5

Part II: 7 + 5 = 12 (Note: 1 is carried forward and added to Part III)

Part III: (4 + 7) + 1 = 12 (Note: 1 is carried forward and added to part IV)

Part IV: (2 + 4) + 1 = 7

Part V: 2

∴ 2475 × 11 = 27225

6. **Multiplication of a number by 111:**

Steps: (Solution is found from right to left)

1. **The unit's digit of the number is written as first part of the solution.**
2. **Unit's and ten's digits are added to get the next part of the solution.**
3. **Last three digits are added to get next part and successively 3 digits are added shifting to left side by 1 digit each time to get next parts.**
4. **First 2 digits are added to get second last part of the solution.**
5. **First digit of the given number becomes last part of the solution.**

Note: One digit (right-hand side) is taken from each part and extra digit, if any, is carried forward and added to next part.

Example 1: 413 × 111 = ?

Solution:

Part I: 3

Part II: 1 + 3 = 4

Part III: 4 + 1 + 3 = 8

Part IV: 4 + 1 = 5

Part V: 4

∴ 413 × 111 = 45843

Example 2: 258 × 111 = ?

Solution:

Part I: 8

Part II: 5 + 8 = 13 (Note: 1 is carried forward to part III)

Part III: (2 + 5 + 8) + 1 = 16 (Note: 1 is carried forward to part IV)

Part IV: (2 + 5) + 1 = 8

Part V: 2

∴ 258 × 111 = 28638

Example 3: 2643 × 111 = ?

Solution:

Part I: 3

Part II: 4 + 3 = 7

Part III: 6 + 4 + 3 = 13 (Note: 1 is carried forward to part IV)

Part IV: (2 + 6 + 4) + 1 = 13 (Note: 1 is carried forward to part V)

Part V: (2 + 6) + 1 = 9

Part VI: 2

∴ 2643 × 111 = 293373

Example 4: 27 × 111 = ?

Solution:

We write the problem as 027 × 111 to convert 27 into 3-digit number.

Part I: 7

Part II: 2 + 7 = 9

Part III: 0 + 2 + 7 = 9

Part IV: 0 + 2 = 2

∴ 27 × 111 = 2997

7. Multiplication of a number **by 1111**:

Steps: (Solution is found from right to left)

1. The unit's digit of the number is written as first part of the solution.
2. Unit's and ten's digits are added to get the next part of the solution.
3. Last three digits are added to get the next part of the solution.
4. Last four digits are added to get the next part and successively digits are added in the pair of four digits each time shifting to left side by one digit to get the next parts.
5. First 3 digits are added to get the next part of the solution.
6. First 2 digits are added to get second last part of the solution.
7. First digit of the given number becomes last part of the solution.

Note: One digit (right-hand side) is taken from each part and extra digit, if any, is carried forward and added to next part.

Example 1: 1324 × 1111 = ?

Solution:

Part I: 4 = 4

Part II: 2 + 4 = 6

Part III: 3 + 2 + 4 = 9

Part IV: 1 + 3 + 2 + 4 = 10 (Note: 1 is carried forward to Part V)

Part V: (1 + 3 + 2) + 1 = 7

Part VI: 1 + 3 = 4

Part VII: 1 = 1

∴ 1324 × 1111 = 1470964

Example 2: 24587 × 1111 = ?

Solution:

Part I: 7 = 7

Part II: 8 + 7 = 15 (Note: 1 is carried forward to Part III)

Part III: (5 + 8 + 7) + 1 = 21 (Note: 2 is carried forward to Part IV)

Part IV: (4 + 5 + 8 + 7) + 2 = 26 (Note: 2 is carried forward to Part V)

Part V: (2 + 4 + 5 + 8) + 2 = 21 (Note: 2 is carried forward to Part VI)

Part VI: (2 + 4 + 5) + 2 = 13 (Note: 1 is carried forward to Part VII)

Part VII: (2 + 4) + 1 = 7

Part VIII: 2 = 2

∴ 24587 × 1111 = 27316157

Example 3: 432 × 1111 = ?

Solution:

We write the question as 0432 × 1111 to convert 432 into 4-digit number.

Part I: 2 = 2

Part II: 3 + 2 = 5

Part III: 4 + 3 + 2 = 9

Part IV: 0 + 4 + 3 + 2 = 9

Part V: 0 + 4 + 3 = 7

Part VI: 0 + 4 = 4

∴ 432 × 1111 = 479952

8. Multiplication of number **by a multiple of 11**, i.e. 22, 33, 44 etc.:

Steps:

1. Find how many times the multiplier is the multiple of 11.
2. Now multiplication can be done in the same manner as in case of 11 with a modification that the sum of digits is to be multiplied by the times the multiplier is multiple of 11.

Note: One digit (right-hand side) is taken from each part and extra digit, if any, is carried forward and added to next part.

Example 1: Find the value of 42 × 22.

Solution:

22 is 2 times of 11.

Part I: (2) × 2 = 4

Part II: (4 + 2) × 2 = 12 (**Note:** 1 is carried forward and added to next part)

Part III: (4) × 2 + 1 = 9

∴ 42 × 22 = 924

Example 2: Find the value of 156 × 33.

Solution:

33 is 3 times of 11.

Part I: (6) × 3 = 18 (Note: Carried forward is 1)

Part II: (5 + 6) × 3 + 1 = 34 (Note: 3 is carried forward)

Part III: (1 + 5) × 3 + 3 = 21 (Note: 2 is carried forward)

Part IV: (1) × 3 + 2 = 5

∴ 156 × 33 = 5148

Example 3: Find the value of 142 × 55.

Solution:

55 is 5 times of 11.

Part I: (2) × 5 = 10 (Note: Carried forward is 1)

Part II: (4 + 2) × 5 + 1 = 31 (Note: 3 is carried forward)

Part III: (1 + 4) × 5 + 3 = 28 (Note: 2 is carried forward)

Part IV: (1) × 5 + 2 = 7

∴ 142 × 55 = 7810

9. Multiplication of two numbers when **both numbers have 5 at unit's place**:

Solution: (from right to left):

1. First two digits will always be 5 × 5 = 25.
2. To get remaining digits:

Multiply the remaining digits of the two numbers and add half of theirs sum to it.

Example:

$$25 \times 65 = (2 \times 6) + \frac{1}{2}(2 + 6) \mid 25 = 16\ 25$$

$$35 \times 55 = (3 \times 5) + \frac{1}{2}(3 + 5) \mid 25 = 19\ 25$$

$$45 \times 65 = (4 \times 6) + \frac{1}{2}(4 + 6) \mid 25 = 29\ 25$$

$$125 \times 105 = (12 \times 10) + \frac{1}{2}(12 + 10) \mid 25 = 131\ 25$$

Note: When second portion leaves 0.5 on solving, we write 75 instead of 25 in the first part.

Example:

$$35 \times 65 = (3 \times 6) + \frac{1}{2}(3 + 6) \mid 25 = 22.5 \mid 25 = 22\ 75$$

$$115 \times 85 = (11 \times 8) + \frac{1}{2}(11 + 8) \mid 25 = 97.5 \mid 25 = 97\ 75$$

10. Multiplication of two numbers when both **numbers are less than base** i.e. less than 10, 100, 1000 etc.

Method:

Let, the two numbers are a_1 and a_2.
Where, $a_1 = 10^n - b_1$ and $a_2 = 10^n - b_2$.
Then, $a_1 \times a_2 = (a_1 + a_2) \mid b_1 \times b_2$.
Note: Subtract 1 from the extreme left digit of $(a_1 + a_2)$.
Note: First part (right hand side) has digits equal to digits in each of numbers multiplied.

Example 1:

Find the value of 98 × 94.

Solution:

$98 = 100 - 2$
$94 = 100 - 6$
$b_1 \times b_2 = 2 \times 6 = 12$
$a_1 + a_2 = 98 + 94 = 192$
$\therefore\ a_1 \times a_2 = 192 \mid 12 = 9212$

Example 2:

Find the value of 97 × 99.

Solution:

$97 = 100 - 3$
$99 = 100 - 1$
$b_1 \times b_2 = 3 \times 1 = 3$
$a_1 + a_2 = 97 + 99 = 196$
$\therefore\ a_1 \times a_2 = 196 \mid 03 = 9603$

More Examples:

8 – 2	97 – 3	995 – 5	9991 – 9
7 – 3	95 – 5	999 – 1	9997 – 3
5 – 6	92 – 15	994 – 005	9988 – 0027
= 5 6	= 92 15	= 994 005	= 9988 0027

Note: When the numbers to be multiplied have equal number of digits, we can use the following method also to get the second part :

$7-2=5 \qquad 95-3=92 \qquad 999-5=994 \qquad 9997-9=9988$

$8-3=5 \qquad 97-5=92 \qquad 995-1=994 \qquad 9991-3=9988$

When the two numbers don't have equal digits:

$$\begin{array}{r} 999-1 \\ 99\ \ -1 \\ \hline 989-01 \\ \hline =989\ \ 01 \end{array} \qquad \begin{array}{r} 991\ \ -9 \\ 96\ \ -4 \\ \hline 951-36 \\ \hline =951\ \ 36 \end{array} \qquad \begin{array}{r} 9991\ \ -9 \\ 92\ \ -8 \\ \hline 9191-72 \\ \hline =9191\ \ 72 \end{array}$$

$$\begin{array}{r} 999\ -1 \\ 8\ \ -2 \\ \hline 799\ -2 \\ \hline =799\ \ 2 \end{array} \qquad \begin{array}{r} 9994-\ 6 \\ 85\ \ -15 \\ \hline 8494-90 \\ \hline =8494\ \ 90 \end{array} \qquad \begin{array}{r} 9989-\ 11 \\ 997\ -\ 3 \\ \hline 9959-033 \\ \hline =9959\ \ 033 \end{array}$$

Note: For multiplication of numbers with unequal digits:

Add first digit (left-hand side) of the first number to first digit (left-hand side) of the second number and so on.

Note: First part (right hand side) contains digits equal to the digits in the smaller number and second part is simply the sum of the numbers.

11. Multiplication of two numbers when both the **numbers are greater than base** i.e. greater than 10, 100, 1000 etc.

Method:

Let, the two numbers are a_1 and a_2.

Where, $a_1 = 10^n + b_1$ and $a_2 = 10^n + b_2$.

Then, $a_1 \times a_2 = (a_1 + a_2) \mid b_1 \times b_2$.

Note: Subtract 1 from the extreme left digit of $(a_1 + a_2)$.

First part contains *n* digits.

Example 1:

Find the value of 102 × 106.

Solution:

$102 = 100 + 2$

$106 = 100 + 6$

$b_1 \times b_2 = 2 \times 6 = 12$

$a_1 + a_2 = 102 + 106 = 208$

$\therefore a_1 \times a_2 = 208 \mid 12 = 10812$

Note: 102 + 106 = 208. We subtract 1 from '2' of 208, we get 108 as second part.

More Examples:

$$\begin{array}{r} 101-\ 1 \\ 102-\ 2 \\ \hline 103-02 \\ \hline =103\ 02 \end{array} \qquad \begin{array}{r} 105-\ 5 \\ 103-\ 3 \\ \hline 108-15 \\ \hline =108\ 15 \end{array} \qquad \begin{array}{r} 107-\ 7 \\ 101-\ 1 \\ \hline 108-07 \\ \hline =108\ 07 \end{array}$$

Note: First part (right-hand side) contains digits one less than the digits in each number.

Note: When the numbers to be multiplied have equal number of digits, we can use the following method also to get the second part:

101 + 2 = 103 105 + 3 = 108 107 + 1 = 108

102 + 1 = 103 103 + 5 = 108 101 + 7 = 108

When the two numbers don't have equal digits:

1001 – 1
12 – 2
1201 – 2
= 1201 2

1009 – 9
109 – 9
1099 – 81
= 1099 81

1009 – 9
12 – 2
1209 – 18
= 1210 8

101 – 1
11 – 1
111 – 1
= 111 1

Note: First part (right-hand side) consists of digits one less than the digits in the smaller number.

12. Multiplication of two numbers – **one** of which **is greater than base** and the **other is less than base**:

Method:

Let, the two numbers are a_1 and a_2.
Where, $a_1 = 10^n + b_1$ and $a_2 = 10^n - b_2$.
Then, $a_1 \times a_2 = (a_1 + a_2) | -(b_1 \times b_2)$

Note: Subtract 1 from the extreme left digit of $(a_1 + a_2)$.

Note: First digit from (left-hand side) of the number less than base is added to the second digit of the number bigger than base and so on.

Example:

102 – 2 (+)
8 – 2 (–)
082 – 0
– 4 (–)
8 1 6

1001 – 1(+)
9 – 1(–)
0901 – 0
– 1 (–)
9009

105 – 5 (+)
989 – 11 (–)
1039 – 00
– 55 (–)
1038 45

11 – 1 (+)
9 9 8 – 2 (–)
1098 – 0
– 2 (–)
1097 8

143 – 43 (+)
999 – 01 (–)
1429 – 00
– 43 (–)
1428 57

101 – 1(+)
99 – 1(–)
100 – 00
– 1 (–)
99 99

Note: Put 0s at the end of the sum of the numbers so that digits in the solution (before subtraction) are equal to the total digits of the numbers multiplied.

13. Multiplication of two numbers which are **more than half of bases** i.e. more than 5, 50, 500, 5000, etc.:

Solution. (from right to left):

1. Multiply the differences and write as first part.
2. Add the two numbers and deduct 5 from extreme left digit. Half the result to obtain the second part.

Note: Digits in the solution are equal to total digits of two numbers multiplied.

Example:

51 – 1	56 – 6	52 – 2	513 – 13
53 – 3	58 – 8	57 – 7	511 – 11
54 – 03	64 – 48	59 – 14	524 – 143
= 27 03	32 48	= 29 64	262 143

Note:

$\frac{1}{2} \times 54 = 27$ $\frac{1}{2} \times 64 = 32$ $\frac{1}{2} \times 59 = 29.5$ $\frac{1}{2} \times 524 = 262$

Note: When the numbers to be multiplied have equal digits, we can use the following method also to get the first part:

51 + 3 = 54	56 + 8 = 64	52 + 7 = 59	513 + 11 = 524
53 + 1 = 54	58 + 6 = 64	57 + 2 = 59	511 + 13 = 524

When the numbers have unequal digits:

<table>
<tr><td>52 – 2</td><td>501 – 1</td><td>5004 – 4</td><td>551 – 51</td></tr>
<tr><td>6 – 1</td><td>51 – 1</td><td>53 – 3</td><td>51 – 1</td></tr>
<tr><td>62 – 2</td><td>511 – 01</td><td>5304 – 12</td><td>561 – 51</td></tr>
<tr><td>= 31 2</td><td>= 255.5 | 01</td><td>= 2652 12</td><td>= 280.5 | 51</td></tr>
<tr><td></td><td>= 255 51</td><td></td><td>= 281 01</td></tr>
</table>

Note:

$\frac{1}{2} \times 62 = 31$ $\frac{1}{2} \times 511 = 255.5$ $\frac{1}{2} \times 5304 = 2652$ $\frac{1}{2} \times 561 = 280.5$

14. Multiplication of two numbers when both are **smaller than half of bases** i.e. less than 5, 50, 500, 5000, etc.:

Solution. (from right to left):

1. Multiply the differences and write as first part.
2. Add the two numbers and deduct 5 from extreme left digit. Half the result to obtain the second part.

Note: Total digits in the solution are equal to total digits of two numbers multiplied.

Example:

47 – 3	41 – 9	49 – 1
49 – 1	47 – 3	48 – 2
46 – 03	38 – 27	47 – 02
= 23 03	19 27	= 23 52

Note:

$\frac{1}{2} \times 46 = 23$	$\frac{1}{2} \times 38 = 19$	$\frac{1}{2} \times 47 = 23.5$

Note: When the numbers to be multiplied have equal number of digits, we can use the following method also to get the first part:

47 – 1 = 46	41 – 3 = 38	49 – 2 = 47
49 – 3 = 46	47 – 9 = 38	48 – 1 = 47

491 – 9	474 – 26	4997 – 3
495 – 5	492 – 8	4996 – 4
486 – 045	466 – 208	4993 – 0012
= 243 045	= 233 208	= 2496 5012

Note:

$\frac{1}{2} \times 486 = 243$	$\frac{1}{2} \times 466 = 233$	$\frac{1}{2} \times 4993 = 2496.5$

Note: When the numbers to be multiplied have equal number of digits, we can use the following method also to get the first part:

491 – 5 = 486	474 – 8 = 466	4997 – 4 = 4993
495 – 9 = 486	492 – 26 = 466	4996 – 3 = 4993

When the two numbers don't have equal digits:

498 – 2	4993 – 7
49 – 1	489 – 11
488 – 02	4883 – 077
= 244 02	= 2441.5|077 = 2441577

Note:

$\frac{1}{2} \times 488 = 244$	$\frac{1}{2} \times 4883 = 2441.5$

15. Multiplication of two numbers – one of which is **more than** and the other is **less than half of base**:

Steps:

1. Add the two numbers. Put 0s at the end of sum of the numbers so that digits of the sum are equal to total digits of the numbers multiplied. Deduct 5 from extreme left digit
2. Half the result obtained in Step 1.
3. Multiply the differences and deduct it from the result obtained in Step 2.

Note: Total digits in the solution are equal to total digits of two numbers multiplied.

Example:

51 – 1 (+)	53 – 3 (+)	507 – 7 (+)
47 – 3 (–)	46 – 4 (–)	498 – 2 (–)
48 – 03	49 – 12	505 – 14
= 24 00	= 24 50	= 252 500
– 3	– 12	– 14
23 97	24 38	252 486

Note:

$\frac{1}{2} \times 48 = 24$ $\frac{1}{2} \times 49 = 24.5$ $\frac{1}{2} \times 505 = 252.5$

Note : When the numbers to be multiplied have equal number of digits, we can use the following method also to get the first part:

$51 - 3 = 48$ $53 - 4 = 49$ $507 - 2 = 505$
$47 + 1 = 48$ $46 + 3 = 49$ $498 + 7 = 505$

When the numbers have unequal digits:

$$\begin{array}{rr} 54 - & 4\,(+) \\ 494 - & 6\,(-) \\ \hline 534 - & 24 \\ \hline = 267 & 00 \\ & -24 \\ \hline 266 & 76 \end{array} \qquad \begin{array}{rr} 503 - & 3\,(+) \\ 49 - & 1\,(-) \\ \hline 493 - & 03 \\ \hline = 246 & 50 \\ & -03 \\ \hline 246 & 47 \end{array} \qquad \begin{array}{rr} 481 - & 19\,(-) \\ 52 - & 2\,(+) \\ \hline 501 - & 38 \\ \hline = 250 & 50 \\ & -38 \\ \hline 250 & 12 \end{array} \qquad \begin{array}{rr} 49988 - & 12\,(-) \\ 511 - & 11\,(+) \\ \hline 51088 - & 132 \\ \hline = 25544 & 000 \\ - & 132 \\ \hline 25543 & 868 \end{array}$$

16. Multiplication of two numbers when their base is '*x*' times of 10^n. :

Solution (from right to left) :

1. Multiply the difference from base and write as the first part.
2. Add first number and difference from base for the second number, *or* second number and difference from base for first number, to get the second part.
3. Multiply the left-hand part with '*x*'.

Example :

$$\begin{array}{c} \underline{\text{Base } 300} \\ 304 + 4 \\ 305 + 5 \\ \hline 309\,/20 \\ \hline = 927\ 20 \end{array} \qquad \begin{array}{c} \underline{\text{Base } 40} \\ 39 - 1 \\ 37 - 3 \\ \hline 36\,/\,3 \\ \hline 144\ 3 \end{array} \qquad \begin{array}{c} \underline{\text{Base } 70} \\ 72 + 2 \\ 76 + 6 \\ \hline 78\,/\,12 \\ \hline = 546\ 12 \end{array}$$

Note :

$3 \times 309 = 927$ $4 \times 36 = 144$ $7 \times 78 = 546$

17. Multiplication of two numbers – one of which is **more than a base** and the other is **more than half of same base**:

Method :

Let the numbers are a_1 and a_2.

Where $a_1 = 10^n + b_1$ and $a_2 = \frac{10^n}{2} + b_2$

Then, $a_1 \times a_2 = a_2 + \frac{1}{2}\, b_1 \mid b_1 \times b_2$

Note :- First part (right-hand side) Contains *n* digits

Example:

$$\begin{array}{c} 102 - 2 \\ 51 - 1 \\ \hline 51 + \frac{1}{2} \times 2 \mid 02 \\ = 52\ 02 \end{array} \qquad \begin{array}{c} 103 - 3 \\ 57 - 7 \\ \hline 57 + \frac{1}{2} \times 3 \mid 21 \\ = 58.5 \mid 21 = 5871 \end{array} \qquad \begin{array}{c} 101 - 1 \\ 51 - 1 \\ \hline 51 + \frac{1}{2} \times 1 \mid 01 \\ = 51.5 \mid 01 = 5151 \end{array} \qquad \begin{array}{c} 1012 - 12 \\ 508 - 8 \\ \hline 508 + \frac{1}{2} \times 12 \mid 096 \\ = 514\ 096 \end{array}$$

18. Multiplication of two numbers – one of which is **less than a base** and the other is **less than half of same base**:

Method:

Let the numbers are a_1 and a_2.

Where $a_1 = 10^n - b_1$ and $a_2 = \frac{10^n}{2} - b_2$

Then, $a_1 \times a_2 = a_2 - \frac{1}{2} b_1 \mid b_1 \times b_2$

Note. First part (right-hand side) contains *n* digits

Example:

96 – 4	999 – 1	9986 – 14
48 – 2	496 – 4	4994 – 6
$48 - \frac{1}{2} \times 4 \mid 08$	$496 - \frac{1}{2} \times 1 \mid 004$	$4994 - \frac{1}{2} \times 14 \mid 0084$
= 46 08	= 495.5 | 004 = 495 504	= 4987 0084

19. Multiplication of two numbers – one of which is **more than a base** and other is **less than half of same base**:

Method:

Let the numbers are a_1 and a_2.

Where $a_1 = 10^n + b_1$ and $a_2 = \frac{10^n}{2} - b_2$

Then, $a_1 \times a_2 = a_2 + \frac{1}{2} b_1 \mid (-)\ b_1 \times b_2$

Note : First part (right-hand side) contains *n* digits

Example:

104 – 4 (+)	1018 – 18 (+)
47 – 3 (–)	497 – 3 (–)
$47 + \frac{1}{2} \times 4 \mid (-)\ 12$	$497 + \frac{1}{2} \times 18 \mid (-)\ 54$
= 49 00	= 506 000
– 12	– 054
48 88	505 946

20. Multiplication of two numbers one of which is **less than a base** and other is **more than half of same base**:

Method:

Let the numbers are a_1 and a_2.

Where $a_1 = 10^n - b_1$ and $a_2 = \frac{10^n}{2} + b_2$

Then, $a_1 \times a_2 = a_2 - \frac{1}{2} b_1 \mid (-)\ b_1 \times b_2$

Note. First part (right-hand side) contains *n* digits

Example:

98 – 2 (–)
53 – 3 (+)

$53 - \frac{1}{2} \times 2 \mid (-)\, 6$

= 52 00
– 06
51 94

993 – 7 (–)
512 – 12 (+)

$512 - \frac{1}{2} \times 7 \mid 084 = 508.5 \mid (-)\, 84$

= 508 500
– 084
508 416

20. Multiplication of two numbers whose **unit's digits total to 10** and the **remaining digits are same**:

Steps:

1. Multiply the unit's digits of the given numbers to get last 2 digits of the solution.
2. Multiply the common digits in the two numbers with the next higher number to get the remaining digits of the solution.

Proof:

Let the two numbers have unit's digits 'm' and '10-m', and same remaining digits 'n'.

∴ The numbers are (10n + m) and [10n + (10 – m)]

Now $(10n + m) \times (10n + 10 - m)$

$= 100n^2 + 100n - 10mn + 10mn + 10m - m^2$

$= 100n^2 + 100n + 10m - m^2$

$= 100n\,(n + 1) + m\,(10 - m)$

i.e. $n \times (n + 1) \mid m\,(10 - m)$

Note: First part i.e. m (10 – m) will have 2 digits.

Example:

62 × 68 = 6 × 7 | 2 × 8 = 42 16
94 × 96 = 9 × 10 | 4 × 6 = 90 24
123 × 127 = 12 × 13 | 3 × 7 = 156 21
31 × 39 = 3 × 4 | 1 × 9 = 12 09

22. Multiplication of two numbers whose **last two digits total to 100** and **remaining digits are same**:

Steps:

1. Multiply the last two digits of both the numbers to get last 4 digits of the solution.
2. Multiply the common digits in the two numbers with the next higher number to get the remaining digits of the solution.

Examples:

301 × 399 = 3 × 4 | 01 × 99 = 12 0099
204 × 296 = 2 × 3 | 04 × 96 = 6 0384
411 × 489 = 4 × 5 |11 × 89 = 20 0979
1103 × 1197 = 11 × 12 | 03 × 97 = 132 0291
1235 × 1265 = 12 × 13 | 35 × 65 = 156 2275

23. Multiplication of two numbers whose **last three digits total to 1000** and **remaining digits are same:**

Steps:

1. Multiply the last three digits of both the numbers to get last 6 digits of the solution.
2. Multiply the common digits in the two numbers with the next higher number to get remaining digits of the solution.

Examples:

$2002 \times 2998 = 2 \times 3 \mid 002 \times 998 = 6\ 001996$

$9001 \times 9999 = 9 \times 10 \mid 001 \times 999 = 90\ 000999$

$13006 \times 13994 = 13 \times 14 \mid 006 \times 994 = 182\ 005964$

24. Multiplication of two numbers whose **ten's digits total to 10** and the **unit's digits are same:**

Steps:

1. Multiply the unit's digits of the given numbers to get last two digits of the solution.
2. Multiply the ten's digits of the two numbers and add common unit's digit to it to get remaining digits of the solution.

Examples:

$43 \times 63 = 4 \times 6 + 3 \mid 3^2 = 27\ 09$

$27 \times 87 = 2 \times 8 + 7 \mid 7^2 = 23\ 49$

$98 \times 18 = 9 \times 1 + 8 \mid 8^2 = 17\ 64$

25. Multiplication of two numbers **total of whose units' digits is 5** and the **remaining digits are same:**

Steps:

1. Multiply the unit's digits of the given numbers to get last 2 digits of the solution.
2. Multiply the common digits in the two numbers and add half of the common digit to get the remaining digits.

Examples:

$22 \times 23 = 2 \times 2 + \frac{1}{2} \times 2 \mid 2 \times 3 = 5\ 06$

$803 \times 802 = 80 \times 80 + \frac{1}{2} \times\ 80 \mid 3 \times 2 = 6440\ 06$

$11 \times 14 = 1 \times 1 + \frac{1}{2} \times 1 \mid 1 \times 4 = 1.5 \mid 04 = 1\ 54$

$52 \times 53 = 5 \times 5 + \frac{1}{2} \times 5 \mid 2 \times 3 = 27.5 \mid 06 = 27\ 56$

26. Multiplication of two numbers **total of whose last two digits is 50** and the **remaining digits are same:**

Steps:

1. Multiply last two digits of the two numbers to get last 4 digits of the solution.
2. Multiply the common digits in the numbers and add half of the common digits to get remaining digits of the solution.

Examples:

$211 \times 239 = 2 \times 2 + \frac{1}{2} \times 2 \mid 11 \times 39 = 5\ 0429$

$604 \times 646 = 6 \times 6 + \frac{1}{2} \times 6 \mid 04 \times 46 = 39\ 0184$

$2521 \times 2529 = 25 \times 25 + \frac{1}{2} \times 25 \mid 21 \times 29 = 637.5 \mid 0609 = 637\ 5609$

27. Multiplication of two numbers whose **last two digits total to 20, 25, 40, 60, 75, 80, 90, 110, 120, 125, 130, 140, 150 etc.** and the remaining digits are same:

Steps:

1. To obtain the last 4 digits of the solution, multiply the last two digits of the given numbers.
2. To obtain remaining digits of the solution:
    - (*a*) Find the sum of the numbers formed by last 2 digits of the given numbers.
    - (*b*) Divide the sum obtained in step (a) by 100.
    - (*c*) Multiply the common digits of the two numbers.
    - (*d*) Multiply the common digit with the fraction obtained in step (b).
    - (*e*) Add the results obtained at Step (c) and (d) to get the right portion of the solution.

Examples:

1. $512 \times 508 = 5 \times 5 + \frac{1}{5} \times 5 \mid 12 \times 08 = 26\ 0096$

Note: $12 + 8 = 20$; And $\frac{20}{100} = \frac{1}{5}$

2. $811 \times 814 = 8 \times 8 + \frac{1}{4} \times 8 \mid 11 \times 14 = 66\ 0154$

Note: $11 + 14 = 25$; And $\frac{25}{100} = \frac{1}{4}$

3. $1016 \times 1024 = 10 \times 10 + \frac{2}{5} \times 10 \mid 16 \times 24 = 104\ 0384$

Note: $16 + 24 = 40$; And $\frac{40}{100} = \frac{2}{5}$

4. $502 \times 558 = 5 \times 5 + \frac{3}{5} \times 5 \mid 02 \times 58 = 28\ 0116$

Note: $2 + 58 = 60$; And $= \frac{60}{100} = \frac{3}{5}$

5. $503 \times 577 = 5 \times 5 + \frac{4}{5} \times 5 \mid 03 \times 77 = 29\ 0231$

Note: $3 + 77 = 80$; And $= \frac{80}{100} = \frac{4}{5}$

6. $1011 \times 1099 = 10 \times 10 + \frac{11}{10} \times 10 \mid 11 \times 99 = 111\ 1089$

Note: $11 + 99 = 110$; And $= \frac{110}{100} = \frac{11}{10}$

7. $851 \times 899 = 8 \times 8 + \frac{3}{2} \times 8 \mid 51 \times 99 = 76\ 5049$

Note: 51 + 99 = 150; And $\frac{150}{100} = \frac{3}{2}$

Note: When the result in first part is in decimals:

1. $102 \times 108 = 1 \times 1 + \frac{1}{10} \times 1 \mid 02 \times 08 = 1.1 \mid 0016 = 1\ 1016$

Note: $2 + 8 = 10$; And $\frac{10}{100} = \frac{1}{10}$

2. $303 \times 327 = 3 \times 3 + \frac{3}{10} \times 3 \mid 03 \times 27 = 9.9 \mid 0081 = 9\ 9081$

Note: $3 + 27 = 30$; And $= \frac{30}{100} = \frac{3}{10}$

28. Multiplication of two numbers when **both numbers are more than 50:**

Method:

If the numbers are (50 + a) and (50 + b).

Then, $(50 + a) \times (50 + b) = 25 + \frac{1}{2}(a + b) \mid ab$

Note: 2 digits are taken from each part.

Examples:

1. Find the value of 53 × 57.

Solution:

Here a = 3, b = 7, a + b = 10

$\therefore 53 \times 57 = 25 + \frac{1}{2} \times 10 \mid 3 \times 7 = 30\ 21$

2. Find the value of 58 × 62.

Solution:

Here a = 8, b = 12, a + b = 20

$\therefore 58 \times 62 = 25 + \frac{1}{2} \times 20 \mid 8 \times 12 = 35\ 96$

3. Find the value of 51 × 52.

Solution:

Here a = 1, b = 2, a + b = 3

$\therefore 51 \times 52 = 25 + \frac{1}{2} \times 3 \mid 1 \times 2 = 26.5 \mid 02 = 26\ 52$

4. Find the value of 53 × 56.

Solution:

Here a = 3, b = 6, a + b = 9

$\therefore 53 \times 56 = 25 + \frac{1}{2} \times 9 \mid 3 \times 6 = 29.5 \mid 18 = 29\ 68$

29. Multiplication of two numbers when **both the numbers are less than 50:**

Method:

If the numbers are (50 – a) and (50 – b).

Then, $(50 - a) \times (50 - b) = 25 - \frac{1}{2}(a + b) \mid ab$

Note: 2 digits are taken from each part.

Examples:

1. Find the value of 48×46.

Solution:

Here: $a = 2$, $b = 4$ and $a + b = 6$

$\therefore 48 \times 46 = 25 - \frac{1}{2} \times 6 \mid 2 \times 4 = 22\ 08$

2. Find the value of 41×47.

Solution:

Here: $a = 9$, $b = 3$ and $a + b = 12$

$\therefore 41 \times 47 = 25 - \frac{1}{2} \times 12 \mid 9 \times 3 = 19\ 27$

3. Find the value of 42×43.

Solution:

Here: $a = 8$, $b = 7$ and $a + b = 15$

$\therefore 42 \times 43 = 25 - \frac{1}{2} \times 15 \mid 8 \times 7 = 17.5 \mid 56 = 18\ 06$

30. Multiplication of two numbers – **one** of which **is bigger than 50** and the **other is less than 50**:

Method:

If the numbers are $(50 + a)$ and $(50 - b)$.

Then, $(50 + a) \times (50 - b) = 25 + \frac{1}{2}(a - b) \mid (-ab)$

Note: 2 digits are taken from each part.

Examples:

1. Find the value of 53×49.

Solution:

Here $a = 3$, $b = 1$, and $a - b = 2$

$\therefore 53 \times 49 = 25 + \frac{1}{2}(3 - 1) \mid -3 = 2600 - 3 = 25\ 97$

2. Find the value of 63×45.

Solution:

Here $a = 13$, $b = 5$, and $a - b = 8$

$\therefore 63 \times 45 = 25 + \frac{1}{2}(13 - 5) \mid -65 = 2900 - 65 = 28\ 35$

3. Find the value of 52×49.

Solution:

Here $a = 2$, $b = 1$, and $a - b = 1$

$\therefore 52 \times 49 = 25 + \frac{1}{2}(2 - 1) \mid -2 = 2550 - 2 = 25\ 48$

4. Find the value of 51×47.

Solution:

Here $a = 1$, $b = 3$, and $a - b = -2$

$\therefore\ 51 \times 47 = 25 + \frac{1}{2}(1-3)\,| - 3 = 2400 - 3 = 23\ 97$

5. Find the value of 52×43.

Solution:

Here $a = 2$, $b = 7$, and $a - b = -5$

$\therefore\ 52 \times 43 = 25 + \frac{1}{2}(2-7)\,| - 14 = 2250 - 14 = 22\ 36$

31. Multiplication of two numbers one of which is **greater than** and other number is **less than 50** and **difference from 50 is same** in both the cases:

Method:

Let, the numbers are $(50 + a)$ and $(50 - a)$.

Then, $(50 - a) \times (50 + a) = 2500 - a^2$

Examples:

1. Find the value of 51×49.

Solution:

Here common number is 50 and difference is 1.

$\therefore\ 51 \times 49 = 2500 - 1^2 = 2500$ ~~$- 1 = 2499$~~

2. Find the value of 52×48.

Solution:

Here common number is 50 and difference is 2.

$\therefore\ 52 \times 48 = 2500 - 2^2 = 2500 - 4 = 2496$

3. Find the value of 55×45.

Solution:

Here common number is 50 and difference is 5.

$\therefore\ 55 \times 45 = 2500 - 5^2 = 2500 - 25 = 2475$

4. Find the value of 58×42.

Solution:

Here common number is 50 and difference is 8.

$\therefore\ 58 \times 42 = 2500 - 8^2 = 2500 - 64 = 2436$

32. Multiplication of two numbers one of which is **greater than** and the other is **less than a common number** and **difference** from that common number **is same** in both the cases:

Method:

Let, the numbers are $(x + a)$ and $(x - a)$.

Then, $(x + a) \times (x - a) = x^2 - a^2$

Examples:

1. Find the value of 42×38.

Solution:

Here common number is 40 and difference is 2.

$\therefore\ 42 \times 38 = 40^2 - 2^2 = 1600 - 4 = 1596$

2. Find the value of 53×47.

Solution:

Here common number is 50 and difference is 3.

$\therefore 53 \times 47 = 50^2 - 3^2 = 2500 - 9 = 2491$

3. Find the value of 61×59.

Solution:

Here common number is 60 and difference is 1.

$\therefore 61 \times 59 = 60^2 - 1^2 = 3600 - 1 = 3599$

4. Find the value of 62×58.

Solution:

Here common number is 60 and difference is 2.

$\therefore 62 \times 58 = 60^2 - 2^2 = 3600 - 4 = 3596$

5. Find the value of 94×86.

Solution:

Here common number is 90 and difference is 4.

$\therefore 94 \times 86 = 90^2 - 4^2 = 8100 - 16 = 8084$

6. Find the value of 224×216.

Solution:

Here common number is 220 and difference is 4.

$\therefore 224 \times 216 = 220^2 - 4^2 = 48400 - 16 = 48384$

Note: Common number must be the number whose square can be calculated easily.

32. Multiplication of **two 2-digit numbers**:

$$\begin{array}{ccc} & A & B \\ \times & C & D \\ \hline \multicolumn{3}{c}{A \times C \mid A \times D + B \times C \mid B \times D} \\ \hline \end{array}$$

Note:

B × D is multiplication of the unit's digits of both the numbers.

A × D + B × C is sum of cross multiplication of the digits of the given numbers.

A × C is multiplication of ten's digits of both the numbers.

Note:

1. We start calculation from right hand side and move to left hand side.
2. One digit is taken from each part. Extra digit (left hand side), if any is carried forward and added to the part immediate left to it.

Example 1:

$$\begin{array}{ccc} & 1 & 2 \\ \times & 4 & 3 \\ \hline \multicolumn{3}{c}{1 \times 4 \mid 1 \times 3 + 2 \times 4 \mid 2 \times 3} \\ \hline \end{array} \qquad = 4 \mid 11 \mid 6 = 516$$

Hint: Here, second part consists of 2 digits. We write down first 1 and carry forward second 1 and add to 4 in the next part to get 4 + 1 = 5.

Example 2:

$$\begin{array}{ccc} & 2 & 3 \\ \times & 7 & 4 \\ \hline \multicolumn{3}{c}{2\times 7\,|\,2\times 4+3\times 7\,|\,3\times 4} \end{array} \qquad = 14\,|\,29\,|\,12 = 1702$$

Here, we write down 2 from first part and carry forward 1 and add to 29 in the next part.

In second part, we get 29 + 1 = 30. 0 is written down and 3 is carried forward and added to 14 in the next part.

We get 14 + 3 = 17 in the last part. Since this is the last part, we can't carry forward 1 to the next part. Therefore, we write down 17.

34. Multiplication of two **numbers with any number of digits**:

$$\begin{array}{ccccc} & A & B & C & D \\ \times & P & Q & R & S \\ \hline \multicolumn{5}{c}{AP\,|\,AQ+BP\,|\,AR+BQ+CP\,|\,AS+BR+CQ+DP\,|\,BS+CR+DQ\,|\,CS+DR\,|\,DS} \end{array}$$

Steps for multiplication (from right to left):

1. Multiply 1st digit of the first number with 1st digit of the second number,
2. Multiply 1st digit of the first number with 2nd digit of the second number,
   Multiply 2nd digit of the first number with 1st digit of the second number,
   Add the results obtained.
3. Multiply 1st digit of the first number with 3rd digit of the second number,
   Multiply 2nd digit of the first number with 2nd digit of the second number,
   Multiply 3rd digit of the first number with 1st digit of the second number.
   Add the results obtained.
4. Multiply 1st digit of the first number with 4th digit of the second number,
   Multiply 2nd digit of the first number with 3rd digit of the second number,
   Multiply 3rd digit of the first number with 2nd digit of the second number,
   Multiply 4th digit of the first number with 1st digit of the second number,
   Add the results obtained.
5. Multiply 2nd digit of the first number with 4th digit of the second number,
   Multiply 3rd digit of the first number with 3rd digit of the second number,
   Multiply 4th digit of the first number with 2nd digit of the second number,
   Add the results obtained.
6. Multiply 3rd digit of the first number with 4th digit of the second number,
   Multiply 4th digit of the first number with 3rd digit of the second number,
   Add the results obtained.
7. Multiply 4th digit of the first number with 4th digit of the second number,

Note:

1. We start calculation from right hand side and move to left hand side.
2. One digit is taken from each part of the result. Extra digits (left hand side), if any, is carried forward and added to the result obtained in immediate left part.

Example 1:

$$\begin{array}{ccccc} & 2 & 5 & 3 & 8 \\ \times & 3 & 1 & 7 & 6 \\ \hline \end{array}$$

First part = 8 × 6 = 48 (4 is carried forward)

Second part = 3 × 6 + 8 × 7 = 18 + 56 = 74

74 + 4 = 78 (7 is carried forward)

Third part = 5 × 6 + 3 × 7 + 8 × 1 = 30 + 21 + 8 = 59

59 + 7 = 66 (6 is carried forward)

Fourth part = 2 × 6 + 5 × 7 + 3 × 1 + 8 × 3 = 12 + 35 + 3 + 24 = 74

74 + 6 = 80 (8 is carried forward)

Fifth part = 2 × 7 + 5 × 1 + 3 × 3 = 14 + 5 + 9 = 28

28 + 8 = 36 (3 is carried forward)

Sixth part = 2 × 1 + 5 × 3 = 2 + 15

17 + 3 = 20 (2 is carried forward)

Seventh part = 2 × 3 = 6

6 + 2 = 8

∴ 2538 × 3176 = 8060688

Example 2:

$$\begin{array}{ccc} 4 & 2 & 7 \\ \hline \times & 4 & 6 \end{array}$$

First part = 7 × 6 = 42 (4 is carried forward)

Second part = 2 × 6 + 7 × 4 = 12 + 28 = 40

40 + 4 = 44 (4 is carried forward)

Third part = 4 × 6 + 2 × 4 = 24 + 8 = 32

32 + 4 = 36 (3 is carried forward)

Fourth part = 4 × 4 = 16

16 + 3 = 19

∴ 427 × 26 = 19642

Example 3:

$$\begin{array}{cccc} 3 & 5 & 6 & 2 \\ \times & 1 & 2 & 3 \\ \hline \end{array}$$

First part = 2 × 3 = 6

Second part = 6 × 3 + 2 × 2 = 18 + 4 = 22 (2 is carried forward)

Third part = 5 × 3 + 6 × 2 + 2 × 1 = 15 + 12 + 2 = 29

29 + 2 = 31 (3 is carried forward)

Fourth part = 3 × 3 + 5 × 2 + 6 × 1 = 9 + 10 + 6 = 25

25 + 3 = 28 (2 is carried forward)

Fifth part = 3 × 2 + 5 × 1 = 6 + 5 = 11

11 + 2 = 13 (1 is carried forward)

Sixth part $= 3 \times 1 = 3$

$3 + 1 = 4$

$\therefore 3562 \times 123 = 438126$

35. Multiplication of **two 2-digit numbers** both **having 1 in the unit's place**:

Using the method of multiplication, we get the value of A1× B1 as follows:

$$\begin{array}{r} A\quad 1 \\ \times \quad B\quad 1 \\ \hline A \times B \mid A + B \mid 1 \times 1 \\ \hline \end{array}$$

∴ Solution (from right to left) is

Unit's place: $1 \times 1 = 1$.

Ten's place: A + B i.e. sum of ten's digits of the numbers multiplied.

Hundred's place: A × B i.e. Multiplication of digits at ten's place of the numbers multiplied.

Note:

1. We start calculation from right hand side and move to left hand side.
2. One digit is taken from each part. Extra digits (left hand side), if any is carried forward and added to the result obtained in immediate left part.

Example:

21	31	61
× 31	× 41	× 91
$= 2 \times 3 \mid 2 + 3 \mid 1$	$= 3 \times 4 \mid 3 + 4 \mid 1$	$= 6 \times 9 \mid 6 + 9 \mid 1$
= 651	= 1271	$= 54 \mid 15 \mid 1 = 5551$

36. Multiplication of **two 2-digit numbers** both **having 1 in the ten's place**.

Using the method of multiplication, we get the value of 1A × 1B as follows:

$$\begin{array}{r} 1\quad A \\ \times \quad 1\quad B \\ \hline 1 \times 1 \mid A + B \mid A \times B \\ \hline \end{array}$$

∴ Solution (from right to left):

Unit's digit: A × B i.e. Multiplication of unit's digits of the given numbers.

Ten's digit: A + B i.e. sum of unit's digits.

Hundred's digit: $1 \times 1 = 1$.

Note:

1. We start calculation from right hand side and move to left hand side.
2. One digit is taken from each part. Extra digit (left hand side), if any, is carried forward and added to the result obtained in immediate left part.

Example:

12	12	13
× 14	× 16	× 18
$= 1 \mid 2 + 4 \mid 2 \times 4$	$= 1 \mid 2 + 6 \mid 2 \times 6$	$= 1 \mid 3 + 8 \mid 3 \times 8$
= 168	$= 1 \mid 8 \mid 12 = 192$	$= 1 \mid 11 \mid 24 = 234$

37. Multiplication of **two 2-digit numbers** both having **same digit in unit's place**.

$$\begin{array}{r} A \quad C \\ \times \quad B \quad C \\ \hline A \times B \mid (A+B) \times C \mid C \times C \\ \hline \end{array}$$

Multiplication of ten's digits	Multiplication of Common digit with sum of ten's digits	Multiplication of unit's digit

Note:

1. We start calculation from right hand side and move to left hand side.
2. One digit is taken from each part. Extra digit (left hand side), if any, is carried forward and added to the result obtained in the immediate left part.

Example:

$$\begin{array}{r} 13 \\ \times 23 \\ \hline = 1\times 2 \mid 3\,(1+2) \mid 3\times 3 \\ = 299 \end{array} \qquad \begin{array}{r} 24 \\ \times 34 \\ \hline = 2\times 3 \mid 4\,(2+3) \mid 4\times 4 \\ = 6 \mid 20 \mid 16 = 816 \end{array} \qquad \begin{array}{r} 58 \\ \times 98 \\ \hline = 5\times 9 \mid 8\,(5+9) \mid 8\times 8 \\ = 45 \mid 112 \mid 64 \;= 5684 \end{array}$$

Note: This method can be used to find multiplication of two 3-digit numbers where last two digits are same in both the numbers.

Example:

1. $312 \times 512 = 3 \times 5 \mid (3 + 5) \times 12 \mid 12 \times 12 = 15 \mid 96 \mid 144 = 15\ 97\ 44$
2. $213 \times 813 = 2 \times 8 \mid (2 + 8) \times 13 \mid 13 \times 13 = 16 \mid 130 \mid 169 = 17\ 31\ 69$
3. $102 \times 202 = 1 \times 2 \mid (1 + 2) \times 2 \mid 2 \times 2 = 2 \mid 6 \mid 4 = 2\ 06\ 04$

Note: In these examples two digits are taken from each part.

38. Multiplication of **two 2-digit numbers** both **having same digit in ten's place**.

$$\begin{array}{r} A \quad B \\ \times \quad A \quad C \\ \hline A \times A \mid A \times (B+C) \mid B \times C \\ \hline \end{array}$$

Multiplication of ten's digit	Multiplication of Common digit with sum of unit's digits	Multiplication of unit's digit

Note:

1. We start calculation from right hand side and move to left hand side.
2. One digit is taken from each part. Extra digit (left hand side), if any, is carried forward and added to the result obtained in the immediate left part.

Example:

$$\begin{array}{r} 23 \\ \times 28 \\ \hline \underline{2\times 2 \mid 2\,(3+8) \mid 3\times 8} \end{array} = 4 \mid 22 \mid 24 = 644$$

Note: This method can be used to multiply two 3-digit numbers where first two digits in both the numbers are same.

Example:

1. 113 × 115 = 11 × 11 | (3 + 5) × 11 | 3 × 5 = 121 | 88 | 15 = 129 9 5

(First two digits are same viz. 11 in both the numbers)

2. 254 × 256 = 25 × 25 | (4 + 6) × 25 | 4 × 6 = 625 | 250 | 24 = 650 2 4

(First two digits are same viz. 25 in both the numbers)

Note: One digit is taken from each part.

39. **Verification of the correctness of the multiplication** of two or more numbers:

Method:

1. Find sum of digits of all the numbers separately. If sum of digits consists of more than one digit, add the digits of the results to get the sum in one digit.
2. Multiply the sum of digits and then find sum of digits of the product.
3. Find sum of the digits of the multiplication of the given numbers.
4. Compare the results obtained in Step 2 and 3. If the two results are same the product is correct.

Example 1:

$$\begin{array}{r} 25 \\ \times 35 \\ \hline 875 \\ \hline \end{array}$$

Step 1: 2 + 5 = 7 and 3 + 5 = 8

Step 2: 7 × 8 = 56 and sum of digits = 5 + 6 = 11; 1 + 1 = 2

Step 3: 8 + 7 + 5 = 20; 2 + 0 = 2

Step 4: Since the two results are same i.e. 2, the solution is correct.

Example 2:

$$\begin{array}{r} 87 \\ \times 71 \\ \hline 6177 \\ \hline \end{array}$$

Step 1: 8 + 7 = 15; 1 + 5 = 6

And 7 + 1 = 8

Step 2: 6 × 8 = 48 and sum of digits = 4 + 8 = 12; 1 + 2 = 3

Step 3: 6 + 1 + 7 + 7 = 21; 2 + 1 = 3

Step 4: Since the two results are same i.e. 3, the solution is correct.

Note: If the results obtained in Step 2 and Step 3 are not same, the product is definitely incorrect but opposite of this is not always true. In other words, equality of two results does not necessarily mean that the product is correct. For example, if in the first example, we change the product as 785 instead of 875, then the sum of digits of the product will remain same and hence will tally but 785 is the wrong answer.

Note: If two or more numbers are multiplied together, sum of digits of atleast one of the numbers is 9, then sum of the digits of the product of the numbers will always be 9.

Example:

$$\begin{array}{r} 18 \\ \times 41 \\ \hline 738 \\ \hline \end{array}$$

Step 1: 1 + 8 = 9 and 4 + 1 = 5

Step 2: Since sum of digits of one of the numbers is 9. This result will always be 9. However, we can verify this result also.

9 × 5 = 45 and sum of digits = 4 + 5 = 9

Step 3: 7 + 3 + 8 = 18; 1 + 8 = 9

Step 4: Since the two results are same i.e. 9, the solution is correct.

40. Multiplication of **two mixed numbers** whose **whole numbers are same** and **sum of the fractions is 1, $\frac{1}{2}$, $\frac{1}{4}$ etc.**

Steps:

1. Multiply the common whole number with the next higher number, if sum of the fractions is 1.

   (If the sum is $\frac{1}{2}$ or $\frac{1}{4}$ etc., square the common number and add to it the product of whole number and sum of the fractions.)
2. Multiply the fractions.

Examples:

$8\frac{1}{4} \times 8\frac{3}{4} = 8 \times 9 \mid \frac{1}{4} \times \frac{3}{4} = 72\,\frac{3}{16}$

(**Note:** sum of the fractions is $\frac{1}{4} + \frac{3}{4} = 1$)

$6\,\frac{3}{8} \times 6\,\frac{1}{8} = 6 \times 6 + \frac{1}{2} \times 6 \mid \frac{3}{8} \times \frac{1}{8} = 39\,\frac{3}{64}$

$7\,\frac{3}{8} \times 7\,\frac{1}{8} = 7 \times 7 + \frac{1}{2} \times 7 \mid \frac{3}{8} \times \frac{1}{8} = 52\,\frac{1}{2} \mid \frac{3}{64} = 52\,\frac{35}{64}$

Note: sum of the fractions is $\frac{3}{8} + \frac{1}{8} = \frac{1}{2}$

In the second example, note: $\frac{1}{2} + \frac{3}{64} = \frac{35}{64}$

$4\,\frac{1}{6} \times 4\,\frac{1}{12} = 4 \times 4 + \frac{1}{4} \times 4 \mid \frac{1}{6} \times \frac{1}{12} = 17\,\frac{1}{72}$

Note: sum of the fractions is $\frac{1}{6} + \frac{1}{12} = \frac{1}{4}$

41. **Multiplication of Mixed Numbers:**

Base:

(x + a) × (y + b) = xy + ab + (xb + ay)
i.e. Product of whole numbers
+ Product of fractions
+ Sum of cross multiplication of whole numbers and fractions

Example 1: $13\frac{1}{4} \times 12\frac{1}{5}$

Solution:

$$13\frac{1}{4} \times 12\frac{1}{5}$$

$$= 13 \times 12 + \frac{1}{4} \times \frac{1}{5} + 12 \times \frac{1}{4} + 13 \times \frac{1}{5}$$

$$= 156 + \frac{1}{20} + 3 + 2\frac{3}{5}$$

$$= 156 + 3 + \frac{1}{20} + \frac{3}{5}$$

$$= 161 + \frac{1}{20} + \frac{3}{5}$$

$$= 161 + \frac{1}{20} + \frac{12}{20} = 161\frac{13}{20}$$

Example 2: $25\frac{1}{6} \times 13\frac{1}{4}$

Solution:

$$= 25 \times 13 + \frac{1}{6} \times \frac{1}{4} + 25 \times \frac{1}{4} + 13 \times \frac{1}{6}$$

$$= 325 + \frac{1}{24} + 6\frac{1}{4} + 2\frac{1}{6}$$

$$= (325 + 6 + 2) + \frac{1}{24} + \frac{1}{4} + \frac{1}{6}$$

$$= 333 + \frac{11}{24} = 333\frac{11}{24}$$

42. More Examples of Multiplication :

Example 1 :
$28 \times 225 = ?$

Solution:

$(28 \times 25) \times 9 = 700 \times 9 = 6300$

Example 2:

$4\frac{1}{3} \times 1.80 = ?$

Solution:

$$4\frac{1}{3} \times 1.80 = 4 \times 1.80 + \frac{1}{3} \times 1.80 = 7.20 + 0.60 = 7.80$$

Example 3 :
$10.25 \times 4.80 = ?$

Solution:

We know that $0.25 = \frac{1}{4}$

$$\therefore\ 10.25 \times 4.80 = 10 \times 4.80 + \frac{1}{4} \times 4.80 = 48 + 1.20 = 49.20$$

SOLVED EXERCISE

1. Find Unit's digit in 217 × 818 × 193.

(a) 0 (b) 1 (c) 7 (d) 8

Solution:

(d) Since 7 × 8 = 56

∴ Unit's digit in 217 × 818 is 6.

Now 6 × 3 = 18

∴ Unit's digit in (217 × 818) × 193 is 8.

2. Find Unit's digit in 200 × 217 × 818 × 193.

(a) 0 (b) 1 (c) 7 (d) 8

Solution:

(a) Since one of the number has two 0s at the end.

∴ Multiplication of these numbers will also have two 0s.

∴ Unit's digit in the product is 0.

3. Find Unit's digit in 313 × 815 × 717 × 126.

(a) 0 (b) 3 (c) 6 (d) 7

Solution:

(a) 3 × 5 = 15

∴ Unit's digit in 313 × 815 is 5.

Now 5 × 7 = 35

∴ Unit's digit in (313 × 815) × 717 is 5.

5 × 6 = 30

∴ Unit's digit in (313 × 815 × 717) × 126 is 0.

Trick:

Here, second number is multiple of 5 and fourth number is multiple of 2.
The product of these numbers is a multiple of 5 × 2 = 10 i.e. a number ending in 0.
∴ The product has '0' at unit's place.

4. 12345679 × 9 = ?

(a) 111111111 (b) 21212121 (c) 111111112 (d) 222222221

Solution:

(a) Unit's digit of the solution must be 1 i.e. unit's digit in 9 × 9.

∴ (c) is ruled out.

Since, 12 × 9 = 108

We note that digits in 12 and 9 are 2 and 1 respectively and digits in 12 × 9 = 108 is 3 (i.e. 2 digits + 1 digit).

∴ Digits in 12345679 × 9 must be 8 + 1 = 9.

(b) has 8 digits.

∴ (b) is ruled out.

(d) is ruled out as first three digits (left-hand side) are much greater than 108.

Sum of digits of solution must be 9 as the sum of digits of one multiplier is 9.

∴ (a) is the correct answer.

5. On multiplying a number by 153, 102325 is obtained as result. It is found that both the 2's are wrong. Find the correct result.

(a) 104345 (b) 107375 (c) 108385 (d) 109395

Solution:

(d) 153 is a multiple of 9 as sum of its digits is 9.

∴ The result must also be a multiple of 9.

Sum of digits (other than the wrong digits) = 1 + 0 + 3 + 5 = 9.

∴ The number 10*3*5 is divisible by 9 (ignoring both *).

∴ The number will be divisible by 9 when * + * is divisible by 9.

Sum of replaced digits = 4 + 4, 7 + 7, 8 + 8, 9 + 9 or 8, 14, 16, 18.

Only 18 is divisible by 9.

∴ * is to be replaced by 9.

∴ 109395 is divisible by 153.

Chapter 5

DIVISION–SHORT CUT METHODS

This chapter deals with division of a number by 5, 25, 125 and by mixed number.

DIVISION OF A NUMBER BY 5, 25, 125, ETC.

Division by 5:

Method:

Multiply the number by 2 and divide the result by 10.

Note: $\frac{10}{2} = 5$

Example:

$135 \div 5 = (135 \times 2) \div 10 = 270 \div 10 = 27$

$570 \div 5 = (570 \times 2) \div 10 = 57 \times 2 = 114$

$713 \div 5 = (713 \times 2) \div 10 = 1426 \div 10 = 142.6$

Short-cut Method:

1. Divide last digit of the number by 5.
2. Multiply the number formed by the remaining digits by 2.
3. Add the results obtained in Step 1 and 2.

Examples:

$135 \div 5 = (13 \times 2) + (5 \div 5) = 26 + 1 = 27$

$570 \div 5 = (57 \times 2) + (0 \div 5) = 114 + 0 = 114$

$713 \div 5 = (71 \times 2) + (3 \div 5) = 142 + 0.6 = 142.6$

Division by 25:

Method:

Multiply the number by 4 and divide the result by 100.

Note: $\frac{100}{4} = 25$

Example:

$725 \div 25 = (725 \times 4) \div 100 = 2900 \div 100 = 29$

Short-cut Method:

1. Divide last 2 digits of the number by 25.
2. Multiply the number formed by the remaining digits by 4.
3. Add the results obtained in Step 1 and 2.

Example:

$725 \div 25 = 7 \times 4 + (25 \div 25) = 28 + 1 = 29$

$3275 \div 25 = 32 \times 4 + (75 \div 25) = 128 + 3 = 131$

Division by 125:

Method:

Multiply the number by 8 and divide the result by 1000.

Note: $\frac{1000}{8} = 125$

Example:

$8375 \div 125 = (8375 \times 8) \div 1000 = 67000 \div 1000 = 67$

Short-cut Method:

1. Divide last 3 digits of the number by 125.
2. Multiply the number formed by the remaining digits by 8.
3. Add the results obtained in Step 1 and 2.

Example:

$8375 \div 125 = (8 \times 8) + (375 \div 125) = 64 + 3 = 67$

$9250 \div 125 = (9 \times 8) + (250 \div 125) = 72 + 2 = 74$

DIVISION OF MIXED NUMBER

Method:

The number is divided into two parts and then divided by the divisor.

Example 1:

$$42\frac{2}{3} \div 6 = 42\frac{2}{3} \times \frac{1}{6} = 42 \times \frac{1}{6} + \frac{2}{3} \times \frac{1}{6} = 7 + \frac{1}{9} = 7\frac{1}{9}.$$

Example 2:

$$25\frac{1}{3} \div 11 = 25\frac{1}{3} \times \frac{1}{11} = 25 \times \frac{1}{11} + \frac{1}{3} \times \frac{1}{11} = 2\frac{3}{11} + \frac{1}{33}$$

$$= 2 + \frac{3 \times 3 + 1}{33} = 2\frac{10}{33}.$$

Example 3:

$$66\frac{3}{8} \div 7 = 66\frac{3}{8} \times \frac{1}{7} = 66 \times \frac{1}{7} + \frac{3}{8} \times \frac{1}{7} = 9\frac{3}{7} + \frac{3}{56}$$

$$= 9 + \frac{3 \times 8 + 3}{56} = 9\frac{27}{56}.$$

Chapter 6

HCF AND LCM

Lowest Common Multiple (LCM) : LCM of two or more numbers is the smallest number, which is exactly divisible by each of the given numbers.

Highest Common Factor (HCF) : HCF of two or more numbers is the greatest number by which each of the given numbers are exactly divisible.

For any set of numbers **LCM is always multiple of HCF**. In other words HCF always exactly divides LCM.

HCF divides not only the given numbers but also difference between any two numbers.

If two different numbers when separately divided by a certain divisor leaves same remainder in each case, then difference between the numbers is exactly divisible by the divisor.

For any set of two numbers, product of the numbers is equal to the product of LCM and HCF of the given numbers,

i.e., $\boxed{\textbf{Product of numbers = HCF × LCM}}$

Note: If the numbers are co-prime, their HCF is 1.

∴ LCM of the numbers = Product of the numbers.

$$\text{HCF of fractions} = \frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$$

$$\text{LCM of fractions} = \frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$$

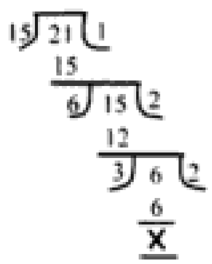
Finding HCF of two numbers by Division Method:

Steps:

1. Divide the bigger number by the smallest number.
2. Divide the smaller number by the remainder obtained in Step 1.
3. Divide the first remainder by the second remainder obtained in Step 2.
4. Divide the second remainder by the third remainder obtained in Step 3.
5. Repeat this process, till final remainder is 0.
6. Last divisor is HCF of the given numbers.

Example: Find HCF of 15 and 21.

Solution:



Divide 21 (bigger number) by 15 (smaller number).

Smaller number (15) is divided by first remainder (6)

First remainder (6) is divided by second remainder (3)
Remainder is 0,

∴ 3, the last divisor is HCF.

Finding HCF of more than two numbers by Division Method:

Steps:

1. Find HCF of any two numbers.

   Note. It is preferable to take two numbers whose difference is the least to make the calculation easy.
2. Find HCF of the third number and of HCF calculated in first Step.
3. Find HCF of next number and HCF found at step 2 and so on.

Example:

Find HCF of 12, 18 and 33.

Solution:

We find HCF of 12 and 18 by using the Division Method explained above.

HCF of 12 and 18 is 6.

Now HCF of first HCF i.e. 6 and the third number i.e. 33 is 3.

Finding HCF of two or more numbers by Factor Method:

Steps:

1. Write the prime factors of each number separately.
2. Find the factor or set of factors which are common in all the numbers.
3. The multiplication of the common factors is HCF.

Note: If no factor is common in all the numbers, then HCF is 1.

Example 1: Find HCF of 15 and 21.

Solution:

$15 = 3 \times 5$

$21 = 3 \times 7$

Now 3 being the only common factor is HCF.

Example 2: Find HCF of 20 and 50.

Solution:

$20 = 2 \times 2 \times 5$

$50 = 2 \times 5 \times 5$

Common factors are 2 and 5.

$\therefore$ HCF $= 2 \times 5 = 10$

Example 3: Find HCF of 8 and 15.

Solution:

$8 = 2 \times 2 \times 2$

$15 = 3 \times 5$

We see that there is no common factor in above two numbers.

$\therefore$ Their HCF is 1.

Note : 8 and 15 are co-prime numbers.

Quicker Method:

1. Instead of finding factors of each number, we find co-prime factors of one number and then check with the help of Divisibility Test Method if these factors divide the other numbers or not.
2. To get HCF, multiply the factors of the first number which divide each of the remaining numbers.

Example 1: Find HCF of 20 and 45.

Solution:

$20 = 4 \times 5$

Now we check Divisibility Test for 45 with the factors of 20 i.e. with 4 and 5.

45 is not divisible by 4 or 2 as the number 45 ends in an odd number.

45 is divisible by 5.

∴ HCF of 20 and 45 is 5.

Example 2: Find the HCF of 42, 63 and 70.

Solution:

$70 = 2 \times 5 \times 7$

Now we check Divisibility Test for 42 and 63 with 2, 5 and 7.

2: 63 is not divisible by 2.

5: 42 is not divisible by 5 (no need to check for 63).

7: 42 and 63 both are divisible by 7.

Since 7 is the only factor which divides each of the given numbers.

∴ HCF = 7.

Use of Factor method for finding LCM of two or more numbers:

Steps:

1. Write factors of one of the numbers.
2. Write factors of the second number. Strike out the factors which are already there in the first number. Multiply the remaining factors of the second number with the factors of the first number to get LCM of the first and the second number.
3. Write factors of the third number. Strike out the factors which are already included in LCM of first two numbers. Multiply the remaining factors to get LCM of first three numbers.
4. Repeat process mentioned in Step 3 for the remaining numbers till all the numbers are covered.

Example 1: Find LCM of 15 and 21.

Solution:

$15 = 3 \times 5$

$21 = 3 \times 7$

(3 in 3×7 is already included in 3×5 and therefore ignored)

∴ LCM of 15 and 21 is $3 \times 5 \times 7 = 105$.

Example 2: Find LCM of 12, 15 and 18.

Solution:

$12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

(3 in 3×5 is already included in $2 \times 2 \times 3$ and therefore ignored)

$\therefore$ LCM of 12 and 15 is $2 \times 2 \times 3 \times 5$.

$18 = 2 \times 3 \times 3$

(2×3 in $2 \times 3 \times 3$ is already included in $2 \times 2 \times 3 \times 5$ and therefore ignored)

$\therefore$ LCM of 12, 15, 18 is $2 \times 2 \times 3 \times 5 \times 3 = 180$.

SOLVED EXERCISE

1. Find HCF of 12 and 18.

(a) 2 (b) 3 (c) 4 (d) 6

Solution:

(d) $12 = 2 \times 2 \times 3$

$18 = 2 \times 3 \times 3$

Here 2×3 is common in both the numbers.

$\therefore$ HCF $= 2 \times 3 = 6$

2. HCF of 18, 27 and 81 is:

(a) 9 (b) 18 (c) 27 (d) 45

Solution:

(a) $18 = 2 \times 3 \times 3$

$27 = 3 \times 3 \times 3$

$81 = 3 \times 3 \times 3 \times 3$

Here 3×3 is common in all the three numbers.

$\therefore$ HCF $= 3 \times 3 = 9$

3. Find HCF of 2268, 2058 and 2100.

(a) 6 (b) 12 (c) 14 (d) 42

Solution:

(d) $2268 - 2058 = 210$ and $2100 - 2058 = 42$

HCF of 210 and 42 is 42.

4. LCM of 25, 30 and 45 is:

(a) 5 (b) 450 (c) 900 (d) 2250

Solution:

(b) $25 = 5 \times 5$

$30 = 2 \times 3 \times 5$

But 5 is already there in factors of 25.

$\therefore$ LCM of 25 and 30 is $5 \times 5 \times 2 \times 3$.

45 = 3 × 3 × 5

But 3 × 5 are already included in 5 × 5 × 2 × 3.

∴ LCM of 25, 30 and 45 is 5 × 5 × 2 × 3 × 3 = 450.

5. The LCM of $\frac{2}{5}, \frac{3}{5}$ and $\frac{1}{2}$ is:

(a) $\frac{1}{10}$ (b) $\frac{6}{10}$ (c) $\frac{3}{25}$ (d) 6

Solution:

(d) $\text{LCM} = \frac{\text{LCM of } 2, 3, 1}{\text{HCF of } 5, 5, 2} = \frac{6}{1} = 6$

6. Three bells toll at the interval of 12, 15 and 18 minutes respectively. If all the three bells toll together at 8 a.m., when will they toll together again?

(a) 9 a.m. (b) 10 a.m. (c) 11 a.m. (d) 1 p.m.

Solution:

(c) 12 = 3 × 4

15 = 3 × 5 (3 is already included in 3 × 4)

∴ LCM of 12 and 15 is 3 × 4 × 5.

18 = 2 × 3 × 3 (2 × 3 is already included in 3 × 4 × 5)

∴ LCM of 12, 15 and 18 is 3 × 4 × 5 × 3 = 180 minutes = 3 hours.

∴ The bells will toll together 3 hours after 8 a.m. i.e. at 11 a.m.

7. Three persons A, B and C can complete one round of a circular track in 6, 15, and 20 minutes respectively. If they start together at 9 a.m. from a certain point, at what time will they be together again at the starting point?

(a) 10 a.m. (b) 10.30 a.m. (c) 11 a.m. (d) 11.30 a.m.

Solution:

(a) 6 = 2 × 3

15 = 3 × 5 (3 is already included in 2 × 3)

∴ LCM of 6 and 15 is 2 × 3 × 5.

20 = 2 × 2 × 5 (2 × 5 is already included in 2 × 3 × 5)

∴ LCM of 6, 15 and 20 is 2 × 3 × 5 × 2 = 60 minutes = 1 hour.

∴ They will be again at starting point 1 hour after 9 a.m. i.e. at 10 a.m.

8. If product of two numbers is 750 and their LCM is 150, what is their HCF?

(a) 5 (b) 50 (c) 55 (d) 100

Solution:

(a) Let HCF of the numbers is x.

Then 150 × x = 750

$\therefore x = \frac{750}{150} = 5$

9. LCM of two numbers is 432 and their HCF is 72. If one number is 144, find the other number.

(a) 214 (b) 215 (c) 216 (d) 218

Solution:

(c) Let the second number is x.

Then 144 × x = 432 × 72

$$\therefore x = \frac{432 \times 72}{144} = 216$$

Trick:

If we compare only the unit's digit:
Unit's digit in the product of HCF and LCM is 2 × 2 = 4.
One number has 4 at the unit's place.
Now, out of the given options only 4 × 6 has 4 in unit's place.
∴ 216 is the required number.

10. What is the least number which when divided by 12, 15, 20 and 25 leaves remainder 7 in each case?

(a) 300 (b) 293 (c) 307 (d) 600

Solution:

(c) 12 = 3 × 4
15 = 3 × 5 (But 3 is already included in 3 × 4)
∴ LCM of 12 and 15 is 3 × 4 × 5.
20 = 4 × 5 (4 × 5 is already included in 3 × 4 × 5)
∴ LCM of 12, 15, 20 is 3 × 4 × 5.
25 = 5 × 5 (5 is already included in 3 × 4 × 5)
∴ LCM of 12, 15, 20, 25 is 3 × 4 × 5 × 5 = 300.
∴ The required number is 300 + 7 = 307.

11. What is the least number which when increased by 5 is exactly divisible by 4, 5, 6 and 7?

(a) 415 (b) 425 (c) 420 (d) 845

Solution:

(a) LCM of 4 and 5 = 4 × 5
LCM of 4, 5 and 6 = 4 × 5 × 3
(Note: 6 = 2 × 3 but 2 is already included in 4 × 5)
LCM of 4, 5, 6 and 7 = 4 × 5 × 3 × 7 = 420
∴ The required number is 420 – 5 = 415.

12. Which is the greatest three-digit number which when divided by 3, 4, 5 and 6 leaves remainder 2 in each case?

(a) 060 (b) 062 (c) 960 (d) 962

Solution:

(d) LCM of 3, 4, 5 and 6 is 3 × 4 × 5 = 60.
Greatest 3-digit number is 999.
On dividing 999 by 60, remainder is 39.
∴ 999 – 39 = 960 is exactly divisible by 60.
∴ 960 + 2 = 962 is the number which when divided by 60 will leave remainder 2.

Trick:

60 and 62, being two-digit numbers can't be the required number.
960 and 962 when reduced by 2 becomes 958 and 960 respectively.
Clearly, 958 is not divisible by 5.
Using Divisibility Test Method, we find 960 is divisible by 3, 4, 5 and 6.

13. Which of the followings is the least square number by which 12, 20 and 25 are exactly divisible:

(a) 100 (b) 300 (c) 900 (d) 3600

Solution:

(c) We start from the smallest number as we are to find the least number.
100 is not divisible by 12.
300 is not a square number.
900 is a square number and is also divisible by 12, 20 and 25.

14. HCF of two different numbers is 24, which one of the following can be their LCM:

(a) 12 (b) 18 (c) 36 (d) 48

Solution:

(d) We know that for any pair of numbers, LCM is always exactly divisible by HCF of the numbers.
Here 48 is the only number which is exactly divisible by 24.
Note: The possible numbers are 24 and 48.
Note: LCM cannot be 24 as the numbers are different.

15. LCM of 3 numbers is 120. Which one of the following cannot be their HCF:

(a) 8 (b) 10 (c) 12 (d) 25

Solution:

(d) We know that LCM is always exactly divisible by HCF.
But, 25 out of the given options does not exactly divide 120, the LCM.

16. Find the greatest number that divides 132 and 77 leaving remainder 2 in each case.

(a) 5 (b) 11 (c) 22 (d) 25

Solution:

(a) Remainder is 2 on dividing 132 by the divisor.
∴ 132 – 2 = 130 is exactly divisible by that divisor.
Similarly, 77 – 2 = 75 is exactly divisible by the divisor.
∴ The required number is HCF of 130 and 75 i.e. 5.

17. Find the greatest number which divides 127 and 41 leaving remainders 7 and 5 respectively.

(a) 5 (b) 7 (c) 8 (d) 12

Solution:

(d) On dividing 127 by the HCF, the remainder is 7.
∴ 127 – 7 = 120 is exactly divisible by the HCF.
Similarly on dividing 41 by the HCF, the remainder is 5.
∴ 41 – 5 = 36 is exactly divisible by the HCF.
Now, HCF of 120 and 36 is 12.

18. Find the largest number which when divides 98, 120 and 153, leaves the same remainder?

(a) 3 (b) 5 (c) 10 (d) 11

Solution:

(d) If two different numbers when separately divided by a certain divisor leaves same remainder in each case, then difference between the numbers is exactly divisible by the divisor.

∴ Required number is HCF of (120 – 98) and (153 – 120) i.e. HCF of 22 and 33 i.e. 11.

Note: Remainder is 10 in each case i.e. remainder obtained on dividing 98, 120 and 153 by 11.

19. What is the least number which when divided by 8, 12, 15 and 18 leaves remainder 5, 9, 12 and 15 respectively?

(a) 180 (b) 357 (c) 360 (d) 363

Solution:

(b) 8 = 8

Since 12 = 3 × 4 and 4 is already included in 8.

∴ LCM of 8 and 12 is 8 × 3.

Since 15 = 3 × 5, but 3 is already included in 8 × 3.

∴ LCM of 8, 12 and 15 is 8 × 3 × 5.

Since 18 = 2 × 3 × 3, but 2 × 3 is already included in 8 × 3 × 5.

∴ LCM of 8, 12, 15, 18 is 8 × 3 × 5 × 3 = 360.

Remainder in each case is 3 less than the respective divisor.

∴ The required number is = 360 – 3 = 357.

20. Find the least number which when divided by 2, 3, 4, 5 and 6 leaves remainder 1 in each case, but when divided by 7 leaves no remainder.

(a) 60 (b) 293 (c) 300 (d) 301

Solution:

(d) LCM of 2, 3, 4, 5 and 6 is 2 × 3 × 2 × 5 = 60.

Since remainder is 1 in each case.

∴ Required number is (60x + 1).

Given, (60x + 1) is divisible by 7.

On dividing (60x + 1) by 7, remainder is 4x + 1.

(**Note:** 60x = 56x + 4x; and 56x is exactly divisible by 7 for all values of x).

Now, 4x + 1 is divisible by 7, when x = 5.

Hence, required number is 60 × 5 + 1 = 301.

21. Find ratio between LCM and HCF of 28 and 42.

(a) 2 : 3 (b) 6 : 1 (c) 7 : 2 (d) 3 : 2

Solution:

(b) 28 = 2 × 2 × 7

42 = 2 × 3 × 7

LCM of 28 and 42 is 2 × 2 × 7 × 3.

HCF of 28 and 42 is 2 × 7.

∴ LCM : HCF = (2 × 2 × 7 × 3) : (2 × 7) = (2 × 3) : 1 = 6 : 1

22. Ratio of two number is 5 : 7 and their LCM is 350, find the smaller number.

(a) 50 (b) 60 (c) 70 (d) 75

Solution:

(a) Let the numbers are 5x and 7x.

Then LCM = 35x

But actual LCM = 350

∴ 35x = 350

∴ x = 10

∴ The smaller number is 5x = 5 × 10 = 50.

23. A number when divided by a certain divisor leaves remainder 50. When double of that number is divided by the same divisor, the remainder is 8. Find the divisor.

(a) 42 (b) 58 (c) 92 (d) 108

Solution:

(c) When the number is doubled, the remainder should also be doubled.

∴ Remainder should be 50 × 2 = 100.

But, under no circumstances, remainder can exceed the divisor.

∴ Actual remainder is 8 which is reduced by the divisor.

∴ Divisor = 100 – 8 = 92.

24. Two numbers when separately divided by a certain divisor, leave remainders 53 and 46 respectively. When sum of the numbers is divided by the same divisor, the remainder is 34. Find the divisor.

(a) 35 (b) 42 (c) 65 (d) 68

Solution:

(c) Remainder should be 53 + 46 = 99.

But remainder can't exceed the divisor.

∴ Remainder is reduced by the divisor.

∴ Divisor = 99 – 34 = 65.

25. A wire when cut into pieces of 14 metre each, leaves a piece of 11 metre. When another wire of same length is cut into pieces of 15 metre each, a piece of 3 metre is left. The minimum length of the wire is:

(a) 18 metre (b) 25 metre (c) 123 metre (d) 221 metre

Solution:

(c) Difference between the pieces left = 11 – 3 = 8 metre.

Difference in length of two types of pieces = 15 – 14 = 1 metre each.

∴ No. of pieces cut from the wire = $\frac{8}{1} = 8$

∴ Length of the wire = 14 × 8 + 11 = 123 metre

Or, length of the wire = 15 × 8 + 3 = 123 metre

26. HCF of two numbers is 29 and their LCM is 4147. Find sum of the numbers.

(*a*) 576 (*b*) 669 (*c*) 696 (*d*) 741

Solution :

(c) Let the number are $29x$ and $29y$

Then $29x \times 29y = 4147 \times 29$

$$\therefore\ x \times y = \frac{4147 \times 29}{29 \times 29} = 143 = 11 \times 13$$

$\therefore$ Sum of numbers = 29 × 11 + 29 × 13 = 29 × (11 + 13) = 29 × 24 = 696.

27. Product of two numbers is 1440 and their HCF is 12. How many pairs of such numbers are possible?

(a) 2 (b) 3 (c) 4 (d) 6

Solution:

(a) Let the numbers are 12x and 12y.

Then, (12x) × (12y) = 1440

$$\therefore xy = \frac{1440}{12 \times 12} = 10$$

$\therefore$ The possible values of x and y for which xy is 10 are (1, 10), (2, 5).

$\therefore$ Two pairs are possible.

28. LCM and HCF of two different numbers are 288 and 24 respectively. How many such pairs are possible?

(a) 1 (b) 2 (c) 3 (d) 4

Solution:

(b) Let the numbers are 24x and 24y.

Then, (24x) × (24y) = 288 × 24.

$$\therefore xy = \frac{288 \times 24}{24 \times 24} = 12$$

The possible values of x and y for which xy is 12 are (1, 12), (2, 6), (3, 4).

But (2, 6) is not possible as they are not co-prime numbers.

$\therefore$ Two pairs are possible.

Note: If x = 2 and y = 6,

Then the numbers are 24 × 2 = 48 and 24 × 6 = 144.

HCF and LCM of 48 and 144 are 48 and 144 respectively which are different from the given HCF and LCM.

29. The sum of two different numbers is 135 and their HCF is 15. How many such pairs are possible?

(a) 1 (b) 2 (c) 3 (d) 4

Solution:

(c) Let the numbers are 15x and 15y.

Then, (15x) + (15y) = 135.

$$\therefore x + y = \frac{135}{15} = 9$$

The possible values of x and y for which (x + y) = 9 are (1, 8), (2, 7), (3, 6), (4, 5).

But (3. 6) is not possible as they are not co-prime numbers.

∴ Three pairs are possible.

30. The LCM of two numbers is 15 times their HCF. The sum of HCF and LCM is 368. If one number is 69, find the other number.

(*a*) 23 (*b*) 92 (*c*) 115 (*d*) 199

Solution :

(*c*) $\text{HCF} = \dfrac{368}{15+1} = 23$

One number = 23 × 3 = 69

∴ Other number $= 23 \times \dfrac{15}{3} = 115$

31. Four prime numbers are written in ascending order. The product of the first three numbers is 385 and that of the last three numbers is 1001. What is the largest number ?

(*a*) 7 (*b*) 11 (*c*) 13 (*d*) 17

Solution :

(*c*) HCF of 385 and 1001 is 77

∴ The largest number is 1001 ÷ 77 = 13.

Chapter 7

BODMAS

If we have to perform a series of mathematical operations, they should be performed in the following order of preference :

B : Bracket:

If more than one type of brackets are found, they are operated in the following order:

First: $(\overline{\ })$ line or bar bracket

Second: ( ) small bracket

Third: { } Mid bracket

Fourth [ ] Large Bracket

O : Of ('Of' means multiplication but is operated before division)

D : Divide

M : Multiply

A : Add

S : Subtract

Note: This rule is know as 'Rule of BODMAS'

SOLVED EXERCISE

1. $15 - 15 \div 15 \times 6 = ?$

(a) 6 (b) 84 (c) 9 (d) 0

Solution:

(c) $15 - 15 \div 15 \times 6$

$$= 15 - 15 \times \frac{1}{15} \times 6$$

$$= 15 - 1 \times 6$$

$$= 15 - 6 = 9$$

2. $5 \times 16 \div 8 \times 2 = ?$

(a) 5 (b) 6 (c) 20 (d) 30

Solution:

(c) $5 \times 16 \div 8 \times 2$

$$= 5 \times 16 \times \frac{1}{8} \times 2 = 20$$

3. $5 \times 16 \div 8$ of $2 = ?$

(a) 5 (b) 6 (c) 20 (d) 30

Solution:

(a) $5 \times 16 \div 8$ of 2

$$= 5 \times 16 \div 16$$

$= 5 \times 16 \times \frac{1}{16} = 5$

4. $4 \times 8 \div 2 \text{ of } 2 + (15 - 3) = ?$

(a) 12 (b) 20 (c) 40 (d) 44

Solution:

(b) $4 \times 8 \div 2 \text{ of } 2 + (15 - 3)$

$= 4 \times 8 \div 2 \text{ of } 2 + 12$

$= 4 \times 8 \div 4 + 12$

$= 4 \times 8 \times \frac{1}{4} + 12$

$= 8 + 12 = 20$

5. $\frac{8 + 2 \text{ of } 5 - 2}{8 - 2 \text{ of } (5 - 2)} = ?$

(a) 2 (b) 6 (c) 8 (d) 24

Solution:

(c) $\frac{8 + 2 \text{ of } 5 - 2}{8 - 2 \text{ of } (5 - 2)} = \frac{8 + 10 - 2}{8 - 2 \text{ of } 3} = \frac{18 - 2}{8 - 6} = \frac{16}{2} = 8$

6. $\left(\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}\right) \div \frac{2}{3} = ?$

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) 4 (d) 5

Solution:

(d) $\left(\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3}\right) \div \frac{2}{3} = 5 \times \frac{2}{3} \div \frac{2}{3} = 5$

7. $1 + \frac{1}{1 + \frac{1}{4}} = ?$

(a) 1 (b) $\frac{9}{5}$ (c) $\frac{5}{4}$ (d) $\frac{9}{4}$

Solution:

(b) $1 + \frac{1}{1 + \frac{1}{4}} = 1 + \frac{1}{\frac{5}{4}} = 1 + \frac{4}{5} = \frac{9}{5}$

8. $\frac{9 \times 16 - 9}{6 \times 11 - 6} = ?$

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{3}{2}$ (d) $\frac{9}{4}$

Solution:

(d) $\frac{9 \times 16 - 9}{6 \times 11 - 6} = \frac{9 \times (16 - 1)}{6 \times (11 - 1)} = \frac{9 \times 15}{6 \times 10} = \frac{9}{4}$

9. $\dfrac{0.048 \times 0.35}{0.006 \times 0.014} = ?$

(a) 2 (b) 20 (c) 200 (d) 2000

Solution:

(c) $\dfrac{0.048 \times 0.35}{0.006 \times 0.014} = \dfrac{48 \times 35 \times 1000 \times 1000}{1000 \times 100 \times 6 \times 14} = 200$

Alternative Method:

First, we solve the problem ignoring the decimal.

$\dfrac{48 \times 35}{6 \times 14} = 20$

Now, we have to decide the place of decimal.

In this question, numerator has $3 + 2 = 5$ digits after the decimal point and denominator has $3 + 3 = 6$ digits after the decimal point.

Since, the denominator has 1 decimal more than the numerator, the result obtained above is to be multiplied by 10.

∴ The answer is $20 \times 10 = 200$.

Chapter 8

SERIES

In a series each term has a definite relation with the next term. In this chapter, we are going to discuss Arithmetic Progression and Geometric Progression.

Arithmetic Progression (A.P.): A constant number is added to each term to get the next term. Hence, difference between each pair of two adjacent terms is same.

$\therefore$ A.P. = a, a + d, a + 2d, a + 3d, ………………………., a + (n – 1)d

n^{th} term = [a + (n – 1)d]

Where 'a' is the first term of the series and 'd' is the common difference between two adjacent terms.

In an A.P., sum of first 'n' terms is

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[a + \{a + (n-1)d\}]$$

$\therefore S_n = \text{Number of terms} \times \frac{\text{First term + Last term}}{2}$

i.e. Number of terms × Average of first and last term.

Geometric Progression (G.P.) : Each term is multiplied by a constant number to get the next term.

G.P. = b, br, br^2, br^3, br^4, ………………..

Note: n^{th} term is $b(r^{n-1})$

Where, 'b' is the first term and every term is multiplied by a constant number 'r' to get the next term.

Sum of first 'n' terms in a Geometric Progression is

$$S_n = \frac{b(1-r^n)}{1-r}$$

Sum of infinite series in a G.P. is

$$S_\infty = \frac{b}{1-r} \quad [\text{If} \mid r \mid < 1]$$

MORE SERIES

1. $1+2+3+4+\ldots\ldots\ldots\ldots\ldots\ldots+n = \frac{n(n+1)}{2}$

2. $1+3+5+7+\ldots\ldots\ldots\ldots\ldots\ldots+(2n-1) = n^2 = \left(\frac{\text{Last Number} + 1}{2}\right)^2$

3. $2+4+6+8+\ldots\ldots\ldots\ldots+2n = n\times(n+1)$

$$= \left(\frac{\text{Last Number}}{2}\right)\times\left(\frac{\text{Last number}}{2}+1\right)$$

4. $1+4+7+10+\ldots\ldots\ldots\ldots+(3n-2) = \dfrac{n(3n-1)}{2}$

5. $1^2+2^2+3^2+\ldots\ldots\ldots\ldots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$

6. $1^2+3^2+5^2+\ldots\ldots\ldots\ldots+(2n-1)^2 = \dfrac{n(2n-1)(2n+1)}{3}$

7. $1^3+2^3+3^3+\ldots\ldots\ldots\ldots+n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

8. $\dfrac{1}{2^1}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+\ldots\ldots\ldots\ldots+\dfrac{1}{2^n} = 1-\dfrac{1}{2^n}$

9. $(1\times 2)+(2\times 3)+(3\times 4)+\ldots\ldots\ldots\ldots+n(n+1) = \dfrac{n(n+1)\times(n+2)}{3}$

10. $\dfrac{1}{1\times 2}+\dfrac{1}{2\times 3}+\dfrac{1}{3\times 4}+\ldots\ldots\ldots\ldots+\dfrac{1}{n(n+1)} = \dfrac{n}{(n+1)}$

11. $2^0+2^1+2^2+2^3+2^4+\ldots\ldots\ldots\ldots+2^n = 2^{n+1}-1$

SOLVED EXERCISE

1. If $1+2+3+\ldots\ldots+10 = 55$, then, $11+12+13+\ldots\ldots+20 = ?$

(a) 145 (b) 155 (c) 210 (d) 255

Solution:

(b) Here, number of terms is 10.

$$\therefore \text{ Sum of the series} = 10\times\frac{11+20}{2} = 10\times\frac{31}{2} = 155$$

Trick:

$11+12+13+\ldots\ldots\ldots+20$
$= (10+1)+(10+2)+(10+3)\ldots\ldots\ldots+(10+10)$
$= 10\times 10+(1+2+3+\ldots\ldots\ldots+10)$
$= 100+55 = 155$

2. Find the value of $21+22+23+\ldots\ldots\ldots+60$, if $1+2+3+\ldots\ldots+40 = 820$.

(a) 1620 (b) 1630 (c) 1830 (d) 3660

Solution:

(a) Here, number of terms is 40.

$$\therefore \text{ Sum of the series} = 40\times\frac{21+60}{2} = 40\times\frac{81}{2} = 1620$$

Trick:

$$21 + 22 + 23 + \ldots\ldots\ldots\ldots + 60$$
$$=(20+1)+(20+2)+(20+3)\ldots\ldots\ldots\ldots\ldots+(20+40)$$
$$= 20 \times 40 + (1 + 2 + 3 + \ldots\ldots\ldots\ldots\ldots + 40)$$
$$= 800 + 820 = 1620$$

3. $99\frac{1}{7}+99\frac{2}{7}+99\frac{3}{7}+\ldots\ldots+99\frac{6}{7}=?$

(*a*) 279 (*b*) 297 (*c*) 579 (*d*) 597

Solution :

(*d*) $99\times6+\left(\frac{1}{7}+\frac{2}{7}+\frac{3}{7}+\ldots\ldots\ldots+\frac{6}{7}\right) = 594+\frac{6\times7}{2\times7}=594+3=597$

4. Find the sum of even numbers between 11 and 31.

(*a*) 200 (*b*) 210 (*c*) 242 (*d*) 262

Solution :

(*b*) Sum of even numbers between 11 and 31

$$=\frac{30}{2}\times\left(\frac{30}{2}+1\right)-\frac{10}{2}\times\left(\frac{10}{2}+1\right)=15\times16-5\times6=210$$

5. Find the sum of odd numbers between 20 and 50.

(*a*) 400 (*b*) 500 (*c*) 525 (*d*) 625

Solution :

(*c*) Sum of even numbers between 20 and 50

$$=\left(\frac{49+1}{2}\right)^2-\left(\frac{19+1}{2}\right)^2=25^2-10^2=525$$

6. If $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots + 10^3 = 3025$,
find the value of $2^3 + 4^3 + 6^3 + \ldots\ldots\ldots + 20^3$.

(*a*) 6050 (*b*) 9075 (*c*) 12100 (*d*) 24200

Solution :

(*d*) $(2^3 + 4^3 + 6^3 + \ldots\ldots\ldots\ldots + 20^3)$
$= 2^3 \times (1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots + 10^3)$
$= 8 \times 3025 = 24200$

7. Find the value of $5^3 + 6^3 + \ldots\ldots\ldots\ldots + 10^3$.

(a) 2295 (b) 2425 (c) 2495 (d) 2925

Solution:

(d) Sum $=(1^3+2^3+3^3+\ldots\ldots\ldots\ldots+10^3)-(1^3+2^3+3^3+4^3)$

$$=\left[\frac{10\times11}{2}\right]^2-\left[\frac{4\times5}{2}\right]^2$$

$$=55^2-10^2=3025-100=2925$$

8. Find the sum of infinite series:

$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

(a) 1 (b) 2 (c) $\frac{3}{4}$ (d) $\frac{5}{4}$

Solution:

(a) Given series is a Geometric Progression with $b = \frac{1}{2}$ and $r = \frac{1}{2}$

$$\text{Sum} = \frac{b}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

9. The sum of all natural numbers between 100 and 200 which are fully divisible by 3 is:

(a) 4900 (b) 4950 (c) 4980 (d) 5000

Solution:

(b) Since $100 = 33 \times 3 + 1$

$\therefore$ Terms (divisible by 3) up to 100 = 33

And $200 = 66 \times 3 + 2$

$\therefore$ Terms (divisible by 3) up to 200 = 66

$\therefore$ Total terms (divisible by 3) between 100 and 200 are $66 - 33 = 33$.

Average = Mid-term = 150.

$\therefore$ Sum of 33 terms is $150 \times 33 = 4950$

Note: Sum of the terms must be divisible by 3 as each of the term is divisible by 3.

$\therefore$ 4950 is divisible by 3.

10. The value of $1-\frac{1}{20}+\frac{1}{20^2}-\frac{1}{20^3}+\dots\dots\dots$ correct up to 5 places of decimals is :

(*a*) 1.0523 (*b*) 9.5241 (*c*) 0.95239 (*d*) 9.5238

Solution :

(*d*) Given series is a Geometric Progression with $b = 1$ and $r = -\frac{1}{20}$

$$\text{Sum} = \frac{b}{1-r} - \frac{1}{1-\left(-\frac{1}{20}\right)} = \frac{20}{21} = 0.95238$$

11. If $1^2+2^2+3^2+\dots\dots\dots\dots+x^2 = \frac{x(x+1)(2x+1)}{6}$

Then $1^2 + 3^2 + 5^2 + \dots\dots\dots\dots + 19^2 = ?$

(*a*) 665 (*b*) 1330 (*c*) 2470 (*d*) 2660

Solution :

(*b*) If $1^2+2^2+3^2+\dots\dots\dots\dots+19^2 = \frac{19\times20\times39}{6} = 2470$

$1^2 + 3^2 + 5^2 + \dots\dots\dots\dots + 19^2$

$= (1^2 + 2^2 + 3^2 + \dots\dots\dots\dots + 19^2) - (2^2 + 4^2 + 6^2 + \dots\dots\dots\dots + 18^2)$

$= 2470 - 4 \times (1^2 + 2^2 + 3^2 + \dots\dots\dots\dots + 9^2)$

$= 2470 - 4 \times \frac{9\times10\times19}{6}$

$= 2470 - 4 \times 285 = 2470 - 1140 = 1330$

Chapter 9

SQUARE NUMBER AND ITS PROPERTIES

Square of a number is equal to product of the number by itself. Square of 'x' is denoted by x^2. Therefore, a number is called a square number if the number can be written as product of a natural number by itself.

Example:

Square of 7 = $7^2 = 7 \times 7 = 49$

Square of 18 = $18^2 = 18 \times 18 = 324$

PROPERTIES OF SQUARE NUMBERS

Unit's digit of a square number can only be one of the following numbers:

1, 4, 5, 6, 9, 0 (Even number of 0s)

This implies a square number can never have the following numbers at unit's place:

2, 3, 7, 8

Square numbers ending in an even number is always divisible by 4.

Proof:

$(2x)^2 = 4x^2$

Note: If a square number ends in an even number, it is always square of an even number.

Square number ending in an odd number when reduced by 1 is exactly divisible by 8.

Proof:

Let the number is $(2x + 1)$.

Then $(2x + 1)^2 - 1 = (4x^2 + 4x + 1) - 1 = 4x^2 + 4x$

$= 4(x^2 + x) = 4(x)(x + 1)$

Either 'x' or $(x + 1)$ is an even number.

Hence, $4(x)(x + 1)$ is divisible by $4 \times 2 = 8$ for all values of x.

Note: If a square number ends in an odd number, it is always square of an odd number.

Square number ending in 5, always ends in 25.

Proof:

A number ending in 5 can be written as $(10x + 5)$.

Now, $(10x + 5)^2 = 100x^2 + 100x + 25 = 100(x^2 + x) + 25$

If a square number is multiple of 3, it is divisible by 9 also.

Proof:

$(3x)^2 = 9x^2$

If a square number is not a multiple of 3, it exceeds multiple of 3 by 1.

Proof:

If a number is not a multiple of 3, it may be 1 or 2 more than the multiple of 3.

Let the numbers are $(3x + 1)$ and $(3x + 2)$.

$(3x + 1)^2 = 9x^2 + 6x + 1 = (9x^2 + 6x) + 1 = 3 \times (3x^2 + 2x) + 1$

$(3x + 2)^2 = 9x^2 + 12x + 4 = (9x^2 + 12x + 3) + 1 = 3 \times (3x^2 + 4x + 1) + 1$

Clearly, square of both numbers exceed multiple of 3 by 1.

UNIT'S AND TEN'S DIGITS OF A SQUARE NUMBER

Unit's digit	*unit's digit of square number*	*Preceded by*
1 or 9	1	An even number or zero
2 or 8	4	An even number or zero
3 or 7	9	An even number or zero
5	25 (last two digits)	An even number or zero
4 or 6	6	An odd number.

REMEMBER THE FOLLOWINGS

$3^2 + 4^2 = 5^2$

$6^2 + 8^2 = 10^2$

$5^2 + 12^2 = 13^2$

$8^2 + 15^2 = 17^2$

$7^2 + 24^2 = 25^2$

$10^2 + 24^2 = 26^2$

$16^2 + 30^2 = 34^2$

$20^2 + 21^2 = 29^2$

UNIT'S DIGIT OF A NUMBER AND ITS CUBE

0 – 0 1 – 1 4 – 4 5 – 5 6 – 6 9 – 9

i.e. same digit in unit's place

2 – 8 3 – 7 7 – 3 8 – 2

i.e. total of unit's digits of the number and its cube's is 10.

Note: Numbers in the second part are the numbers which are not possible at unit's place in a square number.

MORE RULES OF SQUARE NUMBERS

Difference between squares of two even numbers is always divisible by 4.

Proof:

Let the even numbers are 2m and 2n.

Then, difference between their squares

$= (2m)^2 - (2n)^2$

$= 4 (m^2 - n^2)$

$= 4 \times (m + n) \times (m - n)$

Note : Square of an even number is always divisible by 4. Therefore, the difference between any two such numbers is also always divisible by 4.

Difference between squares of any two odd numbers is always divisible by 8.

Proof:

Let the odd numbers are $(2m - 1)$ and $(2n - 1)$.

Then, difference between their squares is

$(2m - 1)^2 - (2n - 1)^2$

$= (4m^2 - 4m + 1) - (4n^2 - 4n + 1)$

$= 4m^2 - 4m + 1 - 4n^2 + 4n - 1$

$= 4 (m^2 - m - n^2 + n)$

$= 4 [(m^2 - n^2) - (m - n)]$

$= 4 [(m - n) (m + n) - (m - n)]$

$= 4 [(m - n)\{(m + n) - 1\}]$

Either, $(m - n)$ or $\{(m + n) - 1\}$ is an even number.

$\therefore$ The result is divisible by $4 \times 2 = 8$.

Product of two consecutive odd or even numbers increased by unity is equal to square of average of the numbers.

Proof:

Let the first number is $(x - 1)$.

Then the next number is $(x - 1) + 2 = x + 1$.

Product of two numbers when increased by 1 is

$(x - 1) \times (x + 1) + 1$

$= x^2 - 1 + 1 = x^2$

Note: Average of the numbers i.e. average of $(x - 1)$ and $(x + 1)$ is x.

Examples:

1. $3 \times 5 + 1 = 16 = 4^2$ (Note: 4 is average of 3 and 5)
2. $4 \times 6 + 1 = 25 = 5^2$ (Note: 5 is average of 4 and 6)
3. $7 \times 9 + 1 = 64 = 8^2$ (Note: 8 is average of 7 and 9)

Sum of cubes of any three consecutive numbers is divisible by the sum of the given numbers and by 9.

Proof:

Let the numbers are $(n - 1)$, (n) and $(n + 1)$, then:

Sum of the cubes of the numbers is

$(n - 1)^3 + (n)^3 + (n + 1)^3$

$= (n^3 - 3n^2 + 3n - 1) + n^3 + (n^3 + 3n^2 + 3n + 1)$

$= 3n^3 + 6n$

$= 3n(n^2 + 2)$

(a) If 'n' is multiple of 3, then 3n is divisible by 3 × 3 = 9

If 'n' is not a multiple of 3, then n^2 exceeds multiple of 3 by 1, therefore, $(n^2 + 2)$ is divisible by 3.

(b) Sum of the numbers = $(n - 1) + (n) + (n + 1) = 3n$.

Clearly, $3n(n^2 + 2)$ is always divisible by 3n.

Sum of square of 3 consecutive numbers is $3n^2 + 2$.
Sum of square of 5 consecutive numbers is $5n^2 + 10$.
Where, n is the middle number.

SOLVED EXERCISE

1. What should be added to 1752 to make it a perfect square number?

(a) 6 (b) 10 (c) 12 (d) 18

Solution:

(c) $41^2 = 1681$, which is smaller than 1752.
And, $42^2 = 1764$, which is slightly bigger than 1752.
∴ Required number is 1764 – 1752 = 12.

Trick:

We take all options one by one, concentrating on unit's digit.
6: Unit's digit is 2 + 6 = 8 (8 is not possible at unit's place).
10: Unit's digit is 2 + 0 = 2 (2 is not possible at unit's place).
12: Unit's digit is 2 + 2 = 4 (4 is possible at unit's digit).
18: Last two digits are 52 + 18 = 70 (Single '0' is not possible at the end of a square number).
∴ 12 is the only possible number out of the given options.

2. What should be added to 16259 to make it a perfect square number?

(a) 107 (b) 125 (c) 212 (d) 423

Solution:

(b) $127^2 = 16129$, which is smaller than 16259.
And, $128^2 = 16384$, which is bigger than 16259.
∴ Required number is 16384 – 16259 = 125.

Trick:

We check the given options one by one:
423: Unit's digit is 9 + 3 = 2 (not possible).
107: Unit's digit is 9 + 7 = 6 (possible)
But 59 + 07 = 66 (The number is an even number but not divisible by 4 and hence not possible).
212: Unit's digit is 9 + 2 = 1 (possible).
Last two digits are 59 + 12 = 71 i.e. 1 at unit's place, preceded by an odd number it can be even number or '0' only and hence not possible.
125: Unit's digit is 9 + 5 = 4 (possible). Last two digits are 59 + 25 = 84.
84 is an even number which is divisible by 4. It being the only option left, is the answer.

3. What should be subtracted from 83908 to make it a perfect square number?

(a) 103 (b) 387 (c) 404 (d) 501

Solution:

(b) $290^2 = 84100$, which is bigger than 83908.

And, $289^2 = 83521$, which is smaller than 83908

∴ Required number is 83908 – 83521 = 387

Trick:

We check the given options one by one:

501: Unit's digit is 8 – 1 = 7 (not possible).

103: Unit's digit is 8 – 3 = 5 (possible). Ten's digit must be 2.
Here, 08 – 03 = 05 (Not possible).

404: Unit's digit is 8 – 4 = 4.
Last two digits = 08 – 04 = 04 (Possible as the number is even and is divisible by 4).
Sum of digits of (83908 – 404) = 1 – 8 = 2 (ignoring 9).
∴ The number when reduced by 1 is not divisible by 3. (Not possible).

387: 83908 – 387 = 83521.
Unit's digit is 1 (Possible).
Ten's digit is 2, an even number (Possible).
Sum of digits of 83521 is 1 i.e. the number which when divided by 3 will leave remainder 1. (Possible)
∴ 387 being the only possibility, is the answer.

4. The sum of the squares of three consecutive natural numbers is 365. What is the middie number?

(a) 10 (b) 11 (c) 12 (d) 13

Solution:

(b) Let, the three consecutive numbers are (n – 1), (n), (n + 1).

Then, sum of their squares = $3n^2 + 2 = 365$.

$$\therefore 3n^2 = 365 - 2 = 363$$

$$\therefore n^2 = \frac{363}{3} = 121$$

$$\therefore n = \sqrt{121} = 11 \text{ (middle-number)}$$

5. Sum of the squares of three consecutive natural numbers is 434. Find the smallest number of the three.

(a) 11 (b) 12 (c) 15 (d) 21

Solution:

(a) Let the numbers are (n – 1), (n), (n + 1).

Then sum of their squares = $3n^2 + 2 = 434$

$$\therefore 3n^2 = 434 - 2 = 432$$

$$\therefore n^2 = \frac{432}{3} = 144$$

$$\therefore n = \sqrt{144} = 12 \text{ (middle-number)}$$

∴ The smallest number is 12 – 1 = 11.

Chapter 10

ALGEBRA

This chapter deals with algebraic equations and their application in simplifying the difficult problems. Algebraic formulae are widely used in solving the problems on various other topics also, therefore, try to remember these formulae.

ALGEBRAIC FORMULAE

$a(b + c) = ab + ac$

$a(b - c) = ab - ac$

$(x + y)^2 = x^2 + y^2 + 2xy$

$(x - y)^2 = x^2 + y^2 - 2xy$

$(x - y)(x + y) = x^2 - y^2$

$(x + y)^3 = x^3 + y^3 + 3xy(x + y) = x^3 + y^3 + 3x^2y + 3xy^2$

$(x - y)^3 = x^3 - y^3 - 3xy(x - y) = x^3 - y^3 - 3x^2y + 3xy^2$

$(x + y)^3 + (x - y)^3 = 2x^3 + 6xy^2$

$(x + y)^3 - (x - y)^3 = 2y^3 + 6x^2y$

$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

$(x + y)^2 - (x - y)^2 = 4xy$

$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$\therefore x^2 + y^2 = \frac{1}{2}[(x + y)^2 + (x - y)^2]$

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2xz$

$= x^2 + y^2 + z^2 + 2(xy + yz + xz)$

if $(x + y + z) = 0$, then $x^2 + y^2 + z^2 = -2(xy + yz + xz)$

$(x^2 + x + 1)(x^2 - x + 1) = (x^4 + x^2 + 1)$

$(x + y)(y + z)(x + z) = x^2(y + z) + y^2(x + z) + z^2(x + y) + 2xyz$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z) \times (x^2 + y^2 + z^2 - xy - yz - xy)$

$= \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2(z - x)^2]$

$(x + y + z)^3 = x^3 + y^3 + z^3 + 3(x + y)(y + z)(x + z)$

$\mathbf{x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)}$

$$\text{if } (x+y+z)=0, \text{ then } x^3+y^3+z^3=3xyz$$

$$x^4+y^4+x^2y^2=(x^2+y^2-xy)(x^2+y^2+xy)$$

$$(x+y)(y+z)(x+z)=x^2(y+z)+y^2(x+z)+z^2(x+y)+2xyz$$

$$(x+a)(x+b)=x^2+(a+b)x+ab$$

$$(x+a)(x+b)(x+c)=x^3+x^2(a+b+c)+(ab+bc+ca)x+abc$$

$$(x-a)(x-b)(x-c)=x^3-x^2(a+b+c)+(ab+bc+ca)x-abc$$

$$\left(x+\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}+2$$

$$\left(x-\frac{1}{x}\right)^2=x^2+\frac{1}{x^2}-2$$

$$\left(x-\frac{1}{x}\right)^3=x^3-\frac{1}{x^3}-3\left(x-\frac{1}{x}\right)$$

$$\left(x+\frac{1}{x}\right)^3=x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)$$

$$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x)$$

$$(x+y+z)(xy+yz+xz)=x^2(y+z)+y^2(z+x)+z^2(x+y)+3xyz$$

RULES

1. If $x+\frac{1}{x}=a$, then $x^2+\frac{1}{x^2}=a^2-2$

Proof:

$$x+\frac{1}{x}=a$$

$$\therefore \left(x+\frac{1}{x}\right)^2=a^2$$

$$\therefore x^2+\frac{1}{x^2}+2=a^2$$

$$\therefore x^2+\frac{1}{x^2}=a^2-2$$

2. If $x-\frac{1}{x}=a$, then $x^2+\frac{1}{x^2}=a^2+2$

Proof:

$$x-\frac{1}{x}=a$$

$$\therefore \left(x-\frac{1}{x}\right)^2 = a^2$$

$$\therefore x^2+\frac{1}{x^2}-2 = a^2$$

$$\therefore x^2+\frac{1}{x^2} = a^2+2$$

3. If $x-\frac{1}{x}=a$, then $x^3-\frac{1}{x^3}=a^3+3a$

Proof:

$$x-\frac{1}{x}=a$$

$$\therefore \left(x-\frac{1}{x}\right)^3 = a^3$$

$$\therefore x^3-\frac{1}{x^3}-3(x)\left(\frac{1}{x}\right)\left(x-\frac{1}{x}\right) = a^3$$

$$\therefore x^3-\frac{1}{x^3} = a^3+3\left(x-\frac{1}{x}\right) = a^3+3a$$

4. If $x+\frac{1}{x}=a$, then $x^3+\frac{1}{x^3}=a^3-3a$.

Proof:

$$x+\frac{1}{x}=a$$

$$\therefore \left(x+\frac{1}{x}\right)^3 = a^3$$

$$\therefore x^3+\frac{1}{x^3}+3x\left(\frac{1}{x}\right)\left(x+\frac{1}{x}\right) = a^3$$

$$\therefore x^3+\frac{1}{x^3} = a^3-3\left(x+\frac{1}{x}\right) = a^3-3a$$

5. (x − a) is a factor of f (x) is f (a) = 0
(x + a) is a factor of f (x) if f (–a) = 0

(ax − b) is a factor of f (x) if $f\left(\frac{b}{a}\right)=0$

(ax + b) is a factor of f (x) if $f\left(-\frac{b}{a}\right)=0$

If $f(x) = 0$ when $x = a$ and when $x = b$ then $f(x)$ is exactly divisible by $(x - a) \times (x - b)$

6. If $f(x)$ is divided by $(x$ − a), the remainder is $f(a)$.
If $f(x)$ is divided by $(x$ + a), the remainder is $f(-a)$.

If $f(x)$ is divided by $(ax - b)$, the remainder is $f\left(\frac{b}{a}\right)$.

If $f(x)$ is divided by $(ax + b)$, the remainder is $f\left(-\frac{b}{a}\right)$.

7. $(x^n - 1)$ is always divisible by $(x - 1)$.

$(x^n - 1)$ is divisible by $(x + 1)$ for even values of 'n'.

$\therefore$ $(x^n - 1)$ is not divisible by $(x + 1)$ for odd values of 'n'.

8. $(x^{ab} - 1)$ is always divisible by $(x^a - 1)$ and $(x^b - 1)$.

$(x^{abc} - 1)$ is always divisible by $(x^a - 1)$, $(x^b - 1)$ and $(x^c - 1)$.

9. $(x^n + 1)$ is divisible by $(x + 1)$ for all odd values of 'n'.

$(x^n + 1)$ is not divisible by $(x + 1)$ for all even values of 'n'.

$(x^n + 1)$ is not divisible by $(x - 1)$ for all values of 'n'.

10. $(x^n - y^n)$ is always divisible by $(x - y)$.

$(x^n - y^n)$ is divisible by $(x + y)$ for even values of 'n'.

$\therefore$ $(x^n - y^n)$ is not divisible by $(x + y)$ for odd values of 'n'.

11. $(x^n + y^n)$ is divisible by $(x + y)$ for odd values of 'n'.

$(x^n + y^n)$ is not divisible by $(x + y)$ for even values of 'n'.

$\therefore$ $(x^n + y^n)$ is not divisible by $(x - y)$ for all values of 'n'.

12. $(x^{2n} - y^{2n})$, is always divisible by $(x^2 - y^2)$.

13. Difference between squares of two consecutive numbers is equal to the sum of the numbers themselves.

Proof:

Let the numbers are 'n' and 'n + 1'.

Difference between their squares = $(n + 1)^2 - n^2 = 2n + 1 = n + (n + 1)$.

Note: The difference is always an odd number.

11. If, sum of two numbers 'a' and 'b' is 'x' and difference between the numbers is 'y'.

Then, $a = \frac{x + y}{2}$

And, $b = \frac{x - y}{2}$

QUADRATIC EQUATIONS

$ax^2 + bx + c = 0$

Finding roots of a quadratic equations :

1. Factor Method :

Let the factor are α and β.

Then $b = \alpha + \beta, ac = \alpha \times \beta$

Example :

$$x^2 + 7x + 12 = 0$$

Solution :

$$x^2 + 7x + 12 = 0$$

$\Rightarrow x^2 + 3x + 4x + 12 = 0$ **(Note : 3 + 4 = 7 and 3 × 4 = 12)**

$\Rightarrow x(x + 3) + 4(x + 3) = 0$

$\Rightarrow (x + 4) \times (x + 3) = 0$

$\Rightarrow x + 4 = 0$ or $x + 3 = 0$

$\Rightarrow x = -4$ or -3

2. Using Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example :

$$x^2 - 7x + 12 = 0$$

Solution :

$$x = \frac{(-7) \pm \sqrt{(-7)^2 - 4 \times 1 \times 12}}{2 \times 1} = \frac{7 \pm 1}{2} = 4 \text{ or } 3$$

SOLVED EXERCISE

1. $19 \times 93 + 19 \times 7 = ?$

(a) 1900 (b) 1993 (c) 2000 (d) 2190

Solution:

(a) $19 \times 93 + 19 \times 7$

$= 19 \times (93 + 7)$

$= 19 \times 100 = 1900$

2. $219 \times 18 - 216 \times 18 = ?$

(a) 54 (b) 72 (c) 7218 (d) 7830

Solution:

(a) $219 \times 18 - 216 \times 18$

$= 18 \times (219 - 216)$

$= 18 \times 3 = 54$

3. $99 \times 28 - 99 \times 18 = ?$

(a) 990 (b) 1100 (c) 999 (d) 1089

Solution:

(a) $99 \times 28 - 99 \times 18$

$= 99 \times (28 - 18) = 99 \times 10 = 990$

4. $996^2 - 995^2 = ?$

(a) 0 (b) 1 (c) 100 (d) 1991

Solution:

(d) We know that, difference between the squares of two consecutive numbers is equal to the sum of the given numbers.

$\therefore 996^2 - 995^2 = 996 + 995 = 1991.$

5. If difference between the squares of two consecutive numbers is 47. Find the smaller number:

(a) 21 (b) 22 (c) 23 (d) 24

Solution:

(c) Let the numbers are x and x + 1.

We know that the difference between square of two consecutive numbers is equal to the sum of the numbers.

$\therefore 2x + 1 = 47$

$\therefore x = \frac{47-1}{2} = 23$

6. $99^2 - 96^2 = ?$

(a) 195 (b) 390 (c) 585 (d) 780

Solution:

(c) We know that, $a^2 - b^2 = (a + b) \times (a - b)$

$\therefore 99^2 - 96^2 = (99 + 96) \times (99 - 96)$

$= 195 \times 3 = 585$

7. $54 \times 54 - 46 \times 46 = ?$

(a) 800 (b) 144 (c) 64 (d) 8

Solution:

(a) $54 \times 54 - 46 \times 46$

$= (54 + 46) \times (54 - 46) = 100 \times 8 = 800.$

Trick:

Unit's digit is 6 in both the cases i.e. 54 × 54 and 46 × 46.

∴ Unit's digit in 54 × 54 – 46 × 46 is 6 – 6 = 0.

∴ (a) is the answer.

8. If $x^3 - 3x^2 + 5x + 1$ is divided by $x - 2$, the remainder is :

(*a*) 0 (*b*) 2 (*c*) 5 (*d*) 7

Solution :

(*d*) $f(2) = 2^3 - 3(2)^2 + 5(2) + 1 = 8 - 12 + 10 + 1 = 7$

9. Sum of two numbers is 20 and difference between the numbers is 4. Find the difference between their squares.

(a) 5 (b) 16 (c) 24 (d) 80

Solution:

(d) Let the numbers are x and y.

Then $x + y = 20$ and $x - y = 4$

$\therefore x^2 - y^2 = (x + y) \times (x - y) = 20 \times 4 = 80$

10. Sum of two numbers is 17 and the difference between their squares is 85. Find the difference between the numbers.

(a) 5 (b) 6 (c) 10 (d) 11

Solution:

(a) Let the numbers are x and y.

Then $x + y = 17$ and $x^2 - y^2 = 85$

$$\therefore x - y = \frac{x^2 - y^2}{x + y} = \frac{85}{17} = 5$$

11. Difference between two numbers is 11 and difference between their squares is 781. Sum of the numbers is:

(a) 71 (b) 770 (c) 792 (d) 8591

Solution:

(a) Let the numbers are x and y.

Then $x - y = 11$ and $x^2 - y^2 = 781$

$$\therefore x + y = \frac{x^2 - y^2}{x - y} = \frac{781}{11} = 71$$

12. If sum of two numbers is 10 and sum of their squares is 58. Find the product of the numbers.

(a) 21 (b) 24 (c) 31 (d) 62

Solution:

(a) $2xy = (x + y)^2 - (x^2 + y^2) = 10^2 - 58 = 100 - 58 = 42$

$$\therefore xy = \frac{42}{2} = 21$$

13. If difference between two numbers is 3 and sum of squares of the numbers is 117. Find the product of the numbers.

(a) 21 (b) 27 (c) 31 (d) 54

Solution:

(d) $2xy = (x^2 + y^2) - (x - y)^2 = 117 - 3^2 = 117 - 9 = 108$

$$\therefore xy = \frac{108}{2} = 54$$

14. If difference between two positive numbers is 3 and sum of square of the numbers is 369, then the sum of the numbers is :

(*a*) 23 (*b*) 27 (*c*) 33 (*d*) 37

Solution :

(*b*) $2xy = (x^2 + y^2) - (x - y)^2 = 369 - 9 = 360$

$(x + y)^2 = (x^2 + y^2) + 2xy = 369 + 360 = 729$

$$\therefore \; x + y = \sqrt{729} = 27$$

15. If sum of two numbers is 25 and difference between them is 11. What is the product of the numbers.

(a) 275 (b) 252 (c) 250 (d) 126

Solution:

(d) We know that, $(x + y)^2 - (x - y)^2 = 4xy$

$\therefore 25^2 - 11^2 = 4xy$

$\therefore 4xy = (25 + 11) \times (25 - 11)$

$\therefore xy = \frac{36 \times 14}{4} = 126$

Trick:

Big number $= \frac{25+11}{2} = 18$

Small number $= \frac{25-11}{2} = 7$

$\therefore$ Multiplication of the numbers $= 18 \times 7 = 126.$

16. $16 \times 16 + 128 + 4 \times 4 = ?$

(a) 20 (b) 100 (c) 400 (d) 500

Solution:

(c) Substituting $x = 16$ and $y = 4$,

The given expression can be written as

$(x^2 + 2xy + y^2)$ $(2xy = 2 \times 16 \times 4 = 128)$

$= (x + y)^2 = (16 + 4)^2 = 400.$

17. Find the value of $\frac{23 \times 23 + 17 \times 17 + 2 \times 23 \times 17}{23 \times 23 - 17 \times 17}$

(a) $\frac{40}{3}$ (b) $\frac{17}{23}$ (c) $\frac{23}{17}$ (d) $\frac{20}{3}$

Solution:

(d) Substituting $x = 23$ and $y = 17$

The given expression can be written as

$$\frac{x^2 + y^2 + 2xy}{x^2 - y^2} = \frac{(x+y)^2}{x^2 - y^2} = \frac{(x+y)(x+y)}{(x+y)(x-y)} = \frac{(x+y)}{(x-y)} = \frac{40}{6} = \frac{20}{3}$$

18. Find the value of $\frac{153 \times 153 \times 153 + 47 \times 47 \times 47}{153 \times 153 - 153 \times 47 + 47 \times 47}$

(a) $\frac{153}{47}$ (b) 106 (c) 200 (d) 7191

Solution:

(c) Substituting $x = 153$ and $y = 47$

The given expression can be written as

$$\frac{x^3 + y^3}{x^2 - xy + y^2} = \frac{(x+y)(x^2 - xy + y^2)}{x^2 - xy + y^2} = x + y$$

$= 153 + 47 = 200$

19. Find the value of $x^3 + y^3 + 3xy$, if $x + y = 1$

(*a*) –1 (*b*) 0 (*c*) 1 (*d*) 3

Solution :

(*c*) $(x + y)^3 = 1^3$

$\therefore\ x^3 + y^3 + 3xy\,(x + y) = 1$

$\therefore\ x^3 + y^3 + 3xy = 1$

20. Find the value of $\dfrac{52 \times 52 \times 52 - 43 \times 43 \times 43}{52 \times 52 + 52 \times 43 + 43 \times 43}$

(a) $\frac{52}{43}$ (b) 2236 (c) 95 (d) 9

Solution:

(d) Substituting x = 52 and y = 43

The given expression can be written as

$$\frac{x^3 - y^3}{x^2 + xy + y^2} = \frac{(x - y)(x^2 + xy + y^2)}{x^2 + xy + y^2} = x - y$$

$= 52 - 43 = 9$

21. Find the value of $\dfrac{0.5 \times 0.5 + 0.1 \times 0.1 - 0.2 \times 0.5}{0.4}$

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.6

Solution:

(c) Substituting a = 0.5 and b = 0.1

The given expression can be written as

$$\frac{a^2 + b^2 - 2ab}{a - b} = \frac{(a - b)^2}{a - b} = a - b$$

$= 0.5 - 0.1 = 0.4$

(**Note:** $2ab = 2 \times 0.1 \times 0.5 = 0.2 \times 0.5$)

22. Find the value of $\dfrac{0.3 \times 0.3 + 0.2 \times 0.2 + 0.3 \times 0.4}{0.5}$

(a) 0.2 (b) 0.3 (c) 0.4 (d) 0.5

Solution:

(d) $\dfrac{a^2 + b^2 + 2ab}{a + b} = \dfrac{(a + b)^2}{a + b} = a + b$

$= 0.3 + 0.2 = 0.5$

(**Note:** $2ab = 2 \times 0.3 \times 0.2 = 0.3 \times 0.4$)

23. Find the value of $15^3 - 6^3 - 9^3 - 5^3$

(*a*) 0 (*b*) –125 (*c*) 2305 (*d*) 2430

Solution:

(*c*) $15^3 - 6^3 - 9^3 - 5^3 = 3 \times 15 \times 6 \times 9 = 2430$ $(\because\ 15 - 6 - 9 = 0)$

$\therefore$ Given Expression = 2430 – 125 = 2305

24. Find the value of $\dfrac{0.8 \times 0.8 \times 0.8 + 0.08 \times 0.08 \times 0.08}{0.2 \times 0.2 \times 0.2 + 0.02 \times 0.02 \times 0.02}$

(a) 2 (b) 4 (c) 12 (d) 64

Solution:

(d) $\dfrac{0.8 \times 0.8 \times 0.8 + 0.08 \times 0.08 \times 0.08}{0.2 \times 0.2 \times 0.2 + 0.02 \times 0.02 \times 0.02}$

$$= \frac{(0.2 \times 4) \times (0.2 \times 4) \times (0.2 \times 4) + (0.02 \times 4) \times (0.02 \times 4) \times (0.02 \times 4)}{0.2 \times 0.2 \times 0.2 + 0.02 \times 0.02 \times 0.02}$$

$$= \frac{4 \times 4 \times 4 \times (0.2 \times 0.2 \times 0.2) + 4 \times 4 \times 4 \times (0.02 \times 0.02 \times 0.02)}{0.2 \times 0.2 \times 0.2 + 0.02 \times 0.02 \times 0.02}$$

$$= \frac{4 \times 4 \times 4 \times (0.2 \times 0.2 \times 0.2 + 0.02 \times 0.02 \times 0.02)}{0.2 \times 0.2 \times 0.2 + 0.02 \times 0.02 \times 0.02}$$

$$= 4 \times 4 \times 4 = 64$$

25. Find the value of $\dfrac{1.49 \times 14.9 - 0.51 \times 5.1}{14.9 - 5.1}$

(*a*) 0.2 (*b*) 2 (*c*) 20 (*d*) 200

Solution :

(*b*) Given Expression $= \dfrac{10 \times (1.49 \times 1.49 - 0.51 \times 0.51)}{10 \times (1.49 - 0.51)} = 1.49 + 0.51 = 2$

26. If $a = 4.965$, $b = 2.343$ and $c = 2.622$, then $a^3 - b^3 - c^2 - 3abc = ?$

(*a*) –1 (*b*) 0 (*c*) 9.93 (*d*) 19.86

Solution :

(*b*) Here $a - b - c = 0$

$\therefore\ a^3 - b^3 - c^2 = 3abc$

$= a^3 - b^3 - c^2 - 3abc = 0.$

27. $\dfrac{8(3.75)^3 + 1}{(7.5)^2 - 6.5} = ?$

(*a*) 3.25 (*b*) 4.25 (*c*) 6.5 (*d*) 8.5

Solution :

(*d*) Given Expression $= \dfrac{2^3 \times (3.75)^3 + 1}{(7.5)^2 - 6.5} = \dfrac{(7.5)^3 + (1)^3}{(7.5)^2 - 6.5}$

$$= \frac{(7.5 + 1) \times [(7.5)^2 - (7.5 \times 1) + (1)^2]}{(7.5)^2 - 6.5} = 7.5 + 1 = 8.5$$

28. Find the value of $\left(\dfrac{5^3 + 5^4 + 5^5 + 5^6 + 5^7}{5^1 + 5^2 + 5^3 + 5^4 + 5^5}\right)^{\frac{1}{2}}$.

(*a*) 5 (*b*) 25 (*c*) 125 (*d*) 625

Solution :

$$(a)\ \left(\frac{5^3+5^4+5^5+5^6+5^7}{5^1+5^2+5^3+5^4+5^5}\right)^{\frac{1}{2}}=\left(\frac{5^2\times(5^1+5^2+5^3+5^4+5^5)}{(5^1+5^2+5^3+5^4+5^5)}\right)^{\frac{1}{2}}=(5^2)^{½}=5.$$

29. Find the value of $\frac{(19+13)^2+(19-13)^2}{19\times19+13\times13}$

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 6

Solution:

(c) We know that $(a+b)^2+(a-b)^2=2\times(a^2+b^2)$

The given expression can be written as

$$=\frac{(a+b)^2+(a-b)^2}{a^2+b^2}$$

$$=\frac{2\times(a^2+b^2)}{a^2+b^2}=2$$

30. $\left(\frac{4}{3}+\frac{3}{4}\right)^2-\left(\frac{4}{3}-\frac{3}{4}\right)^2=?$

(a) 1 (b) 2 (c) 3 (d) 4

Solution:

(d) We know, $(x+y)^2-(x-y)^2=4\,xy$

$\therefore$ The given expression $=4\times\frac{4}{3}\times\frac{3}{4}=4$

31. $(2.5)^3-(1.5)^3=?$

(a) 1 (b) 11.25 (c) 12.25 (d) 12.75

Solution:

(c) $x^3-y^3=(x-y)^3+3xy\,(x-y)$

$\therefore (2.5)^3-(1.5)^3=(1)^3+3\times3.75\times1=1+11.25=12.25$

Hint: $xy=2.5\times1.5=3.75.$

32. $\frac{57\times57+57\times43}{57\times57-57\times43}=?$

(a) $\frac{57}{43}$ (b) $\frac{50}{7}$ (c) 100 (d) 114

Solution:

(b) $\frac{57\times57+57\times43}{57\times57-57\times43}$

$$=\frac{57\times(57+43)}{57\times(57-43)}$$

$$=\frac{57\times100}{57\times14}=\frac{50}{7}$$

33. $\frac{12^3 + 15^3 + 16^3 - 3 \times 12 \times 15 \times 16}{12^2 + 15^2 + 16^2 - 12 \times 15 - 15 \times 16 - 16 \times 12} = ?$

(a) 43 (b) 34 (c) 9 (d) 4

Solution:

(a) The given expression can be written as

$$\frac{a^3 + b^3 + c^3 - 3abc}{a^2 + b^2 + c^2 - ab - bc - ac}$$

$= a + b + c = 12 + 15 + 16 = 43$

34. Find the value of $(x^3 + y^3 + z^3)$, if $(x + y + z) = 0$.

(a) 0 (b) 1 (c) 2xyz (d) 3xyz

Solution:

(d) We know that, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

$= 0 \times (x^2 + y^2 + z^2 - xy - yz - xz) = 0$

$\therefore x^3 + y^3 + z^3 - 3xyz = 0$

$x^3 + y^3 + z^3 = 3xyz$

27. Find the value of $(x^3 + y^3 + z^3 - 3xyz)$, if $(x + y + z) = 9$ and $(xy + yz + xz) = 26$.

(a) 27 (b) 52 (c) 81 (d) 576

Solution:

(a) $(x + y + z) = 9$

$\therefore (x + y + z)^2 = 9^2 = 81$

$\therefore x^2 + y^2 + z^2 + 2(xy + yz + xz) = 81$

$\therefore x^2 + y^2 + z^2 + 2 \times 26 = 81$

$\therefore x^2 + y^2 + z^2 = 81 - 52 = 29$

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)[(x^2 + y^2 + z^2) - (xy + yz + xz)]$

$= 9 \times (29 - 26) = 27.$

36. Find the value of $(x^3 + y^3 + z^3 - 3xyz)$, if $(x + y + z) = 12$ and $x^2 + y^2 + z^2 = 62$.

(*a*) 144 (*b*) 252 (*c*) 256 (*d*) 372

Solution :

(*b*) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + xz) = 144$

Or $2(xy + yz + xz) = 144 - 62 = 82$

Or $(xy + yz + xz) = \frac{1}{2} \times 82 = 41$

Now $(x^3 + y^3 + z^3 - 3xyz) = (x + y + z)[(x^2 + y^2 + z^2) - (xy + yz + xz)]$
$= 12 \times (62 - 41) = 12 \times 21 = 252.$

37. $$\frac{\left(1\frac{1}{4}\right)^4+\left(\frac{4}{5}\right)^4+1}{\left(1\frac{1}{4}\right)^2+\left(\frac{4}{5}\right)^2-1}=?$$

(a) -1 (b) 1 (c) $\frac{41}{20}$ (d) $\frac{1281}{400}$

Solution:

(d) We know, $A^4+B^4+A^2B^2=(A^2+B^2-AB)\times(A^2+B^2+AB)$

$$\therefore \text{Given expression} = \frac{A^4+B^4+A^2B^2}{A^2+B^2-AB}$$

$$=\frac{(A^2+B^2-AB)\times(A^2+B^2+AB)}{A^2+B^2-AB}$$

$$=(A^2+B^2+AB)=\left(\frac{5}{4}\right)^2+\left(\frac{4}{5}\right)^2+1=\frac{25}{16}+\frac{16}{25}+1=\frac{1281}{400}$$

Hint: $a\times b=\frac{5}{4}\times\frac{4}{5}=1$

And $a^2b^2=(ab)^2=(1)^2=1$

38. $$\frac{\left(1\frac{1}{3}\right)^4+\left(1\frac{1}{3}\right)^{-4}+1}{\left(1\frac{1}{3}\right)^2+\left(1\frac{1}{3}\right)^{-2}+1}=?$$

(a) -1 (b) 1 (c) $\frac{193}{144}$ (d) $\frac{481}{144}$

Solution:

(c) $$\frac{\left(1\frac{1}{3}\right)^4+\left(1\frac{1}{3}\right)^{-4}+1}{\left(1\frac{1}{3}\right)^2+\left(1\frac{1}{3}\right)^{-2}+1}=\frac{\left(\frac{4}{3}\right)^4+\left(\frac{3}{4}\right)^4+1}{\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2+1}$$

We know, $A^4+B^4+A^2B^2=(A^2+B^2-AB)\times(A^2+B^2+AB)$

$$\therefore \text{Given expression} = \frac{A^4+B^4+A^2B^2}{A^2+B^2+AB}$$

$$=\frac{(A^2+B^2-AB)\times(A^2+B^2+AB)}{A^2+B^2+AB}$$

$$=(A^2+B^2-AB)=\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2-1=\frac{16}{9}+\frac{9}{16}-1=\frac{193}{144}$$

Hint: $a\times b=\frac{4}{3}\times\frac{3}{4}=1$

And $a^2b^2=(ab)^2=(1)^2=1$

39. If $x + \frac{1}{x} = 8$, then $x^2 + \frac{1}{x^2} = ?$

(a) 62 (b) 64 (c) 66 (d) 72

Solution:

(a) $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$= 8^2 - 2 = 64 - 2 = 62$

40. If $x - \frac{1}{x} = 5$, find the value of $x^2 + \frac{1}{x^2} = ?$

(a) 25 (b) 27 (c) 30 (d) 32

Solution:

(b) $x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

$= 5^2 + 2 = 27$

41. If $x - \frac{1}{x} = 7$, then $x^3 - \frac{1}{x^3} = ?$

(a) 309 (b) 322 (c) 364 (d) 401

Solution:

(c) $x^3 - \frac{1}{x^3} = 7^3 + 3 \times 7 = 343 + 21 = 364$

42. If $x + \frac{1}{x} = 5$, then $x^3 + \frac{1}{x^3} = ?$

(a) 125 (b) 120 (c) 115 (d) 110

Solution:

(d) $x^3 + \frac{1}{x^3} = 5^3 - 3 \times 5 = 125 - 15 = 110$

43. $(10^{21} - 1)$ is not divisible by:

(a) 3 (b) 9 (c) 11 (d) 27

Solution:

(c) We know that $(x^n - 1)$ is not divisible by $(x + 1)$ for odd values of n.

$\therefore$ $(10^{21} - 1)$ is not divisible by $(10 + 1) = 11$ since 21 is an odd number.

Note: $(10^{3n} - 1)$ is always divisible by $9 \times 3 = 27$

$(10^{9n} - 1)$ is always divisible by $9 \times 9 = 81$

44. 26^{16} when divided by 25, the remainder is:

(a) 0 (b) 1 (c) 15 (d) 24

Solution:

(b) We know that $(x^n - 1)$ is divisible by $(x - 1)$ for all values of n.

$\therefore$ $(26^{16} - 1)$ is fully divisible by $(26 - 1)$ i.e. 25.

$\therefore$ 26^{16} when divided by 25 leaves remainder 1.

45. 17^{200} is divided by 18, the remainder is:

(a) 1 (b) 2 (c) 16 (d) 17

Solution:

(a) We know, $(x^n - 1)$ is divisible by $(x + 1)$ for even values of n.

$\therefore (17^{200} - 1)$ is divisible by $(17 + 1)$ as 200 is an even number.

$\therefore$ On dividing 17^{200} by 18, remainder is 1.

46. If $(67^{67} + 37)$ is divided by 68, the remainder is.

(a) 0 (b) 36 (c) 37 (d) 38

Solution:

(b) $(x^n + 1)$ is divisible by $(x + 1)$ for odd values of n.

$\therefore (67^{67} + 1)$ is divisible by $(67 + 1)$ i.e. 68.

$\therefore (67^{67} + 37)$ when divided by 68 leaves remainder $37 - 1 = 36$.

47. 17^{31} is divided by 18, the remainder is:

(a) 0 (b) 1 (c) 16 (d) 17

Solution:

(d) $(x^n + 1)$ is divisible by $(x + 1)$ for all odd values of n.

$\therefore (17^{31} + 1)$ is divisible by $(17 + 1)$ i.e. 18.

$\therefore 17^{31}$ is 1 less than the multiple of 18.

$\therefore 17^{31}$ when divided by 18 will leave $18 - 1 = 17$ as remainder.

48. Find the greatest number by which the expression $7^{2n} - 3^{2n}$ is always exactly divisible.

(a) 4 (b) 10 (c) 20 (d) 40

Solution:

(d) $7^{2n} - 3^{2n}$ is always divisible by $7^2 - 3^2$ i.e. $49 - 9 = 40$.

Alternative Method:

$7^{2n} - 3^{2n}$

$= 49^n - 9^n$ is always divisible by $49 - 9 = 40$.

49. The expression $(5^{2n} - 2^{3n})$ is always divisible by:

(a) 3 (b) 8 (c) 15 (d) 17

Solution:

(d) $5^{2n} - 2^{3n} = (5^2)^n - (2^3)^n = (25^n - 8^n)$

$(25^n - 8^n)$ is always divisible by $25 - 8 = 17$.

50. The expression $(2^{6n} - 4^{2n})$ is always divisible by:

(a) 10 (b) 12 (c) 32 (d) 48

Solution:

(d) $2^{6n} - 4^{2n} = 64^n - 16^n$

$\therefore 2^{6n} - 4^{2n}$ is always divisible by $64 - 16 = 48$.

51. A common factor of $(17^9 + 13^9)$ and $(17^5 + 13^5)$ is

(a) 30 (b) $17^2 + 13^2$ (c) $17^4 + 13^4$ (d) $17^5 + 13^5$

Solution:

(a) We know that $(x^n + y^n)$ is divisible by $(x + y)$ for odd values of n.

$\therefore$ Both $(17^9 + 13^9)$ and $(17^5 + 13^5)$ are divisible by $(17 + 13)$ i.e. by 30 as 9 and 5 are odd numbers.

52. Find unit's digit in $(26^{2n} - 11^{3n})$, if 'n' is a natural number.

(a) 1 (b) 3 (c) 5 (d) 6

Solution:

(c) Unit digit in 26^{2n} is always 6 for all values of 2n.

Unit digit in 11^{3n} is always 1 for all values of 3n.

$\therefore$ Unit's digit in $(26^{2n} - 11^{3n})$ is $6 - 1 = 5$.

53. Find unit's digit in $(7^{2n} - 3^{2n})$, if 'n' is a natural number.

(a) 0 (b) 1 (c) 4 (d) 5

Solution:

(a) The expression is divisible by $7^2 - 3^2 = 49 - 9 = 40$.

Since, the number is divisible by 40, unit's digit is '0'.

Chapter 11

SQUARE AND CUBE – SHORT CUT METHODS

In the exams, we are required to calculate square and cube of many numbers. Conventional Method of finding square and cube of a number is very lengthy and is prone to mistakes as the method involves a lot of calculations. This chapter deals with short-cut methods of finding square and cube of a number in the easiest way. Square and cube of some numbers are so frequently used that it is better to remember them instead of solving them each time. A list of such numbers is given in this chapter.

1. **Number consisting of 1s only:**

Steps:

1. Count 1s in the given number. Let, there are 'n' 1s.
2. Write down counting from 1 to 'n' in the ascending order. Then, write counting in descending order till 1.

Example:

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

2. **Number ending in 5:**

Method:

A number ending in 5 can be written as (10a + 5), where 'a' is ten's place digit.

$(10a + 5)^2 = 100a^2 + 100a + 25$

$= 100a\,(a + 1) + 25$

$= a\,(a + 1) \mid 25$

Example:

$15^2 = 1 \times 2 \mid 25 = 225$

$25^2 = 2 \times 3 \mid 25 = 625$

$35^2 = 3 \times 4 \mid 25 = 1225$

$85^2 = 8 \times 9 \mid 25 = 7225$

$195^2 = 19 \times 20 \mid 25 = 38025$

$1005^2 = 100 \times 101 \mid 25 = 1010025$

Note: Square of these numbers ends at 25 and are preceded by the even numbers.

3. **Numbers less than base:**

Base: $A^2 = A^2 - B^2 + B^2 = (A + B) \times (A - B) + B^2$

[A + B is equal to 10^n]

Rule: If $a = 10^n - b$

Then $a^2 = a - b \mid b^2$

Example 1: Find square of 9.

Solution:

$9 = 10^1 - 1$

$\therefore 9^2 = 9 - 1 \mid 1^2 = 8 \mid 1 = 81$

Example 2: Find square of 98.

Solution:

$98 = 10^2 - 2$

$\therefore 98^2 = 98 - 2 \mid 2^2 = 96 \mid 04 = 9604$

Example 3: Find square of 999.

Solution:

$999 = 10^3 - 1$

$\therefore 999^2 = 999 - 1 \mid 1^2 = 998 \mid 001 = 998001$

Example 4: Find square of 99987.

Solution:

$99987 = 10^5 - 13$

$\therefore 99987^2 = 99987 - 13 \mid 13^2 = 99974 \mid 00169 = 9997400169$

Example 5: Find square of 999973.

Solution:

$999973 = 10^6 - 27$

$\therefore 999973^2 = 999973 - 27 \mid 27^2 = 999946 \mid 000729 = 999946000729$

Note: In the above example, squares of the 'differences from the respective bases' are converted into 'n' digit number where base is 10^n.

Example 6: Find square of 89.

Solution:

$89 = 10^2 - 11$

$\therefore 89^2 = 89 - 11 \mid 11^2 = 78 \mid 121 = 79\ 21$

Note: Here square of 'difference from base' is a 3-digit number whereas, we need two digits from that part.

$\therefore$ We write down 21 and carry forward 1 and add it to the next part.

4. **Numbers greater than base:**

Base: $A^2 = A^2 - B^2 + B^2 = (A + B) \times (A - B) + B^2$

[$A - B$ is equal to 10^n]

Rule: If $a = 10^n + b$,

Then $a^2 = a + b \mid b^2$

Example 1: Find square of 12.

Solution:

$12 = 10^1 + 2$

$\therefore 12^2 = 12 + 2 \mid 2^2 = 14 \mid 4 = 144$

Example 2: Find square of 101.

Solution:

$101 = 10^2 + 1$

$\therefore 101^2 = 101 + 1 \mid 1^2 = 102 \mid 01 = 10201$

Example 3: Find square of 1009.

Solution:

$1009 = 10^3 + 9$

$\therefore 1009^2 = 1009 + 9 \mid 9^2 = 1018 \mid 081 = 1018081$

Example 4: Find square of 10018.

Solution:

$10018 = 10^4 + 18$

$\therefore 10018^2 = 10018 + 18 \mid 18^2 = 10036 \mid 0324 = 100360324$

Note: In the above examples, square of the 'difference from the base' is converted into 'n' digit number where base is 10^n.

Example 5: Find square of 119.

Solution:

$119 = 10^2 + 19$

$\therefore 119^2 = 119 + 19 \mid 19^2 = 138 \mid 361 = 14161$

Note: Here square of 'difference from base' is a 3-digit number whereas, we need two digits from that part.

∴ We write down 61 and carry forward 3 and add it to the next part.

5. **Two digit-number:**

Base: $(a + b)^2 = a^2 + 2ab + b^2$

$\therefore (ab)^2 = a^2 \mid 2ab \mid b^2$

Example:

$26^2 = 2^2 \mid 2 \times 2 \times 6 \mid 6^2 = 4 \mid 24 \mid 36 = 676$

$32^2 = 3^2 \mid 2 \times 3 \times 2 \mid 2^2 = 9 \mid 12 \mid 4 = 1024$

$51^2 = 5^2 \mid 2 \times 5 \times 1 \mid 1^2 = 25 \mid 10 \mid 1 = 2601$

Note: Starting from right-hand side, we write down one digit from each part and extra digit, if any is carried forward and added to the next part.

6. **Any number of digits**

Steps:

1. Sets of digits are made:

   First set consists of unit's digit only, second set of last 2 digits, third set of last 3 digits and similarly one digit is added each time, till all the digits are covered.

   Next set consists of all digits except unit's digit, next set of all digits except last 2 digits and so on.
2. Value of each pair is calculated in the following manner:
   - (a) Pairs of two numbers equidistant from both sides in a set:

     Find twice the product of such numbers.
   - (b) Single middle number (if any):

     Find square of that number.
   - (c) Add the results obtained in (a) and (b).
3. Write the Value of each pair from right-hand side.
4. One digit (right-hand side) is written down from each part and extra digit, if any is carried forward and added to its immediate left part.

Note: This method can be used for squaring a number with any number of digits.

Example 1:

(a) Find square of 432.

Solution:

Step 1:

Sets are 2, 32, 432, 43, 4

Step 2:

Value of 2 is $2^2 = 4$

Value of 32 is $2 \times (3 \times 2) = 12$

Value of 432 is $3^2 + 2\,(4 \times 2) = 9 + 16 = 25$

Value of 43 is $2\,(4 \times 3) = 24$

Value of 4 is $4^2 = 16$

$\therefore\ 432^2 = 16 \mid 24 \mid 25 \mid 12 \mid 4 = 186624$

Note: Carry forward from second, third and fourth parts are 1, 2 and 2 respectively.

(b) Find square of 543.

Solution:

Step 1:

The sets are 3, 43, 543, 54, 5

Step 2:

Value of 3 is $3^2 = 9$

Value of 43 is $2\,(4 \times 3) = 24$

Value of 543 is $4^2 + 2\,(5 \times 3) = 16 + 30 = 46$

Value of 54 is $2\,(5 \times 4) = 40$

Value of 5 is $5^2 = 25$

$\therefore 543^2 = 25 | 40 | 46 | 24 | 9 = 294849$

Note: Carry forward from second, third and fourth parts are 2, 4 and 4 respectively.

(c) Find square of 1427.

Solution:

Step 1:

The sets are 7, 27, 427, 1427, 142, 14, 1

Step 2:

Value of 7 is $7^2 = 49$

Value of 27 is $2 (2 \times 7) = 28$

Value of 427 is $2^2 + 2 (4 \times 7) = 4 + 56 = 60$

Value of 1427 is $2 (1 \times 7) + 2 (4 \times 2) = 14 + 16 = 30$

Value of 142 is $4^2 + 2 (1 \times 2) = 16 + 4 = 20$

Value of 14 is $2 (1 \times 4) = 8$

Value of 1 is $1^2 = 1$

$\therefore 1427^2 = 1 | 8 | 20 | 30 | 60 | 28 | 49 = 2036329.$

Note: Carry forward from first part onward are 4, 3, 6, 3, 2 and 1 respectively.

7. **Numbers around 50:**

Base: $(50 \pm a)^2 = (25 \pm a) \times 100 + a^2$

Rule: If $x = 50 \pm a$;
Then $x^2 = 25 \pm a \mid a^2$

Note: a^2 is written as two-digit number.

Example 1: Find square of 53.

Solution:

$53 = 50 + 3$

$\therefore 53^2 = 25 + 3 | 3^2 = 28 | 09 = 2809$

Example 2: Find square of 60.

Solution:

$60 = 50 + 10$

$\therefore 60^2 = 25 + 10 | 10^2 = 35 | 100 = 3600$

Example 3: Find square of 49.

Solution:

$49 = 50 - 1$

$\therefore 49^2 = 25 - 1 | 1^2 = 24 | 01 = 2401$

Example 4: Find square of 42.

Solution:

$42 = 50 - 8$

$\therefore 42^2 = 25 - 8 | 8^2 = 17 | 64 = 1764$

8. **Numbers around 500**:

Base: $(500 \pm a)^2 = (250 \pm a) \times 1000 + a^2$

Rule: If $x = 500 \pm a$

Then $x^2 = 250 \pm a \mid a^2$

Note: 'a^2' is written as three-digit number.

Example 1: Find square of 501.

Solution:

$501 = 500 + 1$

$\therefore 501^2 = 250 + 1 \mid 1^2 = 251 \mid 001 = 251001$

Example 2: Find square of 511.

Solution:

$511 = 500 + 11$

$\therefore 511^2 = 250 + 11 \mid 11^2 = 261 \mid 121 = 261121$

Example 3: Find square of 498.

Solution:

$498 = 500 - 2$

$\therefore 498^2 = 250 - 2 \mid 2^2 = 248 \mid 004 = 248004$

Example 4: Find square of 491.

Solution:

$491 = 500 - 9$

$\therefore 491^2 = 250 - 9 \mid 9^2 = 241 \mid 081 = 241081$

9. Base : $x^2 = x^2 - a^2 + a^2 = (x + a)(x - a) + a^2$

Method:

Add and subtract the same number so as to make one number round off and then solve as shown above.

Examples:

$31^2 = (31 + 1) \times (31 - 1) + 1^2 = 32 \times 30 + 1^2 = 960 + 1 = 961$

$38^2 = (38 + 2) \times (38 - 2) + 2^2 = 40 \times 36 + 2^2 = 1440 + 4 = 1444$

$83^2 = (83 + 3) \times (83 - 3) + 3^2 = 86 \times 80 + 3^2 = 6880 + 9 = 6889$

10. We can square the numbers ending in 15, 25, 35, 45, etc. with the help of method explained in the Multiplication chapter.

Note: Four digits are taken from first part.

Numbers ending in 15 (Note: Double of last two digits is 30):

$215^2 = 2^2 + 2 \times \frac{3}{10} \mid 15^2 = 4.6 \mid 0225 = 4\ 6225$

$515^2 = 5^2 + 5 \times \frac{3}{10} \mid 15^2 = 26.5 \mid 0225 = 26\ 5225$

Numbers ending in 25 (Note: Double of last two digits is 50):

$$225^2 = 2^2 + \frac{1}{2} \times 2 \,|\, 25^2 = 5 \,|\, 0625 = 5\ 0625$$

$$425^2 = 4^2 + \frac{1}{2} \times 4 \,|\, 25^2 = 18 \,|\, 0625 = 18\ 0625$$

$$825^2 = 8^2 + \frac{1}{2} \times 8 \,|\, 25^2 = 68 \,|\, 0625 = 68\ 0625$$

$$1025^2 = 10^2 + \frac{1}{2} \times 10 \,|\, 25^2 = 105 \,|\, 0625 = 105\ 0625$$

$$725^2 = 7^2 + \frac{1}{2} \times 7 \,|\, 25^2 = 52.5 \,|\, 0625 = 52\ 5625$$

Numbers ending in 35 (Note: Double of last two digits is 70):

$$135^2 = 1^2 + 1 \times \frac{7}{10} \,|\, 35^2 = 1.7 \,|\, 1225 = 1\ 8225$$

$$235^2 = 2^2 + 2 \times \frac{7}{10} \,|\, 35^2 = 5.4 \,|\, 1225 = 5\ 5225$$

Numbers ending in 45 (Note: Double of last two digits is 90):

$$245^2 = 2^2 + 2 \times \frac{9}{10} \,|\, 45^2 = 5.8 \,|\, 2025 = 6\ 0025$$

Numbers ending in 55, 65, 75 etc. :

$$455^2 = 4^2 + 4 \times \frac{11}{10} \,|\, 55^2 = 20.4 \,|\, 3025 = 20\ 7025$$

$$1065^2 = 10^2 + 10 \times \frac{13}{10} \,|\, 65^2 = 113 \,|\, 4225 = 113\ 4225$$

$$675^2 = 6^2 + 6 \times \frac{15}{10} \,|\, 75^2 = 45 \,|\, 5625 = 45\ 5625$$

Note: This method is useful for finding the squares of large numbers ending in 5. Remember we take double of the last two digits of the given number.

11. Square of **three-digit number** when **middle digit is '0'**:

Method:

Square of extreme left digit	Double of multiplication of left & right digits	Square of extreme right digit

Note: Two digits are taken from each part.

Example:

$209^2 = 2^2 \,|\, 2 \times 2 \times 9 \,|\, 9^2 = 4 \,|\, 36 \,|\, 81 = 43681$

$503^2 = 5^2 \,|\, 2 \times 5 \times 3 \,|\, 3^2 = 25 \,|\, 30 \,|\, 09 = 253009$

Note the following examples:

$413^2 = 4^2 \,|\, 2 \times 4 \times 13 \,|\, 13^2 = 16 \,|\, 104 \,|\, 169 = 170569$

$712^2 = 7^2 \,|\, 2 \times 7 \times 12 \,|\, 12^2 = 49 \,|\, 168 \,|\, 144 = 50 \,|\, 69 \,|\, 44 = 506944$

12. **Four-digit number** when **middle two digits are '0'**:

Method:

Square of extreme left digit	Double of multiplication of left & right digits	Square of extreme right digit

Note: Three digits are taken from each part.

Example:

$3002^2 = 3^2 | 2 \times 3 \times 2 | 2^2 = 9 | 012 | 004 = 9012004$

$7005^2 = 7^2 | 2 \times 7 \times 5 | 5^2 = 49 | 070 | 025 = 49070025$

Note the following examples:

$1013^2 = 10^2 | 2 \times 10 \times 13 | 13^2 = 100 | 260 | 169 = 1026169$

$2302^2 = 23^2 | 2 \times 23 \times 2 | 2^2 = 529 | 92 | 04 = 5299204$

$1113^2 = 11^2 | 2 \times 11 \times 13 | 13^2 = 121 | 286 | 169 = 1238769$

Note: 2 digits are taken from each part.

13. **When square of previous or next number is known.**

1. $(a+1)^2 = a^2 + 2a + 1 = a^2 + [a + (a+1)]$

Examples:

$31^2 = 30^2 + (30 + 31) = 900 + 61 = 961$

$91^2 = 90^2 + (90 + 91) = 8100 + 181 = 8281$

$231^2 = 230^2 + (230 + 231) = 52900 + 461 = 53361$

$2501^2 = 2500^2 + (2500 + 2501) = 6250000 + 5001 = 6255001$

2. $(a-1)^2 = a^2 - 2a + 1 = a^2 - [a + (a-1)]$

Examples:

$49^2 = 50^2 - (50 + 49) = 2500 - 99 = 2401$

$179^2 = 180^2 - (179 + 180) = 32400 - 359 = 32041$

$3499^2 = 3500^2 - (3499 + 3500) = 12250000 - 6999 = 12243001$

14. **Mixed number** which can be written as $a\frac{1}{2}$ is $a\,(a+1) | \frac{1}{4}$

Example:

$$\left(2\frac{1}{2}\right)^2 = 2 \times 3 | \frac{1}{4} = 6\frac{1}{4}$$

$$\left(5\frac{1}{2}\right)^2 = 5 \times 6 | \frac{1}{4} = 30\frac{1}{4}$$

15. **Finding cubes:**

Base: $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Examples:

12^3 i.e. $a = 1$ and $b = 2$

$= 1^3 | 3 \times 1^2 \times 2 | 3 \times 1 \times 2^2 | 2^3 = 1 | 6 | 12 | 8 = 1728$

23^3 i.e. $a = 2$ and $b = 3$

$= 2^3 | 3 \times 2^2 \times 3 | 3 \times 2 \times 3^2 | 3^3 = 8 | 36 | 54 | 27 = 12167$

Note: One digit is taken from each part.

104^3 i.e. $a = 1$ and $b = 4$

$= 1^3 | 3 \times 1^2 \times 4 | 3 \times 1 \times 4^2 | 4^3 = 1 | 12 | 48 | 64 = 1124864$

105^3 i.e. $a = 1$ and $b = 5$

$= 1^3 | 3 \times 1^2 \times 5 | 3 \times 1 \times 5^2 | 5^3 = 1 | 15 | 75 | 125 = 1157625$

205^3 i.e. $a = 2$ and $b = 5$

$= 2^3 | 3 \times 2^2 \times 5 | 3 \times 2 \times 5^2 | 5^3 = 8 | 60 | 150 | 125 = 8615125$

Note: Two digits are taken from each part.

1005^3 i.e. $a = 1$ and $b = 5$

$= 1^3 | 3 \times 1^2 \times 5 | 3 \times 1 \times 5^2 | 5^3 = 1 | 015 | 075 | 125 = 1015075125$

Note: Three digits are taken from each part.

SQUARES OF FREQUENTLY USED NUMBERS

11 : 121	12 : 144	13 : 169	14 : 196
15 : 225	16 : 256	17 : 289	18 : 324
19 : 361	21 : 441	22 : 484	23 : 529
24 : 576	25 : 625	26 : 676	27 : 729
29 : 841	31 : 961	32 : 1024	41 : 1681

Note: Difference between square of two consecutive numbers is equal to sum of the numbers themselves.

CUBES OF FREQUENTLY USED NUMBERS

2 : 8	3 : 27	4 : 64	5 : 125
6 : 216	7 : 343	8 : 512	9 : 729
11 : 1331	12 : 1728	13 : 2197	21 : 9261

Note: Difference between cubes of two consecutive numbers

= (3 × smaller number × bigger number) + 1

Chapter 12

SQUARE ROOT AND CUBE ROOT

Square root: Square root of a number is the number, product of which by itself is equal to the given number. Square root is also known as under-root or 2nd root. Square root is denoted by $\sqrt{\ }$.

Examples:

Since, $3 \times 3 = 9$

$\therefore \sqrt{9} = 3$

Since, $11 \times 11 = 121$

$\therefore \sqrt{121} = 11$

Cube root: Cube root of a number is the number, product of which by itself for three times is equal to the given number. In other words, cube root of a number is the number whose cube is equal to the given number itself. Cube root is denoted by $\sqrt[3]{\ }$. It is also called 3rd root.

PRIME FACTOR METHOD FOR FINDING SQUARE ROOT AND CUBE ROOT

Steps :–

1. Write prime factors of the given number.
2. Select the factors which are in pair.
3. Take one number from each pair and multiply them. The product of such numbers followed by the product (written under $\sqrt{\ }$ sign) of the numbers which are not in pair.

Example:

Find square root of 11025 (By Prime factor Method).

Solution:

Prime factors of 11025 are $3 \times 3 \times 5 \times 5 \times 7 \times 7$.

We want to find square-root i.e. 2nd root.

∴ Pairs are made with two same factors.

$11025 = (3 \times 3) \times (5 \times 5) \times (7 \times 7)$

$\therefore \sqrt{11025} = 3 \times 5 \times 7 = 105$

Note: This method can be used to find out any root of a given number.

Example:

Find cube-root of 216.

Solution:

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

As we want to find cube-root i.e. 3rd root.

∴ Pairs are made with three same factors.

216 = (2 × 2 × 2) × (3 × 3 × 3)

∴ Cube-root of 216 is 2 × 3 = 6.

SQUARE ROOT OF 3 OR 4-DIGIT SQUARE NUMBER

Base: $(a + b)^2 = a^2 + 2ab + b^2$

$$\begin{array}{r} a\ \ b \\ \times\ a\ \ b \\ \hline a^2 \mid 2ab \mid b^2 \\ \hline \end{array}$$

Steps:

1. Divide the given number in three parts. First part contains 1 digit (in case of 3-digit number) or 2 digits (in case of 4-digit number) and second and third parts contain one digit each.
2. Find maximum value of one-digit number (say 'a') whose square does not exceed first part from left-hand side.
3. Find one-digit number (say 'b') so that value of (2ab) does not exceed second part.
4. If the third part is equal to b^2, then 'ab' is square-root of the given number.

Note: Remainders, if any, in the first and second parts are carried forward to second and third parts respectively.

Note: Square-root of any three or four-digit number is always a two-digit number.

Note: By using this method, we can find out the square root of a perfect square number only.

Example 1: Find square root of 676.

Solution:

676 is divided into 3 parts: 6 | 7 | 6.

Part I: $6 = a^2 \Rightarrow a = 2$, Remainder = 2

Part II: $27 = 2ab \Rightarrow b = \frac{27}{2 \times 2} = 6$, Remainder = 3.

Part III: $36 = b^2 = 6^2$, Remainder = 0

∴ a = 2 and b = 6

∴ Square-root of 676 is 26.

Example 2: Find square root of 9604.

Solution:

9604 is divided into 3 parts : 96 | 0 | 4

Part I: $96 = a^2 \Rightarrow a = 9$, Remainder = 15

Part II: $150 = 2ab \Rightarrow b = \frac{150}{2 \times 9} = 8$, Remainder = 6

Part III: $64 = b^2 = 8^2$, Remainder = 0

∴ a = 9 and b = 8

∴ Square-root of 9604 is 98.

Example 3: Find square root of 841.

Solution:

The number is divided into three parts: 8 | 4 | 1

Part I: $8 = a^2 \Rightarrow a = 2$, Remainder = 4

Part II: $44 = 2ab \Rightarrow b = \frac{44}{2 \times 2} = 9$, Remainder = 8

Part III: $81 = b^2 = 9^2$, Remainder = 0

$\therefore$ a = 2 and b = 9

$\therefore$ The square-root of 841 is 29.

Note: Maximum value of 'b' is 11. But, we can't take b = 11, for two reasons – firstly because it is a two-digit number, secondly, higher the value of 'b' is, the more remainder is required for 3rd part.

SQUARE-ROOT OF 5 OR 6 DIGIT SQUARE NUMBER

Base: $(a + b + c)^2 = a^2 + 2ab + (2ac + b^2) + 2bc + c^2$

$$\begin{array}{r} a\ b\ c \\ \times\, a\ b\ c \\ \hline a^2 \mid 2ab \mid b^2 + 2ab \mid 2bc \mid c^2 \\ \hline \end{array}$$

Steps:

1. The given number is divided into five parts. First part contains 1 digit (in case of 5-digit number) or 2 digits (in case of 6-digit number) and the remaining parts contain one digit each.
2. The five parts are compared with $a^2 \mid 2ab \mid (2ac + b^2) \mid 2bc \mid c^2$ to find the value of a, b and c.
3. Square root of the given number is 'abc'.

Note: Remainders, if any, in any part is carried forward to the next part.

Note: Root of any five or six-digit number is always a three-digit number.

Example: Find the square root of 15129.

Solution:

The number is divided into 5 parts 1 | 5 | 1 | 2 | 9

Now, $1 = a^2 \Rightarrow a = 1$, Remainder = 0

$5 = 2ab \Rightarrow b = \frac{5}{2 \times 1} = 2$, Remainder = 1

$11 = (2ac + b^2) \Rightarrow 2ac = 11 - 4 = 7 \Rightarrow c = \frac{7}{2 \times 1} = 3$, Remainder = 1

$12 = 2bc \Rightarrow 12 = 2 \times 2 \times 3$, Remainder = 0

$9 = c^2 \Rightarrow 9 = 3^2$, Remainder = 0

Hence a = 1, b = 2, c = 3

$\therefore$ Square-root of 15129 is 123.

SOLVED EXERCISE

1. By what least number 252 must be multiplied to make it a perfect square number?

   (a) 7 (b) 6 (c) 5 (d) 4

Solution:

(a) $252 = 2 \times 2 \times 3 \times 3 \times 7$

Since 7 occurs once in the factors of 252.

∴ 252 must be multiplied by 7 to get a square number.

Note: 252 can be divided by 7 to get a perfect square number.

Trick:

We check unit's digit for all the options one by one:

7: $252 \times 7 = 4$ (Possible)

6: $252 \times 6 = 2$ (Not possible)

5: $252 \times 5 = 10$ (Single '0' is not possible at the end)

4: $252 \times 4 = 8$ (Not possible)

Hence out of the given options, only 7 is possible.

∴ 7 is the answer.

2. Find the least number which when multiplied by 588 gives a perfect square number.

(a) 1 (b) 2 (c) 3 (d) 7

Solution:

(c) $588 = 2 \times 2 \times 3 \times 7 \times 7$

Here except for 3 all other factors are in pairs.

∴ We have to multiply 588 by 3 to make it a perfect square number.

Note: The number 588 can also be divided by 3 to get a square number.

Trick:

We note that 588 is an even number and is divisible by 4. (Possible)

Sum of digits is 21 i.e. the number is divisible by 3 (but not by 9)

∴ The number must be multiplied by 3 to make it a square number.

3. What is the unit's digit in the square root of 23409?

(a) 1 (b) 3 (c) 5 (d) 7

Solution:

(b) The number is divided into 5 parts: 2 | 3 | 4 | 0 | 9

First part: $2 = a^2 \Rightarrow a = 1$, Remainder = 1

Second part: $13 = 2ab \Rightarrow b = \frac{13}{2 \times 1} = 6$, Remainder = 1

Third part: $14 = (2ac + b^2) \Rightarrow$ Here $b^2 = 6^2 = 36$ which exceeds 14

As any number cannot have negative value, we need more remainder in the second part. If we take b = 5 instead of 6, then:

Remainder on dividing 13 by $(2 \times 1 \times 5)$ is 3.

Now a = 1 and b = 5, remainder = 3

Third part: $34 = (2ac + b^2) \Rightarrow 2ac = 34 - 25 = 9$

$c = \frac{9}{2 \times 1} = 4$, Remainder = 1

$\therefore$ a = 1, b = 5 and c = 4

Fourth part: 10 = 2bc, but 2bc = 2 × 5 × 4 = 40 which exceeds 10.

As the value cannot exceed any part, we have to reduce the value of c by 1.

$\therefore$ a = 1, b = 5 and c = 3

Third part: Remainder in 3rd part = $34 - (2 \times 1 \times 3 + 5^2) = 3$

Fourth part: 30 = 2bc = 2 × 5 × 3, Remainder = 0

Fifth part: $9 = c^2 \Rightarrow 9 = 3^2$, Remainder = 0

Hence a = 1, b = 5, c = 3

$\therefore$ Square-root of 15129 is 153.

$\therefore$ Unit's digit in square root of 23409 is 3.

Trick:

Root of first 3 digits of the given number is $\sqrt{234} = 15$, Remainder = 9.

The unit's digit of the square root of 15129 is either 3 or 7.

Since the number has 5 digits, square-root of 15129 will have 3 digits.

$\therefore$ The number is either 153 or 157.

The sum of the digits of 153 and 157 are 9 and 4 respectively.

Sum of the digits of square of 153 must be 9 and sum of digits of square of 157 must exceed multiple of 3 by 1.

Now sum of the digits of the number 23409 is 9.

$\therefore$ The square root is 153.

$\therefore$ The unit's digit is 3.

Chapter 13

POWERS

a^n is read as 'a' raised to the power of 'n', where 'a' is the base and 'n' is the power. a^n is equal to the multiplication of 'a' with 'a' for 'n' times.

$a^n = a \times a \times a \times a \times \ldots\ldots\ldots\ldots\ldots\ldots \times a$ (n times)

FORMULAE

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$\frac{a^m}{a^m} = a^{m-m} = a^0 = 1$ (for non-zero values of a)

$(a^m)^n = a^{mn}$

$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$

$(ab)^m = a^m \times b^m$

$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

$a^{-m} = \frac{1}{a^m}$

$\frac{1}{a^{-m}} = a^m$

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m = \frac{b^m}{a^m}$

Important Rule:

$(4n+1)^{th}$ power of a number always has the same digit at the unit's place which is unit's digit of the given number.

$\therefore$ $(x)^{4n+a}$ has the same unit's digit as is the unit's digit in x^a.

Remember the following Points:

(a) A number with 0, 1, 5 and 6 at unit's place, raised to any power always has 0, 1, 5 and 6 respectively at unit's place.

(b) Unit's digit in $(\ldots.3)^{4n}$, $(\ldots.7)^{4n}$ and $(\ldots.9)^{2n}$ is 1 for all values of n.

(c) Unit's digit in $(\ldots.2)^{4n}$, $(\ldots.4)^{2n}$ $(,\ldots8)^{4n}$ is 6 for all values of n.

Unit's digit in $(\ldots.6)^n$ is 6 for all values of n.

$\therefore$ $(4n)^{th}$ power of an even number always has 6 at unit's place.

POWERS OF 2

$2^1 - 2$ $2^2 - 4$ $2^3 - 8$ $2^4 - 16$ $2^5 - 32$
$2^6 - 64$ $2^7 - 128$ $2^8 - 256$ $2^9 - 512$ $2^{10} - 1024$

POWERS OF 3

$3^1 = 3$ $3^2 = 9$ $3^3 = 27$ $3^4 = 81$ $3^5 = 243$ $3^6 = 729$

POWERS OF 5

$5^1 = 5$ $5^2 = 25$ $5^3 = 125$ $5^4 = 625$ $5^5 = 3125$

SOLVED EXERCISE

1. $3^{13} \times 3^2 = ?$

(a) 3^{15} (b) 9^{15} (c) 3^{26} (d) 9^{26}

Solution:

(a) $3^{13} \times 3^2 = 3^{13+2} = 3^{15}$

2. $2^5 \times 4^3 = ?$

(a) 8^8 (b) 8^{15} (c) 2^{15} (d) 2^{11}

Solution:

(d) $2^5 \times 4^3 = 2^5 \times (2^2)^3 = 2^5 \times 2^6 = 2^{5+6} = 2^{11}$

3. $9^3 \times 27^2 = ?$

(a) 3^{12} (b) 243^5 (c) 9^5 (d) 243

Solution:

(a) $9^3 \times 27^2 = (3^2)^3 \times (3^3)^2 = 3^6 \times 3^6 = 3^{6+6} = 3^{12}$

4. $2^5 \times 3^5 = ?$

(a) 5^5 (b) 6^5 (c) 5^{10} (d) 6^{10}

Solution:

(b) $2^5 \times 3^5 = (2 \times 3)^5 = 6^5$

5. Find the difference between 2^{2^3} and $\left(2^2\right)^3$.

(a) 0 (b) 8 (c) 64 (d) 192

Solution:

(d) $2^{2^3} = 2^{2 \times 2 \times 2} = 2^8 = 256$

$\left(2^2\right)^3 = 2^{2 \times 3} = 2^6 = 64$

$2^{2^3} - \left(2^2\right)^3 = 256 - 64 = 192$

6. $\left(\frac{3}{2}\right)^{-3} = ?$

(a) $\frac{4}{9}$ (b) $\frac{9}{4}$ (c) $\frac{8}{27}$ (d) $\frac{27}{8}$

Solution:

(c) $\left(\frac{3}{2}\right)^{-3} = \left(\frac{2}{3}\right)^{3} = \frac{8}{27}$

7. $(0.04)^{-1.5} = ?$

(*a*) 0.08 (*b*) 25 (*c*) 125 (*d*) 625

Solution :

(*c*) $(0.04)^{-1.5} = (0.04)^{-\frac{3}{2}} = \left(\frac{1}{0.04}\right)^{\frac{3}{2}} = 25^{\frac{3}{2}} = 125$

8. $\frac{4^2 \times 9^4}{8 \times 27^2} = ?$

(a) $\frac{1}{6}$ (b) 1 (c) 6 (d) 18

Solution:

(d) $\frac{4^2 \times 9^4}{8 \times 27^2} = \frac{(2^2)^2 \times (3^2)^4}{(2^3) \times (3^3)^2} = \frac{2^4 \times 3^8}{2^3 \times 3^6}$

$= (2^{4-3}) \times (3^{8-6}) = 2 \times 9 = 18$

9. $8^{0.28} \times 4^{0.08} = ?$

(a) $8^{0.36}$ (b) $4^{0.36}$ (c) 2 (d) 1

Solution:

(c) $8^{0.28} \times 4^{0.08}$

$= (2^3)^{0.28} \times (2^2)^{0.08}$

$= (2^{3 \times 0.28}) \times (2^{2 \times 0.08})$

$= 2^{0.84} \times 2^{0.16} = 2^{0.84 + 0.16} = 2^1 = 2$

10. $\frac{5^{0.25} \times 125^{0.25}}{256^{0.10} \times 256^{0.15}} = ?$

(a) $\frac{4}{5}$ (b) $\frac{5}{4}$ (c) 4 (d) 5

Solution:

(b) $\frac{5^{0.25} \times 125^{0.25}}{256^{0.10} \times 256^{0.15}} = \frac{5^{0.25} \times \left(5^3\right)^{0.25}}{256^{0.25}} = \frac{5^{0.25} \times 5^{0.75}}{\sqrt[4]{256}} = \frac{5}{4}$

11. $\left(\frac{x^a}{x^b}\right)^{a+b} \times \left(\frac{x^b}{x^c}\right)^{b+c} \times \left(\frac{x^c}{x^a}\right)^{c+a} = ?$

(a) 0 (b) 1 (c) x (d) $\frac{1}{x}$

Solution :

(b) Given expression = $x^{(a-b)(a+b)} \times x^{(b-c)(b+c)} \times x^{(c-a)(c+a)}$

$= x^{a^2-b^2} \times x^{b^2-c^2} \times x^{c^2-a^2} = x^{a^2-b^2+b^2-c^2+c^2-a^2} = x^0 = 1$

12. $10^{30} - 10^{29} = ?$

(a) 10 (b) 10^{28} (c) 10^{29} (d) 9×10^{29}

Solution:

(d) $10^{30} - 10^{29} = 10 \times 10^{29} - 10^{29} = 10^{29} \times (10 - 1) = 9 \times 10^{29}$

13. How many prime factors are in $2^8 \times 3^5 \times 5^9$?

(a) 10 (b) 18 (c) 22 (d) 25

Solution:

(c) $2^8 = 2 \times 2 \times 2$ .... (8 times)

$\therefore$ 2^8 has 8 prime factors.

Similarly 3^5 and 5^9 have 5 and 9 prime factors respectively.

$\therefore$ Total prime factors = 8 + 5 + 9 = 22.

14. How many prime factors are in $5^{10} \times 7^5 \times 10^3$?

(a) 18 (b) 21 (c) 23 (d) 28

Solution:

(b) $5^{10} \times 7^5 \times 10^3 = 5^{10} \times 7^5 \times (2 \times 5)^3 = 5^{10} \times 7^5 \times (2^3 \times 5^3)$

$\therefore$ Prime factors = 10 + 5 + (3 + 3) = 21.

15. Find Unit's digit in 25^{1136}

(a) 0 (b) 1 (c) 5 (d) 6

Solution:

(c) We know that a number with 5 at unit's place raised to any power always ends in 5.

$\therefore$ Unit's digit in 25^{1136} is 5.

16. Find Unit's digit in 42^{148}.

(a) 2 (b) 4 (c) 6 (d) 8

Solution:

(c) 148 or 48 is fully divisible by 4.

We know that 4n^th^ power of an even number always have 6 at unit's place.

$\therefore$ Unit's digit in 42^{148} is 6.

17. Find Unit's digit in 34^{325}.

(a) 1 (b) 4 (c) 6 (d) 8

Solution:

(b) On dividing 325 or 25 by 4, remainder is 1.

$\therefore$ 34^{325} have the same unit's digit as is the unit's digit in 4^1 i.e. 4.

18. Find Unit's digit in 7^{1238}.

(a) 1 (b) 3 (c) 7 (d) 9

Solution:

(d) On dividing 1238 or 38 by 4, remainder is 2.

∴ Unit's digit in 7^{1238} will be same as in 7^2 i.e. 49.

∴ Unit's digit in 7^{1238} is 9.

19. Find Unit's digit in $31^{197} + 25^{303} + 50^{55} - 46^{67}$

(a) 0 (b) 1 (c) 5 (d) 6

Solution:

(a) Numbers ending in 0, 1, 5, 6 raised to any power always have same digit in the unit's place.

Unit's digit in the given expression is $1 + 5 + 0 - 6 = 0$.

18. Find Unit's digit in $35^{15} \times 19^{29} \times 21^{25}$.

(a) 0 (b) 1 (c) 5 (d) 9

Solution:

(c) Numbers ending in 5 and 1 raised to any power always have 5 and 1 respectively at the unit's place.

Remainder on dividing 29 by 4 is 1.

∴ 19^{29} will have same unit's digit as in 19^1 i.e. 9.

∴ Unit's digit in the given expression is $5 \times 9 \times 1 = 5$.

21. Find the digit in unit's place in $16^{23} + 17^{29} - 13^{10}$.

(a) 0 (b) 4 (c) 6 (d) 8

Solution:

(b) Any power of a number ending in 6 will always have 6 at unit's place.

On dividing 29 and 10 by 4, remainders are 1 and 2 respectively.

∴ Unit's digit in 17^{29} is same as in 7^1 i.e. 7.

And unit's digit in 13^{10} is same as in 3^2 i.e. 9.

∴ Unit's digit in the given expression is $6 + 7 - 9 = 4$.

22. Find the digit in unit's place in $15^{28} + 11^{22} - 9^{27}$.

(a) 2 (b) 3 (c) 7 (d) 9

Solution:

(c) Any power of a number ending in 5 always have 5 at unit's place and any power of a number ending in 1 always have 1 at unit's place.

On dividing 27 by 4, remainder is 3.

∴ Unit's digit in 9^{27} is same as in 9^3 i.e. 9.

∴ Unit's digit in the given expression is $5 + 1 - 9 = -3$.

But a digit can't have negative value.

∴ Unit's digit in the given expression is $10 - 3 = 7$.

23. If $2^{x-1} + 2^{x+1} = 320$, then x = ?

(a) 4 (b) 5 (c) 6 (d) 7

Solution:

(d) $2^{x-1} + 2^{x+1} = 320$

$\therefore 2^{x-1} + 2^{(x-1)+2} = 320$

$\therefore 2^{x-1} \times (1 + 2^2) = 320$

$\therefore 2^{x-1} \times (1+4) = 320$

$\therefore 2^{x-1} \times 5 = 320$

$\therefore 2^{x-1} = \frac{320}{5} = 64$

$\therefore 2^{x-1} = 2^6$

$\therefore x - 1 = 6$

$\therefore x = 6 + 1 = 7$

Chapter 14

ROOTS

Root is inverse of Power. If $x^n = y$. Then, $x = \sqrt[n]{y} = y^{\frac{1}{n}}$; $y^{\frac{1}{n}}$ is called the surd of nth order.

SQUARE ROOTS OF IMPORTANT NUMBERS

$\sqrt{2} = 1.4142$

$\sqrt{3} = 1.7321$

$\sqrt{5} = 2.2361$

$$\sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2 \ldots\ldots\ldots\ldots}}}} = 1$$

Proof:

Let $x = \sqrt{2-\sqrt{2-\sqrt{2-\sqrt{2 \ldots\ldots\ldots\ldots}}}}$

Squaring both sides, $x^2 = 2 - x$

$\therefore x^2 + x - 2 = 0$

On solving, we get $x = 1, -2$.

$$\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2 \ldots\ldots\ldots\ldots}}}} = 2$$

Proof:

Let $x = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2 \ldots\ldots\ldots\ldots}}}} = 2$

Squaring both sides, $x^2 = 2 + x$

$\therefore x^2 - x - 2 = 0$

On solving, we get $x = 2, -1$.

$$\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6 \ldots\ldots\ldots\ldots}}}} = 3$$

Proof:

Let $x = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6 \ldots\ldots\ldots\ldots}}}}$

Squaring both sides, $x^2 = 6 + x$

$\therefore x^2 - x - 6 = 0$

On solving, we get $x = 3, -2$.

$$\sqrt{12-\sqrt{12-\sqrt{12-\sqrt{12 \ldots\ldots\ldots\ldots\ldots}}}} = 3$$

Proof :

Let $x=\sqrt{12-\sqrt{12-\sqrt{12-\sqrt{12 \ldots\ldots\ldots\ldots}}}}$

Squaring both sides, $x^2 = 12 - x$

$\therefore \quad x^2 + x - 12 = 0$

On solving, we get $x = 3, - 4$.

$$\boxed{\sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12 \ldots\ldots\ldots}}}} = 4}$$

Proof : Let $x = \sqrt{12+\sqrt{12+\sqrt{12+\sqrt{12 \ldots\ldots\ldots}}}}$

Squaring both sides, $x^2 = 12 + x$

$\therefore x^2 - x - 12 = 0$

On solving, we get $x = 4, - 3$.

FORMULAE

1. $\sqrt{x} \times \sqrt{x} = x$
2. $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$
3. $\left(\sqrt{x} + \sqrt{y}\right)^2 = x + y + 2\sqrt{xy}$
4. $\left(\sqrt{x} - \sqrt{y}\right)^2 = x + y - 2\sqrt{xy}$
5. $\left(\sqrt{x} + \sqrt{y}\right)\left(\sqrt{x} - \sqrt{y}\right) = x - y$
6. $\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}}{x}$
7. $\frac{x}{\sqrt{x}} = \frac{x}{\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{x\sqrt{x}}{x} = \sqrt{x}$
8. $\frac{1}{\sqrt{x}+\sqrt{y}} = \frac{1}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{x-y}$
9. $\frac{1}{\sqrt{x}-\sqrt{y}} = \frac{1}{\sqrt{x}-\sqrt{y}} \times \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{x-y}$
10. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}} \times \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\left(\sqrt{x}-\sqrt{y}\right)^2}{x-y} = \frac{x+y-2\sqrt{xy}}{x-y}$
11. $\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} = \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}-\sqrt{y}} \times \frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}} = \frac{\left(\sqrt{x}+\sqrt{y}\right)^2}{x-y} = \frac{x+y+2\sqrt{xy}}{x-y}$
12. $\left(\sqrt{a} + \sqrt{b}\right)\left(\sqrt{c} - \sqrt{d}\right) + \left(\sqrt{a} - \sqrt{b}\right)\left(\sqrt{c} + \sqrt{d}\right) = 2\left(\sqrt{ac} - \sqrt{bd}\right)$

SOLVED EXERCISE

1. $\sqrt{25 + 144} = ?$

(a) 13 (b) 17 (c) 20 (d) 60

Solution:

(a) $\sqrt{25 + 144} = \sqrt{169} = 13$

2. $\sqrt{22 + \sqrt{9}} = ?$

(a) $\sqrt{31}$ (b) 60 (c) 5 (d) 25

Solution:

(c) $\sqrt{22 + \sqrt{9}} = \sqrt{22 + 3} = \sqrt{25} = 5$

3. $\sqrt{10 + \sqrt{25 + \sqrt{121}}} = ?$

(a) 4 (b) 16 (c) 46 (d) 156

Solution:

(a) $\sqrt{10 + \sqrt{25 + \sqrt{121}}} = \sqrt{10 + \sqrt{25 + 11}} = \sqrt{10 + 6} = 4$

4. $\sqrt{\left(1 + \frac{9}{16}\right)} = ?$

(a) $\frac{7}{4}$ (b) $\frac{5}{4}$ (c) $\frac{25}{16}$ (d) $\frac{3}{4}$

Solution:

(b) $\sqrt{\left(1 + \frac{9}{16}\right)} = \sqrt{\frac{25}{16}} = \frac{5}{4}$

5. If $\sqrt{256 \times \sqrt{16 + x}} = 16$, find the value of x.

(a) 4 (b) 8 (c) 12 (d) 16

Solution:

(d) Given, $\sqrt{256 \times \sqrt{16 + x}} = 16$

Squaring both sides, we get:

$256 \times \sqrt{16 + x} = 256$

$\therefore \sqrt{16 + x} = 1$

$\therefore 16 + x = 1$ (Squaring both the sides)

$\therefore x = 16$

6. If $\sqrt{2025} = 45$, then $\sqrt{2025} + \sqrt{20.25} + \sqrt{0.2025} = ?$

(a) 1.35 (b) 13.5 (c) 49.95 (d) 135

Solution:

(c) $\sqrt{2025} + \sqrt{20.25} + \sqrt{0.2025}$

$= 45 + 4.5 + 0.45 = 49.95$

7. Square root of 0.09 is:

(a) 0.03 (b) 0.081 (c) 0.3 (d) 0.81

Solution:

(c) $\sqrt{0.09} = 0.3$

8. The square root of $(272^2 - 128^2)$ is

(a) 200 (b) 240 (c) 144 (d) 400

Solution:

(b) $\sqrt{272^2 - 128^2} = \sqrt{(272 + 128) \times (272 - 128)}$

$= \sqrt{400 \times 144} = 20 \times 12 = 240$

9. Cube root of 64000 is:

(a) 40 (b) 80 (c) 200 (d) 800

Solution:

(a) $\sqrt[3]{64000} = \sqrt[3]{4^3 \times 10^3} = 4 \times 10 = 40$

10. $\sqrt{0.9} = ?$

(a) 0.89 (b) 0.3 (c) 0.03 (d) 0.949

Solution:

(d) $\sqrt{0.9} = 0.949$ (approx.)

11. $\sqrt{27} + \sqrt{75} = ?$

(a) 8 (b) $8\sqrt{3}$ (c) 16 (d) $16\sqrt{3}$

Solution:

(b) $\sqrt{27} + \sqrt{75}$

$= \sqrt{3 \times 3 \times 3} + \sqrt{3 \times 5 \times 5}$

$= 3\sqrt{3} + 5\sqrt{3} = \sqrt{3}\,(3 + 5) = 8\sqrt{3}$

12. $\sqrt{125} - \sqrt{45} - \sqrt{20} = ?$

(a) 0 (b) $\sqrt{5}$ (c) $2\sqrt{5}$ (d) $2\sqrt{15}$

Solution:

(a) $\sqrt{125} - \sqrt{45} - \sqrt{20}$

$= \sqrt{5 \times 5 \times 5} - \sqrt{3 \times 3 \times 5} - \sqrt{2 \times 2 \times 5}$

$= 5\sqrt{5} - 3\sqrt{5} - 2\sqrt{5}$

$= \sqrt{5}(5 - 3 - 2)$

$= \sqrt{5} \times 0 = 0$

13. $\sqrt{80} + \sqrt{45} - \sqrt{125} = ?$

(a) 0 (b) 1 (c) $2\sqrt{5}$ (d) $5\sqrt{5}$

Solution:

(c) $\sqrt{80} + \sqrt{45} - \sqrt{125}$

$= \sqrt{4 \times 4 \times 5} + \sqrt{3 \times 3 \times 5} - \sqrt{5 \times 5 \times 5}$

$= 4\sqrt{5} + 3\sqrt{5} - 5\sqrt{5}$

$= \sqrt{5}(4 + 3 - 5) = 2\sqrt{5}$

14. $\sqrt{60} + \sqrt{15} - \sqrt{135} = ?$

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) $\sqrt{60}$

Solution:

(a) $\sqrt{60} + \sqrt{15} - \sqrt{135}$

$= \sqrt{2 \times 2 \times 3 \times 5} + \sqrt{3 \times 5} - \sqrt{3 \times 3 \times 3 \times 5}$

$= 2\sqrt{15} + \sqrt{15} - 3\sqrt{15}$

$= \sqrt{15}(2 + 1 - 3) = 0$

15. $\sqrt[3]{64} + 3\sqrt{64} = ?$

(a) 28 (b) $\sqrt{128}$ (c) 48 (d) $3\sqrt{128}$

Solution:

(a) $\sqrt[3]{64} + 3\sqrt{64}$

$= \sqrt[3]{4^3} + 3 \times 8$

$= 4 + 24 = 28$

16. $\frac{1}{\sqrt{3}} = ?$

(a) 0.577 (b) 1.732 (c) 0.575 (d) 1.734

Solution:

(a) $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{1.732}{3} = 0.577$

17. $\frac{2}{\sqrt{2}} = ?$

(a) 0 (b) 1 (c) 1.414 (d) 2

Solution:

(c) $\frac{2}{\sqrt{2}} = \sqrt{2} = 1.414$

18. If $\sqrt{6} = 2.450$, then $\sqrt{\frac{3}{2}} = ?$

(a) 1.225 (b) 2.25 (c) 2.45 (d) 5

Solution:

(a) $\sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2} = \frac{2.450}{2} = 1.225$

19. If $\sqrt{6} = 2.450$, then the value of $\frac{3 \times \sqrt{2}}{2 \times \sqrt{3}} = ?$

(a) 1.225 (b) 0.817 (c) 0.408 (d) 0.490

Solution:

(a) $\frac{3 \times \sqrt{2}}{2 \times \sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{2}}{2}$

$= \sqrt{3} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2} = \frac{2.450}{2} = 1.225$

20. $\frac{\sqrt{27}+\sqrt{3}}{\sqrt{27}-\sqrt{3}} = ?$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) $\sqrt{3}$

Solution :

(*b*) $\frac{\sqrt{27}+\sqrt{3}}{\sqrt{27}-\sqrt{3}} = \frac{3\sqrt{3}+\sqrt{3}}{3\sqrt{3}-\sqrt{3}} = \frac{4\sqrt{3}}{2\sqrt{3}} = 2$

21. $\frac{\sqrt{72}-\sqrt{18}}{\sqrt{12}} = ?$

(a) $\sqrt{6}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{2}}{3}$ (d) $\frac{\sqrt{6}}{2}$

Solution:

(d) $\frac{\sqrt{72}-\sqrt{18}}{\sqrt{12}} = \frac{6\sqrt{2}-3\sqrt{2}}{2\sqrt{3}} = \frac{3\sqrt{2}}{2\sqrt{3}} = \sqrt{3} \times \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{2}$

22. $\frac{\sqrt{125}-\sqrt{75}}{\sqrt{45}-\sqrt{27}} = ?$

(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{5}{3}$ (c) $\frac{25}{9}$ (d) $\frac{9}{5}$

Solution:

(b) $\frac{\sqrt{125}-\sqrt{75}}{\sqrt{45}-\sqrt{27}} = \frac{5\sqrt{5}-5\sqrt{3}}{3\sqrt{5}-3\sqrt{3}} = \frac{5(\sqrt{5}-\sqrt{3})}{3(\sqrt{5}-\sqrt{3})} = \frac{5}{3}$

23. $\frac{\sqrt{112}-\sqrt{80}}{\sqrt{63}-\sqrt{45}} = ?$

(a) $\sqrt{\frac{4}{3}}$ (b) $\frac{2}{3}$ (c) $\frac{4}{3}$ (d) $\frac{5}{3}$

Solution:

(c) $\frac{\sqrt{112}-\sqrt{80}}{\sqrt{63}-\sqrt{45}} = \frac{4\sqrt{7}-4\sqrt{5}}{3\sqrt{7}-3\sqrt{5}} = \frac{4(\sqrt{7}-\sqrt{5})}{3(\sqrt{7}-\sqrt{5})} = \frac{4}{3}$

24. $\frac{1}{\sqrt{8}-\sqrt{7}} = ?$

(a) 1 (b) 15 (c) $\sqrt{8}+\sqrt{7}$ (d) $\sqrt{8}-\sqrt{7}$

Solution:

(c) $\frac{1}{\sqrt{8}-\sqrt{7}} = \sqrt{8}+\sqrt{7}$

25. $\frac{1}{\sqrt{6}+\sqrt{5}} = ?$

(a) 1 (b) 11 (c) $\sqrt{6}+\sqrt{5}$ (d) $\sqrt{6}-\sqrt{5}$

Solution:

(d) $\frac{1}{\sqrt{6}+\sqrt{5}} = \sqrt{6}-\sqrt{5}$

26. $\frac{1}{\sqrt{2}+1} = ?$

(a) 0 (b) 1 (c) $\sqrt{2}-1$ (d) $\sqrt{2}+1$

Solution:

(c) $\frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2}-1$

27. $\frac{1}{\sqrt{7}-\sqrt{5}} = ?$

(a) 2 (b) 12 (c) $\frac{\sqrt{7}+\sqrt{5}}{2}$ (d) $\frac{\sqrt{7}-\sqrt{5}}{2}$

Solution:

(c) $\frac{1}{\sqrt{7}-\sqrt{5}} = \frac{\sqrt{7}+\sqrt{5}}{7-5} = \frac{\sqrt{7}+\sqrt{5}}{2}$

28. $\frac{1}{\sqrt{8}+\sqrt{5}} = ?$

(a) 3 (b) 13 (c) $\frac{\sqrt{8}+\sqrt{5}}{3}$ (d) $\frac{\sqrt{8}-\sqrt{5}}{3}$

Solution:

(d) $\frac{1}{\sqrt{8}+\sqrt{5}} = \frac{\sqrt{8}-\sqrt{5}}{8-5} = \frac{\sqrt{8}-\sqrt{5}}{3}$

29. $\frac{2}{\sqrt{5}+\sqrt{3}} = ?$

(a) 4 (b) $\sqrt{5}-\sqrt{3}$

(c) $2\left(\sqrt{5}-\sqrt{3}\right)$ (d) $\sqrt{5}+\sqrt{3}$

Solution:

(b) $\frac{2}{\sqrt{5}+\sqrt{3}} = \frac{2\left(\sqrt{5}-\sqrt{3}\right)}{5-3} = \sqrt{5}-\sqrt{3}$

30. $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}} = ?$

(a) 1 (b) 15 (c) $15 + 2\sqrt{56}$ (d) $15 - 2\sqrt{56}$

Solution:

(c) $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{8}-\sqrt{7}} = \frac{\left(\sqrt{8}+\sqrt{7}\right)^2}{8-7} = \frac{8+7+2\sqrt{56}}{1} = 15 + 2\sqrt{56}$

31. $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} = ?$

(a) 1 (b) $\frac{1}{11}$ (c) $11 + 2\sqrt{30}$ (d) $11 - 2\sqrt{30}$

Solution:

(d) $\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{\left(\sqrt{6}-\sqrt{5}\right)^2}{6-5} = \frac{6+5-2\sqrt{30}}{1} = 11 - 2\sqrt{30}$

32. $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = ?$

(a) 2 (b) 4 (c) $4 - \sqrt{15}$ (d) $4 + \sqrt{15}$

Solution:

(d) $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} = \frac{\left(\sqrt{5}+\sqrt{3}\right)^2}{5-3} = \frac{5+3+2\sqrt{15}}{2} = 4 + \sqrt{15}$

33. $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = ?$

(a) 2 (b) 4 (c) $6 - \sqrt{35}$ (d) $6 + \sqrt{35}$

Solution:

(c) $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{\left(\sqrt{7}-\sqrt{5}\right)^2}{7-5} = \frac{7+5-2\sqrt{35}}{2} = 6 - \sqrt{35}$

34. $\frac{2-\sqrt{3}}{2+\sqrt{3}} = ?$

(a) 1 (b) $\frac{2}{3}$ (c) $7 + 4\sqrt{3}$ (d) $7 - 4\sqrt{3}$

Solution:

(d) $\frac{2-\sqrt{3}}{2+\sqrt{3}} = \frac{\sqrt{4}-\sqrt{3}}{\sqrt{4}+\sqrt{3}} = \frac{\left(\sqrt{4}-\sqrt{3}\right)^2}{4-3} = \frac{4+3-4\sqrt{3}}{1} = 7-4\sqrt{3}$

35. $\frac{2+\sqrt{6}}{\sqrt{2}+\sqrt{3}} = ?$

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) 2 (d) 3

Solution:

(a) $\frac{2+\sqrt{6}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{2\times 2}+\sqrt{2\times 3}}{\sqrt{2}+\sqrt{3}} = \frac{\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)}{\sqrt{2}+\sqrt{3}} = \sqrt{2}$

36. Find the value of

$$\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$$

(a) 0 (b) 1 (c) 5 (d) 6

Solution:

(c) $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$

$$=\frac{1}{\sqrt{9}-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-\sqrt{4}}$$

$$=\left(\sqrt{9}+\sqrt{8}\right)-\left(\sqrt{8}+\sqrt{7}\right)+\left(\sqrt{7}+\sqrt{6}\right)-\left(\sqrt{6}+\sqrt{5}\right)+\left(\sqrt{5}+\sqrt{4}\right)$$

$$=\sqrt{9}+\sqrt{8}-\sqrt{8}-\sqrt{7}+\sqrt{7}+\sqrt{6}-\sqrt{6}-\sqrt{5}+\sqrt{5}+\sqrt{4}$$

$$=\sqrt{9}+\sqrt{4}=3+2=5$$

36. Find the value of

$$2+\sqrt{2}+\frac{1}{\sqrt{2}+2}+\frac{1}{\sqrt{2}-2}$$

(*a*) 0 (*b*) $\sqrt{2}$ (*c*) 2 (*d*) $2+\sqrt{2}$

Solution :

(*c*) $2+\sqrt{2}+\frac{1}{\sqrt{2}+2}+\frac{1}{\sqrt{2}-2}=2+\sqrt{2}+\frac{\sqrt{2}-2+\sqrt{2}+2}{\left(\sqrt{2}+2\right)\times\left(\sqrt{2}-2\right)}$

$$=2+\sqrt{2}+\frac{2\sqrt{2}}{2-4}=2+\sqrt{2}-\sqrt{2}=2$$

38. $\sqrt{\frac{\sqrt{12}+\sqrt{3}}{\sqrt{12}-\sqrt{3}}}=?$

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{6}$ (d) 3

Solution:

(b) $\sqrt{\frac{\sqrt{12}+\sqrt{3}}{\sqrt{12}-\sqrt{3}}}=\sqrt{\frac{\left(\sqrt{12}+\sqrt{3}\right)^2}{12-3}}=\frac{2\sqrt{3}+\sqrt{3}}{3}=\frac{3\sqrt{3}}{3}=\sqrt{3}$

39. $\frac{2+\sqrt{3}}{2-\sqrt{3}}+\frac{2-\sqrt{3}}{2+\sqrt{3}}=?$

(a) 1 (b) 7 (c) 14 (d) 28

Solution:

(c) Substituting 2 with x and $\sqrt{3}$ with y, the given expression can be written as

$$\frac{x+y}{x-y}+\frac{x-y}{x+y}=\frac{(x+y)^2+(x-y)^2}{x^2-y^2}=\frac{2\left(x^2+y^2\right)}{x^2-y^2}$$

$\therefore$ Given expression $=\frac{2(4+3)}{4-3}=14$

40. $\frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}}+\frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}=?$

(a) $\sqrt{5}-\sqrt{6}$ (b) $\sqrt{5}+\sqrt{6}$

(c) $2\left(\sqrt{5}+\sqrt{6}\right)$ (d) $3\left(\sqrt{5}+\sqrt{6}\right)$

Solution:

(a) Substituting a = 1, b = 2, c = 5 and d = 3, the given expression can be written as

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{c}+\sqrt{d}}+\frac{\sqrt{a}-\sqrt{b}}{\sqrt{c}-\sqrt{d}}$$

$$=\frac{\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}-\sqrt{d}\right)+\left(\sqrt{a}-\sqrt{b}\right)\left(\sqrt{c}+\sqrt{d}\right)}{\left(\sqrt{c}+\sqrt{d}\right)\left(\sqrt{c}-\sqrt{d}\right)}$$

$$=\frac{2\left(\sqrt{ac}-\sqrt{bd}\right)}{c-d}=\frac{2\left(\sqrt{5}-\sqrt{6}\right)}{2}=\sqrt{5}-\sqrt{6}$$

41. $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2+\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2=?$

(a) 31 (b) 34 (c) 62 (d) 68

Solution:

(c) $\left(\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}\right)^2+\left(\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}\right)^2$

$$=\left(\frac{8+2\sqrt{15}}{5-3}\right)^2+\left(\frac{8-2\sqrt{15}}{5-3}\right)^2$$

$$=\left(4+\sqrt{15}\right)^2+\left(4-\sqrt{15}\right)^2$$

$$= 2 \times (16 + 15) = 62$$

Hint: $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

42. If $x = 7-4\sqrt{3}$, then $x+\frac{1}{x}=?$

(a) $3\sqrt{3}$ (b) $8\sqrt{3}$ (c) $14+8\sqrt{3}$ (d) 14

Solution:

(d) $\frac{1}{x}=\frac{1}{7-4\sqrt{3}}=7+4\sqrt{3}$ (**Hint**: $7=\sqrt{49}$ and $4\sqrt{3}=\sqrt{48}$)

$$\therefore\ x+\frac{1}{x}=\left(7-4\sqrt{3}\right)+\left(7+4\sqrt{3}\right)=14$$

43. If $x = 6-\sqrt{35}$, then $x^2+\frac{1}{x^2}=?$

(a) 1 (b) 36 (c) 142 (d) 144

Solution:

(c) Here, $\frac{1}{x} = \frac{1}{6 - \sqrt{35}} = 6 + \sqrt{35}$ **(Hint: 6 =** $\sqrt{36}$)

$$x + \frac{1}{x} = \left(6 - \sqrt{35}\right) + \left(6 + \sqrt{35}\right) = 12$$

We know, $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$

$$\therefore\ x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = 12^2 - 2 = 144 - 2 = 142$$

44. If $x = 2 - \sqrt{3}$, find the value of $x^3 + \frac{1}{x^3} = ?$

(*a*) 4 (*b*) 16 (*c*) 52 (*d*) 64

Solution :

(*c*) $\frac{1}{x} = \frac{1}{2-\sqrt{3}} = 2+\sqrt{3}$

$$x + \frac{1}{x} = (2-\sqrt{3}) + (2+\sqrt{3}) = 4$$

$$\left(x+\frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$$

$$x^3 + \frac{1}{x^3} = 4^3 - 3\times 4 = 64 - 12 = 52$$

45. Find the value of $(3+2\sqrt{2})^{-3} + (3-2\sqrt{2})^{-3}$

(*a*) 72 (*b*) 162 (*c*) 189 (*d*) 198

Solution :

(*d*) $(3+2\sqrt{2})^{-3} + (3-2\sqrt{2})^{-3} = \frac{1}{\left(3+2\sqrt{2}\right)^3} + \frac{1}{\left(3-2\sqrt{2}\right)^3}$

$$= \frac{\left(3-2\sqrt{2}\right)^3 + \left(3+2\sqrt{2}\right)^3}{\left(3+2\sqrt{2}\right)^3 \times \left(3-2\sqrt{2}\right)^3} = \frac{2\times 3^3 + 6\times 3\times 8}{\left[\left(3+2\sqrt{2}\right)\times\left(3-2\sqrt{2}\right)\right]^3} = \frac{54+144}{(1)^3} = 198$$

46. If $x = 3 + 2\sqrt{2}$, find the value of $x^4 + \frac{1}{x^4} = ?$

(*a*) 17 (*b*) 32 (*c*) 34 (*d*) 1154

Solution :

(*d*) $x^2 = (3+2\sqrt{2})^2 = 9 + 8 + 12\sqrt{2} = 17 + 12\sqrt{2}$

$$\frac{1}{x^2} = \frac{1}{17+12\sqrt{2}} = 17 - 12\sqrt{2}$$

$$\therefore \quad \left(x^2+\frac{1}{x^2}\right)^2 = x^4+\frac{1}{x^4}+2$$

$$\therefore \quad x^4+\frac{1}{x^4} = (17+12\sqrt{2}+17-12\sqrt{2})^2-2$$

$$= 34^2 - 2 = 1156 - 2 = 1154$$

Note : $17^2 = 289$ and $(12\sqrt{2})^2 = 288$

47. $\dfrac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \dfrac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \dfrac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}} = ?$

(*a*) 0 (*b*) $\sqrt{6}$ (*c*) $\sqrt{15}$ (*d*) $2\sqrt{15}$

Solution :

(*a*) $\dfrac{\sqrt{5}}{\sqrt{3}+\sqrt{2}} - \dfrac{3\sqrt{3}}{\sqrt{5}+\sqrt{2}} + \dfrac{2\sqrt{2}}{\sqrt{5}+\sqrt{3}}$

$$= \frac{\sqrt{5}\left(\sqrt{3}-\sqrt{2}\right)}{3-2} - \frac{3\sqrt{3}\left(\sqrt{5}-\sqrt{2}\right)}{5-2} + \frac{2\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{5-3}$$

$$= \sqrt{5}\times(\sqrt{3}-\sqrt{2}) - \sqrt{3}\times(\sqrt{5}-\sqrt{2}) + \sqrt{2}\times(\sqrt{5}-\sqrt{3})$$

$$= \sqrt{15}-\sqrt{10}-\sqrt{15}+\sqrt{6}+\sqrt{10}-\sqrt{6} = 0$$

48. $\sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{4+3+4\sqrt{3}}}} = ?$

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 4

Solution :

(*c*) Given Expression $= \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{4+3+4\sqrt{3}}}}$

$$= \sqrt{-\sqrt{3}+\sqrt{3+8\sqrt{\left(2+\sqrt{3}\right)^2}}} = \sqrt{-\sqrt{3}+\sqrt{3+8\left(2+\sqrt{3}\right)}}$$

$$= \sqrt{-\sqrt{3}+\sqrt{3+16+8\sqrt{3}}} = \sqrt{-\sqrt{3}+\sqrt{\left(4+\sqrt{3}\right)^2}}$$

$$= \sqrt{4} = 2$$

49. If x = $7-4\sqrt{3}$, then $\sqrt{x}+\dfrac{1}{\sqrt{x}} = ?$

(a) 1 (b) 2 (c) 4 (d) 16

Solution:

(c) $\dfrac{1}{x} = \dfrac{1}{7-4\sqrt{3}} = 7+4\sqrt{3}$ **(Hint:** $7 = \sqrt{49}$ and $4\sqrt{3} = \sqrt{48}$)

$$x+\frac{1}{x} = \left(7-4\sqrt{3}\right)+\left(7+4\sqrt{3}\right) = 14$$

$$\therefore \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^2 = x+\frac{1}{x}+2 = 14+2 = 16$$

$$\therefore \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right) = \sqrt{16} = 4$$

50. $\sqrt[3]{4} \times \sqrt[4]{8} = ?$

(a) 2 (b) $\sqrt[12]{32}$ (c) $2 \times \sqrt[12]{32}$ (d) $\sqrt[12]{32^7}$

Solution:

(c) $\sqrt[3]{4} \times \sqrt[4]{8} = \sqrt[3]{2^2} \times \sqrt[4]{2^3}$

$= 2^{\frac{2}{3}} \times 2^{\frac{3}{4}} = 2^{\frac{2}{3}+\frac{3}{4}} = 2^{\frac{17}{12}}$

$= 2 \times 2^{\frac{5}{12}} = 2 \times \sqrt[12]{2^5} = 2 \times \sqrt[12]{32}$

51. $\sqrt[3]{7} \times \sqrt{2} = ?$

(a) $\sqrt{14}$ (b) $\sqrt[3]{14}$ (c) 5 (d) $\sqrt[6]{392}$

Solution:

(d) LCM of 3 and 2, the root indices is 6

$\therefore$ We reduce the given number to the 6th order.

Now, $\sqrt[3]{7} = \sqrt[6]{7^2} = \sqrt[6]{49}$

And, $\sqrt{2} = \sqrt[6]{2^3} = \sqrt[6]{8}$

$\therefore \sqrt[3]{7} \times \sqrt{2} = \sqrt[6]{49} \times \sqrt[6]{8} = \sqrt[6]{49 \times 8} = \sqrt[6]{392}$

52. Which is bigger: $\sqrt[2]{3}$, $\sqrt[3]{4}$

(a) $\sqrt[2]{3}$ (b) $\sqrt[3]{4}$

(c) Both are equal (d) Can't say

Solution:

(a) LCM of 2 and 3, the root indices is 6.

$\therefore$ We reduce the given number to the 6th order.

Now, $\sqrt[2]{3} = \sqrt[6]{3^3} = \sqrt[6]{27}$

$\sqrt[3]{4} = \sqrt[6]{4^2} = \sqrt[6]{16}$

Since, $\sqrt[6]{27} > \sqrt[6]{16}$

$\therefore \sqrt[2]{3} > \sqrt[3]{4}$

53. $16\sqrt[3]{9} \div 8\sqrt{3} = ?$

(a) $2\sqrt{3}$ (b) $2\sqrt[4]{3}$ (c) $2\sqrt[6]{3}$ (d) $2\sqrt{27}$

Solution:

(c) $\frac{16\sqrt[3]{9}}{8\sqrt{3}} = \frac{16}{8}\times\frac{9^{\frac{1}{3}}}{3^{\frac{1}{2}}} = 2\times\frac{\left(9^2\right)^{\frac{1}{6}}}{\left(3^3\right)^{\frac{1}{6}}} = 2\times\left(\frac{81}{27}\right)^{\frac{1}{6}} = 2\times 3^{\frac{1}{6}} = 2\sqrt[6]{3}$

54. $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}} = ?$

(a) $\sqrt{2}$ (b) 4 (c) $2^{\frac{7}{8}}$ (d) $2^{\frac{15}{16}}$

Solution:

(d) $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}} = \sqrt{2\sqrt{2\sqrt{2\times 2^{\frac{1}{2}}}}} = \sqrt{2\sqrt{2\sqrt{2^{\frac{3}{2}}}}}$

$\sqrt{2\sqrt{2\times 2^{\frac{3}{4}}}} = \sqrt{2\sqrt{2^{\frac{7}{4}}}} = \sqrt{2\times 2^{\frac{7}{8}}} = \sqrt{2^{\frac{15}{8}}} = 2^{\frac{15}{16}}$

Chapter 15

RATIO AND PROPORTION - I

In this chapter, we will discuss the basic rules of 'Ratio and Proportion' and their applications.

Ratio between two numbers 'x' and 'y' is written as x : y. It can also be written as $\frac{x}{y}$. The first term in a ratio is called **antecedent** and the second term is called **consequent**.

Note: Ratio between two quantities is always expressed in the same units.

Note: Multiplication or division of each term of a ratio by the same non-zero number does not effect the ratio. But the same is not true for Addition or Subtraction.

Example:

3 : 5 is same as 6 : 10 or 30 : 50.

Proportion: The equality of two ratios is called Proportion. a, b, c and d are in proportion, if a : b = c : d. It can also be written as a : b :: c : d

In any proportion, the first and fourth proportional are called the **extreme terms** while the second and the third proportional are called **mean terms**.

In a proportion, **Product of Extremes = Product of Means**

i.e. $a \times d = b \times c$

Inverse Ratio: If ratio between two numbers is a : b.

Then the inverse ratio is $\frac{1}{a} : \frac{1}{b}$ or b : a.

Direct Proportion: When one quantity is increased (or decreased), the other quantity also increases (or decreases) in the same ratio.

Examples:

Number of pens and total cost of pens.

Cost per pen and total cost of certain number of pens.

Indirect or Inverse proportion : When one quantity is increased, the other quantity decreases in the same ratio and vice-versa.

Examples :

Number of workers and days taken to do a work.

Price of a pen and number of pens that can be purchased for a certain amount.

Speed and time taken to cover a distance.

Mean Proportional: If 'x' is Mean or Second Proportion of 'a' and 'b'.

Then a : x = x : b

$\therefore x^2 = ab$

$\therefore x = \sqrt{ab}$

Third Proportional: If 'x' is Third Proportion of 'a' and 'b'.

Then a : b = b : x

$\therefore\ x = \frac{b^2}{a}$

Fourth Proportional: If 'x' is Fourth Proportion of 'a', 'b' and 'c'.

Then a : b = c : x

$\therefore\ x = \frac{bc}{a}$

RULES

1. If a : b = m : n and b : c = p : q

   Then, $a : c = \frac{m}{n} \times \frac{p}{q}$

   Note: This rule can be extended for any number of ratios.

2. If 'x' times of a number is equal to 'y' times of second number

   Then ratio between two numbers $= \frac{1}{x} : \frac{1}{y} = y : x$

3. What must be added to each of the four numbers *a*, *b*, *c* and *d* so that the resultant numbers are in proportion ?

   Solution :

   Let the number to be added is *x*.
   Then $(a + x) : (b + x) = (c + x) : (d + x)$
   $\therefore\ (a + x) \times (d + x) = (b + x) \times (c + x)$
   $\therefore\ ad + (a + d)x + x^2 = bc + (b + c)x + x^2$
   $\therefore\ [(a + d) - (b + c)]x = bc - ad$

   $\therefore\ x = \frac{bc - ad}{(a+d)-(b+c)}$

4. What must be subtracted from each of the four numbers *a*, *b*, *c* and *d* so that the resultant numbers are in proportion. ?

   Solution :

   Let the number to be added is *x*.
   Then $(a - x) : (b - x) = (c - x) : (d - x)$
   $\therefore\ (a - x) \times (d - x) = (b - x) \times (c - x)$
   $\therefore\ ad - (a + d)x + x^2 = bc - (b + c)x + x^2$
   $\therefore\ ad - bc = [(a + d) - (b + c)]x$

   $\therefore\ x = \frac{ad - bc}{(a+d)-(b+c)}$

SOLVED EXERCISE

1. A man's expenditure on food is $\frac{3}{5}$ of his expenditure on clothes, find ratio between his expenditure on the two items.

   (a) 2 : 5 (b) 3 : 5 (c) 2 : 3 (d) 5 : 3

Solution:

(b) Let his expenditure on clothes is Rs. 5.

Then expenditure on food is Rs. 3.

$\therefore$ Ratio of expenditure on food and clothes = 3 : 5.

2. Mean proportional of 32 and 2 is:

(a) 1 (b) 8 (c) 16 (d) 64

Solution:

(b) Let mean proportional to 32 and 2 is x.

Then $32 : x = x : 2$

$\therefore\ x = \sqrt{32 \times 2} = 8$

3. Third proportional to 16 and 4 is:

(a) 1 (b) 4 (c) 12 (d) 64

Solution:

(a) Let third proportional to 16 and 4 is x.

Then $16 : 4 = 4 : x$

$$\therefore\ x = \frac{4 \times 4}{16} = 1$$

4. Fourth proportional to 6, 9 and 20 is:

(a) 5 (b) 23 (c) 30 (d) 45

Solution:

(c) Let fourth proportional to 6, 9 and 20 is x.

Then $6 : 9 = 20 : x$

$$\therefore\ x = \frac{9 \times 20}{6} = 30$$

5. What must be added to each of the numbers 7, 10, 19 and 25 so that the resultant numbers are in proportion ?

(*a*) 2 (*b*) 3 (*c*) 4 (*d*) 5

Solution :

(*d*) Let the number is *x*.

$$x = \frac{10 \times 19 - 7 \times 25}{(7+25)-(10+19)} = \frac{15}{3} = 5$$

6. What must be deducted from each of the number 7, 10, 12 and 18 so that the resultant numbers are in proportion ?

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

Solution :

(*b*) Let the number is *x*.

$$x = \frac{7 \times 18 - 10 \times 12}{(7+18)-(10+12)} = \frac{6}{3} = 2$$

7. 400 chocolates were distributed among the students of a class. If each student got as many chocolates as the number of students in the class. Find the number of students in the class.

(a) 10 (b) 20 (c) 40 (d) 50

Solution:

(b) Let Number of students = Chocolate each student got = x

Then $x^2 = 400$

$x = \sqrt{400} = 20$

8. 2700 chocolates were distributed among the students of a class. If each student got 3 times as many chocolates as the number of students in the class. Find the number of students in the class.

(a) 30 (b) 45 (c) 60 (d) 90

Solution:

(a) Let students = x

Then each student get = 3x chocolates

$\therefore 3x^2 = 2700$

$\therefore x = \sqrt{900} = 30$

9. Find the ratio between $2^{2.5}$ and $2^{0.5}$.

(*a*) 2 : 1 (*b*) 3 : 1 (*c*) 4 : 1 (*d*) 5 : 1

Solution :

(*c*) $2^{2.5} : 2^{0.5} = 2^2 \times 2^{0.5} : 2^{0.5} = 2^2 : 1 = 4 : 1$

10. If 21 pens cost Rs. 35, find the cost of 30 pens.

(a) Rs. 18 (b) Rs. 26 (c) Rs. 44 (d) Rs. 50

Solution:

(d) Let cost of 30 pens is Rs. x.

Then 21 : 35 = 30 : x (Direct Proportion)

$\therefore x = \frac{35 \times 30}{21} =$ Rs.50

11. A 6 metre long pole casts 15 metre long shadow. Find the length of second pole if its shadow is 25metre.

(a) 10 metre (b) 12.5 metre (c) 25 metre (d) 62.5 metre

Solution:

(a) Let the length of second pole = x metre.

Then 6 : 15 = x : 25 (Direct Proportion)

$\therefore x = \frac{6 \times 25}{15} = 10$ metre

12. If 5 times a number is equal to 3 times another number. Find ratio between two numbers.

(a) 2 : 3 (b) 4 : 5 (c) 3 : 5 (d) 5 : 3

Solution:

(c) Ratio between two numbers $= \frac{1}{5} : \frac{1}{3} = 3 : 5$

13. If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d = ?

(a) 2 : 7 (b) 16 : 35 (c) 12 : 21 (d) None of these

Solution:

(b) $\frac{a}{d} = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} = \frac{16}{35}$

$\therefore$ a : d = 16 : 35

14. If a : b = 2 : 3 and b : c = 4 : 5, then a : b : c = ?

(a) 8 : 12 : 15 (b) 2 : 3 : 5 (c) 2 : 4 : 5 (d) 8 : 12 : 20

Solution:

(a) In such questions, common ratios are made equal to their LCM.

Here 'b' is common in a : b and b : c .

The values of 'b' are 3 and 4 in the given ratios.

LCM of 3 and 4 is 12.

We multiply (a : b) by 4 and (b : c) by 3 to get value of 'b' equal to 12 in both the ratios.

a : b = 2 : 3 = 4 × (2 : 3) = 8 : 12

b : c = 4 : 5 = 3 × (4 : 5) = 12 : 15

$\therefore$ a : b : c = 8 : 12 : 15

15. If a : b = 3 : 4 and b : c = 8 : 9, then a : b : c = ?

(a) 3 : 8 : 9 (b) 6 : 8 : 9 (c) 3 : 6 : 9 (d) 3 : 4 : 9

Solution:

(b) Here 'b' is common in a : b and b : c.

The values of 'b' are 4 and 8 in the given ratios.

LCM of 4 and 8 is 8.

We multiply (a : b) by 2 to get value of 'b' equal to 8 and (b : c) remains same.

a : b = 3 : 4 = 2 × (3 : 4) = 6 : 8

b : c = 8 : 9

$\therefore$ a : b : c = 6 : 8 : 9

16. If a : b = 3 : 4 and b : c = 6 : 7, then a : b : c = ?

(a) 9 : 12 : 14 (b) 6 : 8 : 21 (c) 3 : 6 : 7 (d) 3 : 4 : 7

Solution:

(a) LCM of 4 and 6 is 12.

We multiply (a : b) by 3 and (b : c) by 2 to get values of 'b' equal to 12 in both the ratios.

a : b = 3 : 4 = 3 × (3 : 4) = 9 : 12

b : c = 6 : 7 = 2 × (6 : 7) = 12 : 14

$\therefore$ a : b : c = 9 : 12 : 14

17. If $a : c = 2 : 3$ and $b : c = 8 : 9$, then $a : b : c$ = ?

(*a*) 2 : 8 : 9 (*b*) 2 : 8 : 3 (*c*) 4 : 8 : 9 (*d*) 6 : 8 : 9

Solution :

(*d*) $a : c = 2 : 3 = 3 \times (2 : 3) = 6 : 9$

$b : c = 8 : 9$

$\therefore a : b : c = 6 : 8 : 9$

18. A's share is twice of B's. 3 times B's share is equal to 4 times C's share. Find A : B : C.

(a) 1 : 2 : 4 (b) 1 : 3 : 4 (c) 6 : 3 : 4 (d) 8 : 4 : 3

Solution:

(d) $A = 2B \Rightarrow A : B = 2 : 1$

$3B = 4C \Rightarrow B : C = 4 : 3$

LCM of 1 and 4 is 4.

$A : B = 2 : 1 = 4 \times (2 : 1) = 8 : 4$

$B : C = 4 : 3$

$\therefore A : B : C = 8 : 4 : 3$

19. If $\frac{A}{B} = \frac{B}{C} = \frac{2}{3}$, then A : B : C = ?

(a) 2 : 2 : 3 (b) 2 : 3 : 4 (c) 2 : 3 : 3 (d) 4 : 6 : 9

Solution:

(d) $A : B = 2 : 3 = 2 \times (2 : 3) = 4 : 6$

$B : C = 2 : 3 = 3 \times (2 : 3) = 6 : 9$

$\therefore A : B : C = 4 : 6 : 9$

20. If $\frac{A}{2} = \frac{B}{3} = \frac{C}{4}$, Then A : B : C =?

(a) 2 : 3 : 4 (b) 3 : 4 : 5 (c) 4 : 3 : 2 (d) 6 : 4 : 3

Solution:

(a) Let $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x$

Then $A = 2x$, $B = 3x$, $C = 4x$

$\therefore A : B : C = 2x : 3x : 4x = 2 : 3 : 4$

21. $\frac{1}{2} : \frac{1}{3} : \frac{1}{4} = ?$

(a) 3 : 4 : 5 (b) 2 : 3 : 4 (c) 4 : 3 : 2 (d) 6 : 4 : 3

Solution:

(d) LCM of denominators of the fractions i.e. LCM of 2, 3 and 4 is 12.

$\therefore$ We multiply each ratio by 12.

The ratio becomes $\frac{1}{2} \times 12 : \frac{1}{3} \times 12 : \frac{1}{4} \times 12 = 6 : 4 : 3$

22. $\frac{1}{2} : \frac{2}{3} : \frac{3}{4} = ?$

(a) 1 : 2 : 3 (b) 2 : 3 : 4 (c) 4 : 3 : 2 (d) 6 : 8 : 9

Solution:

(d) LCM of denominators of the fractions i.e. LCM of 2, 3 and 4 is 12.

∴ We multiply each ratio by 12.

The ratio becomes $\frac{1}{2} \times 12 : \frac{2}{3} \times 12 : \frac{3}{4} \times 12 = 6 : 8 : 9$

23. If 2A = 3B = 4C, then A : B : C = ?

(a) 3 : 4 : 5 (b) 2 : 3 : 4 (c) 4 : 3 : 2 (d) 6 : 4 : 3

Solution:

(d) Given 2A = 3B = 4C

LCM of 2, 3, 4 is 12.

∴ We make each part equal to the LCM i.e. 12.

Then $2A = 12 \Rightarrow A = \frac{12}{2} = 6$

And $3B = 12 \Rightarrow B = \frac{12}{3} = 4$

And $4C = 12 \Rightarrow C = \frac{12}{4} = 3$

∴ A : B : C = 6 : 4 : 3

24. Rs. 700 is divided between A and B in the ratio of 3 : 4. Find A's share.

(a) Rs. 300 (b) Rs. 350 (c) Rs. 400 (d) Rs. 450

Solution:

(a) Sum of ratios = 3 + 4 = 7

∴ A's share = $\frac{3}{7} \times 700$ = Rs. 300

25. If Rs. 700 is divided between A and B in the ratio of $\frac{1}{3} : \frac{1}{4}$. Find A's share.

(a) Rs. 300 (b) Rs. 325 (c) Rs. 350 (d) Rs. 400

Solution:

(d) Ratio between A and B is $\frac{1}{3} : \frac{1}{4} = 4 : 3$

∴ A's share = $\frac{4}{7} \times 700$ = Rs. 400

26. Rs. 561 is to be divided among A, B, C in the ratio of 2 : 4 : 5. How much will C get more than B?

(a) Rs. 51 (b) Rs. 52 (c) Rs. 53 (d) Rs. 255

Solution:

(a) Sum of ratios = 2 + 4 + 5 = 11.

∴ C's share exceeds B's by 5 – 4 = 1 part for every 11 parts.

∴ C's share exceeds B's by $\frac{1}{11} \times 561$ = Rs. 51

27. A certain sum of money is divided between A and B in the ratio of 3 : 7. If B gets Rs. 800 more than A, find the total sum.

(a) Rs. 240 (b) Rs. 320 (c) Rs. 560 (d) Rs. 2000

Solution:

(d) Sum of ratios = 3 + 7 = 10.

B's share exceeds A's by 7 – 3 = 4 parts for every 10 parts.

∴ Total sum = Rs. 800 × $\frac{10}{4}$ = Rs. 2000

28. Rs. 430 is divided among A, B and C in such a way that if A gets Rs. 2, B gets Rs. 3 and if B gets Rs. 5, C gets Rs. 6. Find the share of B.

(a) Rs. 150 (b) Rs. 300 (c) Rs. 225 (d) Rs. 400

Solution:

(a) A : B = 2 : 3 and B : C = 5 : 6

LCM of 3 and 5 is 15.

A : B = 2 : 3 = 5 × (2 : 3) = 10 : 15

B : C = 5 : 6 = 3 × (5 : 6) = 15 : 18

∴ A : B : C = 10 : 15 : 18

Sum of ratios = 10 + 15 + 18 = 43

∴ B's share = $\frac{15}{43}$ × Rs. 430 = Rs. 150

29. A sum of Rs. 1200 is divided among A, B and C, so that A receives half of what B and C receive together. What is A's share?

(a) Rs. 600 (b) Rs. 400 (c) Rs. 500 (d) Rs. 480

Solution:

(b) Let B and C together receive Rs. 2.

Then A's share = Rs. 2 × $\frac{1}{2}$ = Re. 1

A : (B + C) = 1 : 2

Sum of ratios = 1 + 2 = 3

∴ A's share = $\frac{1}{3}$ × 1200 = Rs. 400

30. Rs. 7000 is divided among A, B and C in such a way that A receives $\frac{3}{7}$ of what B and C receive together. What is the share of A?

(a) Rs. 4000 (b) Rs. 3000 (c) Rs. 2100 (d) Rs. 1400

Solution:

(c) Let B and C together receive Rs.7.

Then A's share = Rs. 7 × $\frac{3}{7}$ = Rs. 3

∴ A : (B + C) = 3 : 7

∴ A's share = $\frac{3}{10}$ × 7000 = Rs. 2100

31. Rs. 4060 is to be divided among A, B and C in such a way that A gets $\frac{1}{4}$ of what B and C together get. Find A's share.

(a) Rs. 812 (b) Rs. 1015 (c) Rs. 1025 (d) Rs. 1035

Solution:

(a) A : (B + C) = 1 : 4

Sum of the ratios is 1 + 4 = 5

∴ A's share = $\frac{1}{5} \times 4060$ = Rs. 812

32. A certain sum is divided among A, B and C in such a way that A gets 3 times as much as B, B receives 2 times as much as C. If A receives Rs. 1500 more than C, what is the sum divided?

(a) Rs. 1500 (b) Rs. 2500 (c) Rs. 2700 (d) Rs. 3000

Solution:

(c) A = 3B and B = 2C

∴ A : B = 3 : 1 and B : C = 2 : 1

∴ A : B : C = 6 : 2 : 1

Sum of ratios = 6 + 2 + 1 = 9.

Difference between the shares of A and C = 6 – 1 = 5.

If difference between shares of A and C is Rs. 5, total sum = Rs. 9.

If difference between shares of A and C is Rs. 1500, then total sum

= Rs. 1500 × $\frac{9}{5}$ = Rs. 2700

33. Divide Rs. 1570 between A and B so that Rs.25 being deducted from A's share and Rs. 45 from B's share, their share becomes 2 : 3. Find the amount received by A.

(a) Rs. 625 (b) Rs. 628 (c) Rs. 942 (d) Rs. 945

Solution:

(a) Total amount to be deducted = Rs. 25 + Rs. 45 = Rs. 70

Remaining amount = Rs. 1570 – Rs. 70 = Rs. 1500 is to be distributed between A and B in the ratio of 2 : 3.

∴ A's share in Rs. 1500 = $\frac{2}{5}$ × Rs. 1500 = Rs. 600

∴ A's share in Rs. 1570 = Rs. 600 + Rs. 25 = Rs. 625

34. A sum of Rs. 2550 was divided among A, B and C in such a way that if Rs. 10, Rs. 20 and Rs. 20 respectively deducted from their shares, the balances are in the ratio of 5 : 7 : 13. Find the share received by *A*.

(*a*) Rs. 450 (*b*) Rs. 510 (*c*) Rs. 580 (*d*) Rs. 670

Solution :

(*b*) Divide Rs. 2550 – (10 + 20 + 20) = Rs. 2500 among *A*, *B* and *C* in the ratio of 5 : 7 : 13.

∴ *A*'s share in Rs. 2500 = Rs. $2500 \times \frac{5}{5+7+13}$ = Rs. $2500 \times \frac{5}{25}$ = Rs. 500

∴ *A*'s total share = Rs. 500 + Rs. 10 = Rs. 510

35. The cost of 3 horses is same as the cost of 5 cows. If total cost of 4 horses and 6 cows is Rs. 1900, find the cost of one horse.

(*a*) Rs. 50 (*b*) Rs. 150 (*c*) Rs. 200 (*d*) Rs. 250

Solution :

(*d*) Ratio of cost of horse and cow $= \frac{1}{3} : \frac{1}{5} = 5 : 3$

Let cost of horse and cow is $5x$ and $3x$ respectively.

Then $4 \times 5x + 6 \times 3x =$ Rs. 1900

$$\therefore\ x = \frac{1900}{38} = \text{Rs.}50$$

$\therefore$ Cost of one horse = 5 × Rs. 50 = Rs. 250

36. When a ball bounces, it rises to $\frac{2}{3}$ of the height from which it falls. If the ball is dropped from a height of 27 metre, how high will it rise in the third bounce?

(a) 8 metre (b) 9 metre (c) 15 metre (d) 18 metre

Solution:

(a) Height at third bounce $= 27 \times \left(\frac{2}{3}\right)^3 = 8$ metre

37. A container has 500 litres of milk. From the container , 100 litres of milk is taken out and is replaced by water. This process is repeated twice. How much milk is left in the container now ?

(*a*) 200 litres (*b*) 228 litres (*c*) 256 litres (*d*) 312 litres

Solution :

(*c*) When 100 litres of milk is taken out, the remaining milk in the container

$= \frac{500-100}{500} = \frac{4}{5}$ of the original quantity

$\therefore$ Milk after 3 replacements $= 500 \times \left(\frac{4}{5}\right)^3 = 256$ litres

38. If $\frac{a}{b} = \frac{5}{2}$, then $\frac{a+b}{a-b} = ?$

(a) $\frac{3}{7}$ (b) $\frac{7}{3}$ (c) $\frac{2}{5}$ (d) $\frac{7}{2}$

Solution:

(b) $\frac{a+b}{a-b} = \frac{5+2}{5-2} = \frac{7}{3}$

39. If a : b = 4 : 3, then (5a + 2b) : (5a – 2b) = ?

(a) 8 : 7 (b) 13 : 7 (c) 7 : 3 (d) 5 : 2

Solution:

(b) $\frac{5a+2b}{5a-2b} = \frac{5\times4+2\times3}{5\times4-2\times3} = \frac{20+6}{20-6} = \frac{26}{14} = \frac{13}{7} = 13 : 7$

40. If $\frac{a}{b} = \frac{4}{3}$, then, (3a + 2b) : (3a – 2b) = ?

(a) 4 : 3 (b) 3 : 2 (c) 3 : 1 (d) 1 : 1

Solution:

(c) $\frac{3a + 2b}{3a - 2b} = \frac{3 \times 4 + 2 \times 3}{3 \times 4 - 2 \times 3} = \frac{12 + 6}{12 - 6} = \frac{18}{6} = 3 : 1$

41. If $\frac{a}{2} = \frac{b}{3} = \frac{c}{5}$; then, $\frac{a + b + c}{c} = ?$

(a) 2 (b) 3 (c) 5 (d) 10

Solution:

(a) Let $\frac{a}{2} = \frac{b}{3} = \frac{c}{5} = k$

Then a = 2k, b = 3k and c = 5k

$\therefore \frac{a + b + c}{c} = \frac{2k + 3k + 5k}{5k} = \frac{10k}{5k} = 2$

42. If $\frac{a}{b} = \frac{2}{3}$ and $\frac{b}{c} = \frac{4}{5}$, then (a + b) : (b + c) = ?.

(a) 3 : 4 (b) 4 : 5 (c) 5 : 9 (d) 20 : 27

Solution:

(d) a : b = 2 : 3 and b : c = 4 : 5.

$\therefore$ a : b : c = 8 : 12 : 15

$\therefore \frac{a + b}{b + c} = \frac{8 + 12}{12 + 15} = \frac{20}{27} = 20 : 27$

43. If a : b = c : d = e : f = 1 : 2, then (ra + sc + te) : (rb + sd + tf) = ?

(a) 1 : 2 (b) 1 : 4 (c) 1 : 8 (d) 2 : 3

Solution:

(a) Given, $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{1}{2}$

$\therefore$ (ra + sc + te) : (rb + sd + tf)

= (r + s + t) × 1 : (r + s + t) × 2 = 1 : 2

44. If a : b = c : d; then $\frac{ma + nc}{mb + nd} = ?$

(a) m : n (b) a : b (c) b : a (d) n : m

Solution:

(b) Let $\frac{a}{b} = \frac{c}{d} = k$

Then a = bk, and c = dk

$\therefore \frac{ma + nc}{mb + nd} = \frac{mbk + ndk}{mb + nd} = k = \frac{a}{b} = a : b$

45. If ratio of boys and girls in a class is 7 : 5, which of the following cannot be the total number of students in the class?

(a) 36 (b) 50 (c) 60 (d) 120

Solution:

(b) Sum of the ratios = 7 + 5 = 12.

50 is the only number which is not divisible by 12 (i.e. sum of the ratios) and students cannot be in fractions.

46. A man spends $\frac{3}{5}$ of his income and still saves Rs. 600. His income is:

(a) Rs. 240 (b) Rs. 360 (c) Rs. 1000 (d) Rs. 1500

Solution:

(d) His savings $= 1-\frac{3}{5}=\frac{2}{5}$ of income.

But actual savings = Rs. 600

$\therefore$ Income = Rs. 600 × $\frac{5}{2}$ = Rs. 1500

47. A man spends $\frac{1}{5}$ of his income on food and $\frac{3}{5}$ on clothing. If he still saves Rs. 100, what is his total income?

(a) Rs. 20 (b) Rs. 80 (c) Rs. 400 (d) Rs. 500

Solution:

(d) Total expenditure $=\frac{1}{5}+\frac{3}{5}=\frac{4}{5}$

His savings $= 1-\frac{4}{5}=\frac{1}{5}$ of his income

But actual savings = Rs. 100

$\therefore$ His income = Rs. 100 × $\frac{5}{1}$ = Rs. 500

48. A man won a lottery of Rs. 2000000. He gave $\frac{2}{5}$ of the amount to his wife and $\frac{1}{4}$ of the balance to his son. He donated $\frac{2}{9}$ of the balance amount. How much money was donated ?

(*a*) Rs. 100000 (*b*) Rs. 150000 (*c*) Rs. 200000 (*d*) Rs. 250000

Solution :

(*c*) Amount donated $= \text{Rs. } 2000000\times\left(1-\frac{2}{5}\right)\times\left(1-\frac{1}{4}\right)\times\frac{2}{9}$

$= \text{Rs. } 2000000\times\frac{3}{5}\times\frac{3}{4}\times\frac{2}{9} = \text{Rs. } 200000$

49. A man spends $\frac{2}{5}$ of his income on food and $\frac{4}{5}$ of the remaining income on clothing. If he still saves Rs. 600, what is his total income?

(a) Rs. 480 (b) Rs. 500 (c) Rs. 1920 (d) Rs. 5000

Solution:

(d) Expenditure on food $= \frac{2}{5}$

Balance $= 1 - \frac{2}{5} = \frac{3}{5}$

Expenditure on clothes $= \frac{4}{5} \times \frac{3}{5} = \frac{12}{25}$

Savings $= \frac{3}{5} - \frac{12}{25} = \frac{3}{25}$

$\therefore$ Income $= \frac{25}{3} \times$ Rs. 600 = Rs. 5000

Alternative Method :

Expenditure on food $= \frac{2}{5}$

Balance $= 1 - \frac{2}{5} = \frac{3}{5}$

Expenditure on clothing is $\frac{4}{5}$ of the balance amount.

Savings $= 1 - \frac{4}{5} = \frac{1}{5}$

$\therefore$ Income = Rs. $600 \times \frac{5}{1} \times \frac{5}{3}$ = Rs. 5000

50. A man gives $\frac{1}{3}$ of his money to his wife, $\frac{1}{4}$ of the remaining to his son and $\frac{2}{5}$ of the remaining to his daughter. If he still have a balance of Rs. 3000, how much money does he had originally?

(a) Rs. 100 (b) Rs. 2250 (c) Rs. 6000 (d) Rs. 10000

Solution:

(d) Table showing distribution of amount

	Share	Balance	Inverse
Wife	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{2}$
Son	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{4}{3}$
Daughter	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{5}{3}$

Income = Rs. $3000 \times \frac{5}{3} \times \frac{4}{3} \times \frac{3}{2}$ = Rs. 10000

51. A vessel contains 100 litres of pure milk. 10 litres of milk is taken from the vessel and is replaced by 10 litres of water. Now, 20 litres of the mixture is taken from the vessel and is replaced by water. What is the quantity of milk left in the vessel?

(a) 67 litres (b) 70 litres (c) 72 litres (d) 75 litres

Solution:

(c) Quantity of milk left in the vessel

$$= 100 \times \frac{100\text{-}10}{100} \times \frac{100-20}{100}$$

$$= 100 \times \frac{9}{10} \times \frac{8}{10} = 72 \text{ litres}$$

52. A and B joined a partnership business by investing Rs. 30000 and Rs. 50000 respectively. If they earn a profit of Rs. 4000, find A's share in the profit.

(a) Rs. 1500 (b) Rs. 2000 (c) Rs. 2500 (d) Rs. 1000

Solution:

(a) Since period for which the two amounts is invested is same.

∴ Ratio in which profit is to distributed between A and B is

30000 : 50000 = 3 : 5

∴ A's share in profit = $\frac{3}{8} \times 4000$ = Rs. 1500

53. A and B joined a partnership business. In which A invested Rs. 10000 for 6 months and B invested Rs. 5000 for 8 months. If they earn a profit of Rs. 3000, find B's share in the profit.

(a) Rs. 1800 (b) Rs. 1500 (c) Rs. 1200 (d) Rs. 1000

Solution:

(c) Since periods for which the amounts were invested are not same.

∴ Ratio in which profit is to distributed between A and B is

$10000 \times 6 : 5000 \times 8 = 3 : 2$

∴ B's share = $\frac{2}{5} \times 3000 = 1200$

54. A, B and C rented a pasture. A puts in 15 cows for 4 months, B puts in 14 cows for 5 months and C puts in 20 cows for 2 months. If they pay Rs. 170 towards rent of the pasture, find the amount A will pay as his share of rent.

(a) Rs. 40 (b) Rs. 50 (c) Rs. 60 (d) Rs. 70

Solution:

(c) Ratio of shares of A, B and C in the rent of pasture is

$15 \times 4 : 14 \times 5 : 20 \times 2 = 6 : 7 : 4$

∴ A's share in the rent = Rs. $170 \times \frac{6}{17}$ = Rs. 60

55. Ratio between two numbers is 3 : 4 and sum of their squares is 625. The numbers are:

(a) 6, 8 (b) 9, 12 (c) 12, 16 (d) 15, 20

Solution:

(d) Let the numbers are 3x and 4x.

Then $(3x)^2 + (4x)^2 = 625$

$\therefore 9x^2 + 16x^2 = 625$

$\therefore x = \sqrt{\frac{625}{25}} = \sqrt{25} = 5$

$\therefore$ The numbers are 3 × 5 and 4 × 5 i.e. 15 and 20.

56. **Ratio between two numbers is 5 : 3 and difference between their squares is 144. Find the numbers.**

(*a*) 5, 3 (*b*) 10, 6 (*c*) 15, 9 (*d*) 20, 12

Solution :

(*c*) Let the numbers are $5x$ and $3x$.

Then difference between their squares = $(5x)^2 - (3x)^2$

$\therefore \quad 25x^2 - 9x^2 = 16x^2 = 144$

$\therefore \quad x^2 = \frac{144}{16} = 9$

$\therefore \quad x = \sqrt{9} = 3$

$\therefore$ The numbers are 5 × 3 and 3 × 3 i.e. 15 and 9 respectively.

Chapter 16

RATIO AND PROPORTION - II

In this chapter, we will discuss some advance rules of Ratio and Proportion and how their application makes solution of a problem very easy.

FORMULAE AND RULES

1. If sum of two numbers is 'x' and their difference is 'y'.

   Then the numbers are in the ratio of (x + y) : (x – y).

Proof:

Let the numbers are 'a' and 'b'.

Then a + b = x, and a – b = y

Now $(a + b) + (a - b) = x + y \Rightarrow a = \frac{x+y}{2}$

And $(a + b) - (a - b) = x - y \Rightarrow b = \frac{x-y}{2}$

$\therefore a : b = \frac{x+y}{2} : \frac{x-y}{2} = (x + y) : (x - y)$

Note: The two numbers 'a' and 'b' are

$\frac{x+y}{2}$ and $\frac{x-y}{2}$ respectively,

2. If two numbers are in the ratio x : y.

   On **adding** a constant number **'a' to both the numbers**, the **new ratio becomes (x + 1) : (y + 1), Or**

   On subtracting 'a' from both the numbers, the ratio becomes (x – 1) : (y – 1)

   Then the original numbers are 'ax' and 'ay' respectively.

3. If two numbers are in the ratio x : y.

   On **adding** a constant number **'a' to the first number**, the **new ratio becomes (x + 1) : (y)**, Or

   On subtracting 'a' from the first number, the ratio becomes (x – 1) : (y)

   Then the original numbers are 'ax' and 'ay' respectively.

4. Two numbers are in the ratio a : b where a > b. On adding k, a constant number to second number, new ratio becomes 1 : 1.

   Then the original numbers are ax and ay respectively.

   Where $x = \frac{k}{a-b}$ **and** $k = (a - b) \times x$

Proof :

Let the original numbers are ax and bx

Then $\frac{ax}{bx+k} = \frac{1}{1}$

$\Rightarrow$ ax = bx + k $\Rightarrow$ (a − b)x = k

$\therefore$ $x = \frac{k}{a-b}$

5. If two numbers are in the ratio a : b.

   On **adding 'k'**, a constant number **to one of the numbers, new ratio becomes m : n,** i.e. both the ratios changes instead of one ratio only.

Method:

1. Multiply the ratios (old, new or both) so that the old and new ratios become same for the number, to which 'k' is not added to or subtracted from.
2. Find increase or decrease in the revised new ratio in comparison to revised old ratio.
3. Number added to get the desired change in the ratio is:

$$\frac{\text{Difference in revised old and new ratios}}{\text{Sum of the revised old ratio}} \times \text{Total quantity (initial)}$$

6. Two numbers are in the ratio a : b.
   If **'k'**, a constant number is **added to both the numbers, new ratio of the numbers becomes m : n.**

   Then the original numbers are 'ax' and 'bx',

   Where $x = \frac{(m-n)k}{an-bm}$

$$= \frac{\text{Difference between new ratios} \times \text{Number added to both numbers}}{\text{Difference between cross multiplication of the two ratios}}$$

Proof:

Let the original numbers are 'ax' and 'bx'.

Then $\frac{ax+k}{bx+k} = \frac{m}{n}$

$\therefore$ anx + nk = bmx + mk

$\therefore$ (an − bm)x = (m − n)k

$\therefore$ $x = \frac{(m-n)k}{an-bm}$

Note: The incomes of two persons are in the ratio of a : b and their expenditure are in the ratio of m : n. If both saves Rs. S, then their incomes are ax and bx

$$\text{where } x = \frac{S(n-m)}{an-bm}$$

Note: If the number is deducted, then 'k' takes negative value.

Constant number added to both the numbers is

$$k = \frac{an - bm}{m - n} \times x$$

7. Two numbers are in the ratio a : b.
If **'k'**, a constant number is **added to the first number** and **deducted from the second, new ratio becomes m : n.**
Then the original numbers are 'ax' and 'bx',

$$\text{Where } x = \frac{(m+n)k}{bm - an}$$

$$= \frac{\text{Sum of new ratios} \times \text{Number added to / substracted from both numbers}}{\text{Difference between cross multiplication of the two ratios}}$$

Proof:

Let the original numbers are 'ax' and 'bx'.

$$\text{Then} \quad \frac{ax+k}{bx-k} = \frac{m}{n}$$

$$\therefore \quad anx + nk = bmx - mk$$

$$\therefore \quad (an - bm)x = -(m + n) k$$

$$\therefore \quad x = \frac{-(m+n)k}{an - bm}$$

$$\therefore \quad x = \frac{(m+n)k}{bm - an}$$

8. Two numbers are in the ratio a : b.

If **'p'** and **'q'**, two constant numbers are **added to first number and second number respectively, new ratio becomes m : n**.
Then, the original numbers are 'ax' and 'bx',

$$\text{Where } x = \frac{mq - np}{an - bm}$$

$$= \frac{\text{Difference between cross multiplication of new ratio and numbers added}}{\text{Difference between cross multiplicaton of the two ratios}}$$

Proof:

Let the original numbers are 'ax' and 'bx'.

$$\text{Then} \quad \frac{ax + p}{bx + q} = \frac{m}{n}$$

$$\therefore \quad anx + np = bmx + mq$$

$$\therefore \quad (an - bm)\, x = mq - np$$

$$\therefore \quad x = \frac{mq - np}{an - bm}$$

SOLVED EXERCISE

1. Sum of two numbers is 48 and difference between the numbers is 8. Find ratio between the numbers.
(a) 7 : 5 (b) 6 : 1 (c) 5 : 1 (d) 7 : 1

Solution:

(a) Ratio between the numbers = (48 + 8) : (48 – 8) = 56 : 40 = 7 : 5

2. If the sum of two numbers is 30 and their difference is 12. Find ratio between the numbers.

(a) 2 : 5 (b) 5 : 2 (c) 3 : 7 (d) 7 : 3

Solution:

(d) Ratio between the numbers = (30 + 12) : (30 – 12) = 42 : 18 = 7 : 3

3. Ratio between sum and difference of two numbers is 5 : 1. Find ratio between the numbers.

(a) 5 : 1 (b) 1 : 5 (c) 2 : 3 (d) 3 : 2

Solution:

(d) Ratio between the numbers is (5 + 1) : (5 – 1) = 6 : 4 = 3 : 2

4. The sum of two numbers is twice the difference between them. If the smaller number is 20, find the bigger number.

(a) 5 (b) 40 (c) 60 (d) 80

Solution:

(c) Ratio between sum and difference of the numbers is 2 : 1.

∴ Ratio between the numbers = (2 + 1) : (2 – 1) i.e. 3 : 1.

∴ Bigger number = 3 × 20 = 60.

5. A spends 90% of his salary and B spends 85% of his salary. But savings of both are equal. Find the income of B, if sum of their incomes is Rs. 5000.

(a) Rs. 2000 (b) Rs. 2500 (c) Rs. 3000 (d) Rs. 3500

Solution:

(a) Savings of A = (100 – 90)% = 10% of his salary.

Savings of B = (100 – 85)% = 15% of his salary.

But both saves equal amount.

∴ 10% of A's salary = 15% of B's salary

∴ A's salary : B's salary = 15 : 10 = 3 : 2

∴ B's salary = $\frac{2}{5} \times 5000$ = Rs. 2000.

6. A man has some hens and some cows. If the number of heads is 50 and number of feet is 142. The number of cows is:

(a) 21 (b) 25 (c) 27 (d) 29

Solution:

(a) Let the man has hens only.

Then total heads = 50 × 1 = 50.

And legs = 50 × 2 = 100 which is short by 42 from the actual legs i.e. 142.

Replacement of one cow with one hen means same number of heads and two more legs.

∴ Hens replaced with cows = $\frac{42}{2} = 21$

∴ Cows = 21

7. A sum of Rs. 350 made up of 110 coins, which are of either Re. 1 or Rs. 5 denomination. How many coins are of Rs. 5?

(a) 50 (b) 55 (c) 60 (d) 175

Solution:

(c) Let e coins are of Re.1 denomination.

Th al value of 110 coins = 110 × 1 = Rs. 110 which is short from Rs. 350 by Rs. 350 0 = Rs. 240.

ng 1 one-rupee coin with five-rupee coin means Rs. 4 extra.

-rupee coins = $\frac{240}{4}$ = 60 coins

8. Rs. 80 made up of 100 coins, which are either of Re.1 or 50 paise denomination. y coins are of 50 paise?

(b) 75 (c) 60 (d) 40

all the coins are of 50 paise denomination.

en total value of 100 coins = 100 × $\frac{1}{2}$ = Rs. 50 i.e. Rs. 30 less than Rs. 80.

eplacement of one fifty paise coin with one-rupee coin means 50 paise extra.

One-rupee coins = $\frac{30}{0.50}$ = 30 × 2 = 60 coins

∴ 50 paise coins = 100 – 60 = 40

20 is divided into two parts in such a way that 6 times of first part when added to 4 times of second part gives 104. Find the first part.

(a) 8 (b) 10 (c) 11 (d) 12

Solution:

(d) If we take 4 times of all the numbers, we get 80 which is short than 104 by 104 – 80 = 24.

We can increase the total by 2 (*i.e.* 6 – 4) by shifting one number from second part to first part.

∴ First part = $\frac{24}{2}$ = 12

10. The population of a village is 10000. In one year, male population increase by 6% and female population by 4%. If population at the end of the year is 10520, find size of male population in the village (originally).

(a) 4000 (b) 5000 (c) 6000 (d) 7500

Solution:

(c) Let the population consists of females only.

Then increase in the population is 4% of 10000 = 400

∴ Increased population = 10000 + 400 = 10400

But actual increased population = 10520

Difference = 10520 – 10400 = 120

120 is 2% (i.e. 6% – 4%) of the male population.

$\therefore$ Male population $= \frac{120}{2} \times 100 = 6000$

11. A group of 15 persons spends Rs. 4500 in 6 months, find total expenditure of a group of 25 persons in 4 months, if expenditure per person is same in both the cases

(a) Rs. 2000 (b) Rs. 3000 (c) Rs. 4000 (d) Rs. 5000

Solution:

(d) Let expenditure of 25 persons is x, then

15 : 25 Persons | Expenditure 4500 : x
6 : 4 Months |

More persons, more expenditure i.e. direct proportion

Less months, less expenditure i.e. direct proportion

$\therefore x = 4500 \times \frac{25}{15} \times \frac{4}{6} = \text{Rs. } 5000$

12. Two numbers are in the ratio 5 : 6. If 7 is added to each number, new ratio becomes 6 original numbers are:

(a) 30, 42 (b) 35, 36 (c) 35, 42 (d) 42, 49

Solution:

(c) Here on adding 7 to both the numbers, both ratios are increased by one.

$\therefore$ The numbers are 5 × 7 and 6 × 7 i.e. 35 and 42.

13. Two numbers are in the ratio 8 : 11. If we subtract 6 from each number, then new rati becomes 7 : 10. The numbers are:

(a) 48, 66 (b) 42, 60 (c) 40, 55 (d) 56, 77

Solution:

(a) On subtracting 6 from both the numbers, the new ratio is decreased by same ratio i.e. 1 in each case.

$\therefore$ The numbers are 8 × 6 and 11 × 6 = 48 and 66.

14. In a mixture of 63 litres, the milk and water are in the ratio 7 : 2. If 7 litres of water is added to the mixture, find the ratio in the resulting mixture.

(a) 3 : 1 (b) 4 : 1 (c) 7 : 3 (d) 7 : 5

Solution :

(c) Sum of ratios = 9

$\therefore$ Value of one ratio $= \frac{63}{9} = 7$

$\therefore$ New ratio = 7 : (2 + 1) = 7 : 3

15. The ratio of milk and water in a mixture of 84 litres is 3 : 4. What will be the new ratio if 3 litres each of milk and water is added to the mixture ?

(a) 7 : 9 (b) 4 : 5 (c) 13 : 17 (d) 9 : 16

Solution :

(c) $x = \frac{84}{3+4} = 12$

Quantity added $= \frac{3}{12}$ or $\frac{1}{4}$ of x

$\therefore$ New ratio $= \left(3+\frac{1}{4}\right):\left(4+\frac{1}{4}\right) = 13:17$

16. The ratio of numbers of boys and girls of a school with 504 students is 13 : 11. What will be the new ratio if 12 more girls are admitted ?

(a) 1 : 1 (b) 13 : 12 (c) 91 : 81 (d) 7 : 6

Solution :

(c) $x = \frac{504}{13+11} = \frac{504}{24} = 21$

$\therefore$ Girls admitted $= \frac{12}{21}$ or $\frac{4}{7}$ of x

$\therefore$ New ratio $= 13:\left(11+\frac{4}{7}\right) = 91:81$

17. Ratio of milk and water in a mixture is 4 : 1. If 5 litre of milk is added in the mixture, new ratio becomes 5 : 1. The quantity of milk in the mixture originally was:

(a) 10 litres (b) 15 litres (c) 20 litres (d) 25 litres

Solution:

(c) On adding 5 litres of milk, its ratio in the mixture is increased by 1.

$\therefore$ Original quantity of milk in the mixture was 4 × 5 = 20 litres.

18. Students in Class I, II and III of a school are in the ratio 3 : 5 : 8. Had 15 more students admitted to each class, the ratio would have become 6 : 8 : 11. How many total students were there in the beginning?

(a) 32 (b) 48 (c) 80 (d) 125

Solution:

(c) Increase in ratio for 3 classes is 6 – 3 = 8 – 5 = 11 – 8 = 3.

15 more students are admitted to each class.

$\therefore 3 \equiv 15 \Rightarrow 1 \equiv 5$

$\therefore 3 + 5 + 8 = 16 \equiv 16 \times 5 = 80.$

Hence total students in the beginning were 80.

19. Ratio between the annual income of A and B is 9 : 8 and between their expenditure is 8 : 7. If they save Rs. 500 each. Find A's annual income.

(a) Rs. 3500 (b) Rs. 4000 (c) Rs. 4500 (d) Rs. 5000

Solution:

(c)	A		B
Income	9	:	8
Expenditure	8	:	7
Difference	1	:	1

Hence both are spending 1 ratio less than their incomes' ratio and hence saving Rs. 500 each.

∴ A's annual income = 9 × Rs. 500 = Rs. 4500

20. Incomes of A and B are in the ratio 3 : 2 and their expenditure are in the ratio 5 : 3. If they save Rs. 400 each, find A's income.

(a) Rs. 1200 (b) Rs. 1600 (c) Rs. 2400 (d) Rs. 3000

Solution:

(c)

	A		B
Income ratio = 3 : 2 =	6	:	4
Expenditure ratio	5	:	3
Difference =	1	:	1

Hence, both are spending 1 ratio less than their incomes' ratio and hence saving Rs. 400 each.

∴ A's income = 6 × Rs. 400 = Rs. 2400

21. A mixture of 70 litres contains milk and water in the ratio 3 : 4. How many litres of milk must be added to the mixture so as to make the ratio 5 : 4?

(a) 10 litres (b) 15 litres (c) 20 litres (d) 25 litres

Solution:

(c) Ratio of water is same i.e. 4 in both the cases.

	Milk	:	Water
New ratio	5	:	4
Old ratio =	3	:	4
Difference =	2	:	0

Sum of old ratios = 3 + 4 = 7

∴ Milk added = $\frac{2}{7} \times 70 = 20$ litres

22. 70 litres of a mixture contains milk and water in the ratio 7 : 3. Find the quantity of the water which must be added to the mixture to make the resultant mixture in the ratio of 7 : 5?

(a) 14 litres (b) 20 litres (c) 22 litres (d) 25 litres

Solution:

(a)

	Milk	:	Water
New ratio =	7	:	5
Old ratio =	7	:	3
Difference =	0	:	2

Sum of old ratios = 7 + 3 = 10

∴ Water to be added = $\frac{2}{10} \times 70 = 14$ litres

23. The ratio of males and females in a group is 3 : 2. How many females must join the group so that the ratio becomes 1 : 1, if there were 195 members originally?

(a) 19 (b) 39 (c) 190 (d) 200

Solution:

(b)

	Male	:	Female
New ratio = 1: 1 =	3	:	3
Old ratio =	3	:	2
Difference =	0	:	1

$\therefore$ Females joined = $\frac{1}{5} \times 195 = 39$

Trick:

Females must join = $\frac{3-2}{3+2} \times 195 = 39$

Hint: This formula can be used only when new ratio is 1 : 1.

24. The ratio of boys and girls in a class of 80 students is 5 : 3. How many more girls should join the class to make the ratio 1 : 1?

(a) 10 (b) 15 (c) 20 (d) 25

Solution:

(c)

	Boys	:	Girls
New ratio = 1 : 1 =	5	:	5
Old ratio =	5	:	3
Difference =	0	:	2

Sum of old ratios = 5 + 3 = 8

$\therefore$ Girls should join = $\frac{2}{8} \times 80 = 20$

Trick:

Girls should join = $\frac{5-3}{5+3} \times 80 = 20$

Hint: This formula can be used only when new ratio is 1 : 1.

25. 132 litres of a mixture contains milk and water in the ratio 8 : 3. How much water must be added to the mixture so as to make the ratio of milk and water 2 : 1?

(a) 11 litres (b) 12 litres (c) 13 litres (d) 15 litres

Solution:

(b) LCM of ratios of milk i.e. LCM of 8 and 2 is 8.

$\therefore$ We make ratio of milk equal to 8 in the two ratios.

	Milk	:	Water
New ratio = 2 : 1 =	8	:	4
Old ratio =	8	:	3
Difference =	0	:	1

$\therefore$ Water to be added = $\frac{1}{8+3} \times 132$ = 12 litres

26. A mixture of 36 litres contains milk and water in the ratio 5 : 4. On adding some water in the mixture the ratio becomes 2 : 3, find the quantity of water added to the mixture.

(a) 14 litres (b) 16 litres (c) 20 litres (d) 24 litres

Solution:

(a) LCM of ratios of milk i.e. LCM of 5 and 2 is 10.

∴ We make ratio of milk equal to 10 in the two ratios.

	Milk	:	Water
New ratio = 2 : 3 =	10	:	15
Old ratio = 5 : 4 =	10	:	8
Difference =	0	:	7

∴ Water added $= \frac{7}{10+8} \times 36 = 14$ litres

27. A mixture of 40 litres contains milk and water in the ratio 5 : 3. How many litres of milk must be added to the mixture so that the ratio becomes 2 : 1 in the resultant mixture?

(a) 2 litres (b) 4 litres (c) 5 litres (d) 10 litres

Solution:

(c) LCM of ratios of water i.e. LCM of 3 and 1 is 3.

	Milk	:	Water
New ratio = 2 : 1 =	6	:	3
Old ratio = 5 : 3 =	5	:	3
Difference =	1	:	0

∴ Milk added $= \frac{1}{5+3} \times 40 = 5$ litres

28. A mixture of 50 litres contains milk and water in the ratio 3 : 2. On adding some water in the mixture the ratio becomes 2 : 3, find the quantity of water added to the mixture.

(a) 10 litres (b) 15 litres (c) 20 litres (d) 25 litres

Solution:

(d) LCM of ratios of Milk i.e. LCM of 3 and 2 is 6.

	Milk	:	Water
New ratio = 2 : 3 =	6	:	9
Old ratio = 3 : 2 =	6	:	4
Difference =	0	:	5

∴ Water added $= \frac{5}{6+4} \times 50 = 25$ litres

29. In a mixture of 48 litres, milk and water are in the ratio 5 : 3. Find the quantity of water to be added to make the ratio 3 : 5.

(a) 12 litres (b) 24 litres (c) 32 litres (d) 40 litres

Solution:

(c) LCM of ratios of milk i.e. LCM of 5 and 3 is 15.

	Milk	:	Water
New ratio = 3 : 5 =	15	:	25
Old ratio = 5 : 3 =	15	:	9
Difference =	0	:	16

$\therefore$ Water to be added = $\frac{16}{15+9} \times 48 = 32$ litres

30. The ratio of males and females in a party is 3 : 2. When 20 more females joined, the ratio was reversed. How many males were there at the party?

(a) 16 (b) 24 (c) 32 (d) 40

Solution:

(b) Old ratio of males and females is 3 : 2.

$\therefore$ New ratio of males and females is 2 : 3.

LCM of ratios of male i.e. LCM of 3 and 2 is 6.

	Male	:	Female
New ratio = 2 : 3 =	6	:	9
Old ratio = 3 : 2 =	6	:	4
Difference =	0	:	5

$\therefore$ Female ratio is increased by 5 when 20 more females have joined the group and ratio of males (originally) was 6.

$\therefore$ Males in the beginning = $\frac{6}{5} \times 20 = 24$

31. The ratio between two numbers is 5 : 3. If 3 is added to both the numbers, the ratio becomes 14 : 9. Find the smaller number.

(a) 15 (b) 18 (c) 25 (d) 28

Solution:

(a) $x = \frac{(14-9) \times 3}{5 \times 9 - 3 \times 14} = \frac{5 \times 3}{45 - 42} = \frac{5 \times 3}{3} = 5$

$\therefore$ Smaller number = $3x = 3 \times 5 = 15$

32. The ratio between two numbers is 2 : 3. If 3 is added to both the numbers, the ratio becomes 5 : 7. Find the smaller number.

(a) 10 (b) 12 (c) 18 (d) 21

Solution:

(b) $x = \frac{(5-7) \times 3}{2 \times 7 - 3 \times 5} = \frac{-2 \times 3}{14 - 15} = \frac{-6}{-1} = 6$

$\therefore$ Smaller number = $2 \times 6 = 12$

33. The ratio between two natural numbers is 3 : 5. Find the smallest number which must be added to these numbers so that the resultant numbers are in the ratio of 9 : 14.

(a) 2 (b) 3 (c) 5 (d) 7

Solution :

(b) $x = \frac{(9-14) \times k}{3 \times 14 - 5 \times 9} = \frac{5}{3}k$

The smallest value of k is 3 for which x is a natural number.

34. The ratio between two numbers is 5 : 8. If 8 is subtracted from both the numbers, the ratio becomes 1 : 2. The original numbers are:

(a) 15, 24 (b) 20, 32 (c) 25, 40 (d) 30, 48

Solution:

(b) $x = \dfrac{(1-2)\times(-8)}{5\times2-8\times1} = \dfrac{(-1)(-8)}{10-8} = \dfrac{8}{2} = 4$

$\therefore$ The numbers are 4 × 5 and 4 × 8 *i.e.* 20 and 32.

35. A mixture of 55 litres contains milk and water in the ratio 7 : 4. How many litres of milk and water each must be added to the mixture to make the ratio 3 : 2 ?

(a) 2 litres (b) 5 litres (c) 8 litres (d) 10 litres

Solution:

(d) Let milk and water in the mixture are 7x and 4x litre respectively.

Then $7x + 4x = 55$

$\therefore\ x = \dfrac{55}{11} = 5$

Quantity of milk and water added is

$k = \dfrac{7\times2-4\times3}{3-2}\times5 = \dfrac{14-12}{1}\times5 = 2\times5 = 10$ litres

36. A vessel contains milk and water in the ratio 5 : 7. Had it contained 2 litres more of milk and 2 litres less of water, the ratio would have become 4 : 5. What is the quantity of milk in the vessel?

(a) 28 litres (b) 30 litres (c) 32 litres (d) 42 litres

Solution:

(b) $x = \dfrac{(4+5)\times2}{7\times4-5\times5} = \dfrac{9\times2}{3} = 6$

$\therefore$ Milk = 5 × 6 = 30 litres

37. Two numbers are in the ratio 3 : 5. If 2 is added to the first number and 5 to the second, the ratio between the numbers becomes 4 : 7. Find the smaller number.

(a) 12 (b) 18 (c) 25 (d) 30

Solution:

(b) $x = \dfrac{4\times5-7\times2}{3\times7-5\times4} = \dfrac{20-14}{21-20} = \dfrac{6}{1} = 6$

$\therefore$ Smaller number = 3 × 6 = 18

38. Two vessels A and B contain milk and water in the ratio 5 : 3 and 8 : 7 respectively. Find the ratio in which the quantities be taken from the two vessels so that the ratio of milk and water in the new mixture is 13 : 10.

(a) 8 : 15 (b) 15 : 8 (c) 10 : 13 (d) 13 : 10

Solution:

(a)

Ratio of milk and water in vessel A =	5	:	3
Ratio of milk and water in vessel B =	8	:	7
Total	13	:	10

Since total of two ratios is equal to the required ratio.

$\therefore$ Ratio of mixtures taken from two vessels = (5 + 3) : (8 + 7) = 8 : 15

39. Two vessels A and B contain milk and water in the ratio 7 : 3 and 3 : 2 respectively. Find the ratio in which the quantities be taken from the two vessels so that the ratio of milk and water in the new mixture is 2 : 1.

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 2 : 3

Solution :

(c) Vessel A contains 1 litre more of milk than the required ratio, for every 10 litres of mixture.
Vessel B contains 1 litre less of mik than the required ratio, for every 5 litres of mixture.

∴ More milk in vessel $A = \frac{1}{10}$

Less milk in vessel $B = \frac{1}{5}$

∴ Ratio in which mixtures are required to be taken from vessels A and B is $\frac{1}{5} : \frac{1}{10} = 2 : 1$

40. Two vessels contain mixture of milk and water in the ratio of 5 : 2 and 3 : 1 respectively. Find the ratio of milk and water in the new solution, if two mixtures are mixed in equal amount.

(a) 5 : 2 (b) 3 : 1 (c) 8 : 3 (d) 41 : 15

Solution:

(d) Sum of the ratios are 5 + 2 and 3 + 1 or 7 and 4.

LCM of sum of the ratios i.e. LCM of 7 and 4 is 28.

∴ We assume that 28 litres of mixture is taken from each vessel.

Milk in first vessel = $\frac{5}{7} \times 28 = 20$ litres

∴ Water in first vessel = 28 – 20 = 8 litres

And milk in second vessel = $\frac{3}{4} \times 28 = 21$ litres

∴ Water in second vessel = 28 – 21 = 7 litres

Total quantity of milk in the resultant mixture = 20 + 21 = 41.

Total quantity of water in the resultant mixture = 8 + 7 = 15.

∴ Ratio of milk and water in the new solution = 41 : 15.

Alternative Method :

Milk in the resultant mixture $= \frac{5}{7} + \frac{3}{4} = \frac{41}{28}$

Water in the resultant mixture $= \frac{2}{7} + \frac{1}{4} = \frac{15}{28}$

∴ Ratio of milk and water $= \frac{41}{28} : \frac{15}{28} = 41 : 15$

41. Three equal vessels are filled with milk and water in the ratio 2 : 1, 3 : 2 and 3 : 1 respectively. The contents of the three vessels are emptied into a big vessel, what is the ratio of milk and water in the big vessel?

(a) 2 : 1 (b) 2 : 3 (c) 121 : 59 (d) 9 : 1

Solution:

(c) Sum of the ratios are 2 + 1, 3 + 2, 3 + 1 or 3, 5 and 4.

LCM of the sum of ratios i.e. LCM of 3, 5 and 4 is 60.

∴ We assume that 60 litre of mixture is taken from each vessel.

Vessel	Milk	:	Water
I	40	:	20
II	36	:	24
III	45	:	15
Total	121	:	59

∴ Ratio in the resultant mixture = 121 : 59.

Alternative Method :

Milk in the resultant mixture $= \frac{2}{3} + \frac{3}{5} + \frac{3}{4} = \frac{121}{60}$

Water in the resultant mixture $= \frac{1}{3} + \frac{2}{5} + \frac{1}{4} = \frac{59}{60}$

∴ Ratio of milk and water $= \frac{121}{60} : \frac{59}{60} = 121 : 59$

Note : $121 + 59 = 180 = 3 \times 60$

42. A bag contains coins of Re. 1, Rs. 2 and Rs. 5 denomination in the ratio 2 : 3 : 4. If the total amount is Rs. 280. Find the number of coins of Rs.5 denomination in the bag.

(a) 20 (b) 30 (c) 35 (d) 40

Solution:

(d) Denomination of coins (in Rs.) are 1, 2 and 5.

Ratio of coins = 2 : 3 : 4

∴ Value of coins (in Rs.) = Denomination of coins × Ratio of coins

$= 1 \times 2 : 2 \times 3 : 5 \times 4 = 2 : 6 : 20 = 1 : 3 : 10$

Sum of ratios of value of coins = 1 + 3 + 10 = 14.

But total value is Rs. 280

∴ Value of 5-rupee coins $= \frac{10}{14} \times 280$ = Rs. 200

∴ Number of five-rupee coins $= \frac{200}{5}$ = 40 coins

43. A sum of Rs. 600 is divided among 8 men, 9 women and 12 boys. If share of one man, one woman and one boy is in the ratio 6 : 4 : 3, find share of each boy.

(a) Rs. 15 (b) Rs. 50 (c) Rs. 100 (d) Rs. 180

Solution:

(a) Ratio of money received by men, women and boys

$= 8 \times 6 : 9 \times 4 : 12 \times 3 = 48 : 36 : 36 = 4 : 3 : 3$

Sum of ratios of money received = 4 + 3 + 3 = 10

$\therefore$ 12 boys' share = $\frac{3}{10} \times 600$ = Rs. 180

$\therefore$ 1 boy's share = $\frac{180}{12}$ = Rs. 15

44. The ratio of incomes of two persons A and B is 5 : 3 and that of their expenditure is 9 : 5. If they save Rs. 1300 and Rs. 900 respectively, find the income of A.

(a) Rs. 1500 (b) Rs. 900 (c) Rs. 2400 (d) Rs. 4000

Solution :

(d) $5x - 9y$ = Rs. 1300

$3x - 5y$ = Rs. 900

Solving the equations, x = Rs. 800

$\therefore$ A's income = 5 × Rs. 800 = Rs. 4000

Alternative Method : (using Rule 8)

$$x = \frac{mq - np}{an - bm} = \frac{9 \times (-900) - 5 \times (-1300)}{5 \times 5 - 3 \times 9} = \frac{-1600}{-2} = \text{Rs. } 800$$

$\therefore$ A's income = 5 × Rs. 800 = Rs. 4000

Chapter 17

PROBLEMS ON AGES

In problems on ages, ratios of ages of two persons at two different points of time are given and we are required to calculate their ages at a given point of time. In this chapter, we will learn to solve these problems **without making the equations** with the help of Rules learnt in **Ratio and Proportion** chapter.

FORMULAE

Ages of A and B are in the ratio of a : b. After 'y' years, the ratio of ages becomes m : n.

Then their present ages are 'ax' and 'bx' years, where

$$x = \frac{y(m-n)}{an-bm} = \boxed{\frac{\text{Difference in time} \times \text{Difference in new ratio}}{\text{Difference in cross multiplication of new and old ratios}}}$$

SOLVED EXERCISE

1. Five years ago, A was 10 years younger than B. What will be the difference between their ages after 2 years from now?

   (a) 3 years (b) 8 years (c) 10 years (d) 15 years

Solution:

(c) Difference between the ages of A and B will always remain same i.e. 10 years.

2. Ratio of present ages of A and B is 2 : 3. After 6 years, ratio of their ages is 3 : 4. Find the present age of A.

   (a) 6 years (b) 12 years (c) 18 years (d) 24 years

Solution:

(b) After 6 years both the ratios are increased by 1.

$\therefore$ Present age of A = 2 × 6 years = 12 years.

3. In 30 years, age of a person becomes 3 times of his present age. What is his present age?

   (a) 7.5 years (b) 10 years (c) 12.5 years (d) 15 years

Solution:

(d) Let his present age is x years.

Then his age after 30 years will be 3x years.

Given $x + 30 = 3x$

$\therefore 2x = 30$

$\therefore$ Present age $= x = \frac{30}{2} = 15$ years

Trick:

In 30 years, his age will become 3 times i.e. 2 times more than his present age.

$\therefore$ His present age $= \frac{30}{2} = 15$ years

4. Mohan got married 9 years ago. Today his age is $1\frac{1}{3}$ times of his age at the time of his marriage. Find his present age.

(a) 12 years (b) 27 years (c) 36 years (d) 45 years

Solution:

(c) 9 years = $\frac{1}{3}$ of his age at the time of marriage

∴ Mohan's age at the time of marriage = 3 × 9 years = 27 years

∴ His present age = 27 + 9 = 36 years

5. The sum of ages of Mohan and his father is 35 years. When Mohan's age will be equal to present age of his father, then sum of their ages will be 85 years. Find present age of the father.

(a) 20 years (b) 25 years (c) 30 years (d) 32 years

Solution :

(c) Sum of present ages = 35 years.

Sum of their ages will be 85 years after $\frac{85-35}{2} = 25$ years from now.

∴ The father is 25 years older than his son.

∴ Present age of Mohan $= \frac{35-25}{2} = 5$ years.

∴ Present age of father = 35 – 5 = 30 years.

6. The sum of ages of Sunil and his father is 46 years. 3 years back father's age was 4 times Sunil's age. Find present age of Sunil.

(a) 7 years (b) 8 years (c) 11 years (d) 12 years

Solution :

(c) Sum of their ages, 3 years ago = 46 – 2 × 3 = 40 years

∴ Sunil's Age $= \frac{1}{5} \times 40 = 8$ years

Sunil's present age = 8 + 3 = 11 years

7. The sum of ages of Suresh and his father is 78 years. 6 years hence father's age will be twice of Suresh's age. Find present age of father.

(a) 30 years (b) 52 years (c) 54 years (d) 60 years

Solution :

(c) Sum of ages, 6 years hence = 78 + 2 × 6 = 90 years

∴ Father's age after 6 years $= \frac{2}{3} \times 90 = 60$ years

∴ Fathers's present age = 60 – 6 = 54 years.

8. A's age is 50 years and B's age is 18 years. In how many years will A be twice as old as B ?

(a) 14 years (b) 15 years (c) 16 years (d) 18 years

Solution :

(a) Double of B's age = 2 × 18 = 36 years

Difference between A's and B's age = 50 – 36 = 14 years

∴ A will be twice of B's age after 14 years.

9. Sum of present ages of A, B and C is 72 years. If 4 years ago, their ages were in the ratio of 1 : 2 : 3, find A's present age.

(a) 7 years (b) 10 years (c) 12 years (d) 14 years

Solution:

(d) Sum of present ages of A, B and C is 72 years.

∴ Sum of their ages (4 years ago) = 72 – 3 × 4 = 60 years.

4 years ago, ratio of ages of A, B and C was 1 : 2 : 3.

∴ A's age 4 years ago = $\frac{1}{6} \times 60 = 10$ years

∴ A's present age = 10 + 4 = 14 years

10. Sum of present ages of A and B is 60 years. 5 years hence their ages will be in the ratio of 3 : 4. Find A's present age.

(a) 25 years (b) 30 years (c) 35 years (d) 40 years

Solution:

(a) Sum of their present ages = 60 years.

∴ Sum of their ages after 5 years = 60 + (5 × 2) = 70 years.

After 5 years, ratio of ages of A and B will be 3 : 4.

∴ A's age after 5 years = $\frac{3}{7} \times 70 = 30$ years

∴ A's present age = 30 – 5 = 25 years

11. The ages of A and B are in the ratio of 8 : 5. If the sum of their ages is 39 years, what will be the ratio of their ages after 9 years ?

(a) 3 : 2 (b) 8 : 7 (c) 10 : 7 (d) 11 : 8

Solution :

(d) Sum of ratios = 8 + 5 = 13

Sum of ages = 39 years

∴ 13 ≡ 39 years ⇒ 1 ≡ 3 years

∴ 9years ≡ 3

New ratio = (8 + 3) : (5 + 3) = 11 : 8

12. At the time of marriage, a man was 6 years elder to his wife. 12 years after their marriage, his age is $\frac{6}{5}$ times the age of his wife. What was wife's age at the time of marriage ?

(a) 18 years (b) 24 years (c) 30 years (d) 36 years

Solution :

(a) Ratio of their ages, 12 years after = 6 : 5

Difference in ages = 6 years

∴ Wife's age = 5 × 6 years = 30 years

∴ Wife's age at the time of marriage = 30 – 12 = 18 years

13. Present age of Ram's father is 4 times Ram's age. 5 years from now, father's age will become 3 times of Ram's age, find present age of Ram's father.

(a) 40 years (b) 45 years (c) 50 years (d) 60 years

Solution:

(a) Father Son

Old Ratio = 4 : 1

New Ratio = 3 : 1

$$\therefore\ x = \frac{5 \times (3-1)}{4 \times 1 - 3 \times 1} = \frac{5 \times 2}{1} = 10$$

∴ Father's age = 4 × 10 = 40 years

14. A and B are 3 years and 2 years old respectively. Their father is 40 years old. After how many years, father's age would be twice of combined age of A and B ?

(a) 5 years (b) 10 years (c) 15 years (d) 20 years

Solution :

(b) Father's age = 40 years

Children's combined age = 3 + 2 = 5 years

Double of their ages = 2 × 5 = 10 years

Difference between father's age and combined age of children

= 40 – 10 = 30 years

$$\therefore\ \text{Required time} = \frac{1}{2+1} \times 30 = 10 \text{ years}$$

15. If ratio of Mohan's age to that of his father is 1 : 5. After 10 years, their ages will be in the ratio of 1 : 3. What is the father's present age?

(a) 25 years (b) 30 years (c) 40 years (d) 50 years

Solution:

(d) Father Son

Old Ratio = 5 : 1

New Ratio = 3 : 1

$$\therefore\ x = \frac{10 \times (3-1)}{5 \times 1 - 3 \times 1} = \frac{10 \times 2}{2} = 10.$$

∴ Father's age = 5 × 10 years = 50 years

Trick:

Old ratio = 1 : 5.

New ratio = 1 : 3 or 2 : 6 i.e. both ratios are increased by 1.

∴ Father's age = 5 × 10 = 50 years

16. Ratio of present ages of father and son is 8 : 3. If 10 years from now, the ratio becomes 2 : 1, find present age of the son.

(a) 15 years (b) 20 years (c) 40 years (d) 55 years

Solution:

(a)	Father		Son
Old Ratio =	8	:	3
New Ratio =	2	:	1

$$\therefore\ x = \frac{10 \times (2-1)}{8 \times 1 - 3 \times 2} = \frac{10 \times 1}{2} = 5$$

∴ Present age of son = 3 × 5 years = 15 years

Trick:

Old ratio = 8 : 3

New ratio = 2 : 1 or 10 : 5 i.e. both ratios are increased by 2.

∴ 2 ≡ 10 years or 1 ≡ 5 years

∴ Present age of son = 3 × 5 = 15 years

17. 3 years ago, the age of a father was 7 times of his son's age. If the father's present age is 5 times his son's age. Find the present age of the father.

(a) 35 years (b) 39 years (c) 42 years (d) 45 years

Solution:

(d)	Father		Son
Old Ratio =	7	:	1
New Ratio =	5	:	1

Let 3 years ago, the father's age was 7x years, where

$$x = \frac{3 \times (5-1)}{7-5} = \frac{3 \times 4}{2} = 6$$

∴ Father's age was = 7 × 6 = 42 years

∴ His present age = 42 + 3 = 45 years

18. Ratio between present ages of A and B is 3 : 4. What is the present age of A, if 4 years ago, the ratio was 5 : 7?

(a) 20 years (b) 24 years (c) 28 years (d) 32 years

Solution:

(b)	A		B
Old ratio =	5	:	7
New ratio =	3	:	4

Difference in time = 4 years

$$\therefore\ x = \frac{4 \times (3-4)}{5 \times 4 - 7 \times 3} = \frac{4 \times (-1)}{20-21} = \frac{-4}{-1} = 4$$

∴ A's age (4 years ago) = 5 × 4 = 20 years

∴ A's present age = 20 + 4 = 24 years

19. 5 years ago, the father's age was 3 times of his son's age. 5 years hence the ratio between the ages of father and the son becomes 11 : 5. Find the father's present age.

(a) 40 years (b) 45 years (c) 50 years (d) 55 years

Solution:

(c) Father Son

Old ratio = 3 : 1

New ratio = 11 : 5

Let 5 years ago father's age was 3x years, where

$$x = \frac{(5+5)\times(11-5)}{3\times 5-1\times 11} = \frac{10\times 6}{4} = 15$$

∴ Father's age (5 years ago) = 3 × 15 = 45 years

∴ His present age = 45 + 5 = 50 years

Trick:

Old ratio = 3 : 1 = 9 : 3
New ratio = = 11 : 5

$$\frac{\quad}{\;2 : 2\;}$$

Difference in time = 10 years

$\therefore 2 \equiv 10 \Rightarrow 1 \equiv 5$

∴ Father's age (5 years ago) = 5 × 9 = 45 years

∴ His present age = 45 + 5 = 50 years

20. 2 years ago, the father's age was 6 times of his son's age. 6 years hence the ratio between the ages of father and the son becomes 10 : 3. What is the father's present age?

(a) 40 years (b) 42 years (c) 44 years (d) 48 years

Solution:

(c) Father Son

Old ratio = 6 : 1

New ratio = 10 : 3

Let 2 years ago, age of son and father were x and 6x respectively.

$$\therefore\ x = \frac{(2+6)\times(10-3)}{6\times 3-1\times 10} = \frac{8\times 7}{8} = 7 \text{ years}$$

∴ 2 years ago, father's age was 6 × 7 = 42 years.

∴ Father's present age = 42 + 2 = 44 years.

21. 3 years ago, a father's age was twice the ages of his 4 sons. In 3 years time, the father's age will be equal to sum of ages of the sons. Find the present age of father.

(a) 24 years (b) 36 years (c) 39 years (d) 45 years

Solution :

(c) 3 years ago, let sum of ages of sons = x

Then father's age = 2x

Father's age after 6 years = 2x + 6

Sum of ages of sons = x + 6 × 4 = x + 24

∴ 2x + 6 = x + 24

∴ x = 18

∴ Father's age, 3 years ago = 2 × 18 = 36 years

∴ Father's present age = 36 + 3 = 39 years.

Chapter 18

PERCENTAGE – I

The rules mentioned in this chapter are widely used in Discount, Profit and Loss, Simple Interest, Compound Interest, Mensuration etc.

Percent: Percent means 'per hundered'. It is denoted by % sign. If we say x%, it implies 'x' out of 100. In fraction, x% can be written as $\frac{x}{100}$.

Example:

27% means 27 out of 100. In fraction, 27% can be written as $\frac{27}{100}$ and in decimal it can be written as 0.27.

Convert per cent into a fraction or decimal

Method:

Divide the given number by 100

Example:

$20\% = \frac{20}{100} = \frac{1}{5}$ (or 0.20 in decimal)

Convert a fraction or decimal into percentage

Method:

Multiply the given fraction or decimal by 100

Example:

$\frac{1}{4} = \frac{1}{4} \times 100 = 25\%$

$0.38 = 0.38 \times 100 = 38\%$

FREQUENTLY USED FRACTIONS AND THEIR EQUIVALENT PERCENT:

$1 = 100\%$	$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33.33\%$
$\frac{1}{4} = 25\%$	$\frac{1}{5} = 20\%$	$\frac{1}{6} = 16.67\%$
$\frac{1}{8} = 12.5\%$	$\frac{1}{9} = 11.11\%$	$\frac{1}{10} = 10\%$
$\frac{1}{11} = 9.09\%$	$\frac{1}{12} = 8.33\%$	$\frac{1}{16} = 6.25\%$

$\frac{1}{20} = 5\%$ $\frac{1}{25} = 4\%$ $\frac{1}{50} = 2\%$

$\frac{2}{3} = 66.67\%$ $\frac{2}{5} = 40\%$ $\frac{3}{5} = 60\%$

FORMULAE

1. If a person spends x% of his income.

   Then his savings is (100 – x)% of the income.

2. If x% of a number is y, then the number is $y \times \frac{100}{x}$

   If 'y' can be written as 'ax', then the number = a × 100.

Example:

If 25% of a number is 200. Find the number.

Solution:

Here x is 25% and y is 200.

$\therefore$ The number is $200 \times \frac{100}{25} = 800$.

Alternative Method:

200 is 8 times of 25.

$\therefore$ The number is 8 × 100 = 800.

3. If x% of a number is equal to y% of second number.

   Then ratio between first and second number = $\frac{1}{x} : \frac{1}{y}$ or y : x.

4. Two numbers are in the ratio x : y. If first number is increased by a % and second number is increased by b %, what is percentage increase in the sum of two numbers ?

   Solution :

   Sum of original numbers = x + y

   Sum of new numbers $= x + \frac{ax}{100} + y + \frac{by}{100}$

   $\therefore$ Increase in sum $= \frac{ax}{100} + \frac{by}{100} = \frac{ax+by}{100}$

   $\therefore$ Percentage increase $= \frac{ax+by}{100} \times \frac{100}{x+y} = \frac{ax+by}{x+y}$

SOLVED EXERCISE

1. 25% of 75% of 80 = ?

(a) 15 (b) 20 (c) 60 (d) 80

Solution:

(a) 25% of 75% of 80 = $\frac{25}{100} \times \frac{75}{100} \times 80 = 15$

Alternative Method:

We know that 25% = $\frac{1}{4}$ and 75% = $\frac{3}{4}$

$\therefore$ 25% of 75% of 80 = $\frac{1}{4} \times \frac{3}{4} \times 80 = 15$

2. What rate per cent is 1 minute 12 seconds to an hour?

(a) 1% (b) 2% (c) 3% (d) 4%

Solution:

(b) 1 minute 12 seconds = (60 + 12) seconds = 72 seconds.

1 hour = 3600 seconds.

$\therefore$ Rate percent = $\frac{72}{3600} \times 100 = 2\%$.

3. The population of a town is increased from 25000 to 30000 in a year. Find the rate of increase.

(a) 5% (b) 10% (c) 20% (d) 25%

Solution:

(c) Total increase in the population = 30000 – 25000 = 5000.

$\therefore$ Percentage increase in population = $\frac{5000}{25000} \times 100 = 20\%$.

4. If rate of income tax is increased from 20% to 25%. Find the additional amount of tax, a person has to pay, if his total income is Rs. 40000.

(a) Rs. 2000 (b) Rs. 2500 (c) Rs. 8000 (d) Rs. 10000

Solution:

(a) Increase in tax-rate = (25 – 20)% = 5% = $\frac{1}{20}$

$\therefore$ Increase in tax amount on Rs. 40000 = $\frac{1}{20} \times 40000$ = Rs. 2000

5. Salaries of A and B are in the ratio of 3 : 4. Both of them spend 75% of their salaries and save rest of the money. Find the ratio of their savings.

(a) 3 : 4 (b) 4 : 3 (c) 1: 1 (d) Can't say

Solution:

(a) Ratio of savings = 25% of 3 : 25% of 4 = 3 : 4

6. In an examination every candidate took Math or English or both. If 65% took Math and 60% English, how many took Math and English both?

(a) 5% (b) 25% (c) 62.5% (d) 125%

Solution:

(b) Let total students = 100

Students who took Math = 65

Students who took English = 60

$\therefore$ Students who took both the subjects = 65 + 60 – 100 = 25

$\therefore$ 25% students took both the subjects.

7. In a group of students, 70% can speak English and 75% can speak Hindi. If 10% of the students can speak none of the two languages, what percentage of students can speak both the languages ?

(a) 45% (b) 55% (c) 60% (d) 65%

Solution :

(b) Let total students = 100

Students who know none of the two languages = 10

Remaining students = 100 – 10 = 90

∴ Students who know both languages = 70 + 75 – 90 = 55

Note : (70 – 55) + (75 – 55) + 10 + 55 = 100

8. 1300 boys and 700 girls appeared in an examination, 80% of the boys and 70% of the girls passed in the exam, find the total percentage of failed students.

(a) 12.5% (b) 23.5% (c) 47% (d) 76.5%

Solution :

(b) Boys failed = 1300 × (100 – 80)% = 260

Girls failed = 700 × (100 – 70)% = 210

$$\therefore \text{Students failed} = \frac{260+210}{2000} \times 100 = 23.5\%$$

9. The prices of sugar and rice are in the ratio 4 : 5. If the price of sugar is increased by 10% and that of rice by 20%, find the ratio between increased prices of sugar and rice.

(a) 1 : 2 (b) 5 : 6 (c) 11 : 15 (d) 11 : 12

Solution :

(c) After 10% increase, the price of sugar will become $\frac{110}{100}$ or 1.1 of the original price.

Similarly the price of rice will become 1.2 of the original price.

∴ The ratio of new prices = 4 × 1.1 : 5 × 1.2 = 4.4 : 6.0 = 11 : 15

10. If 30% of a number is 90, the number is:

(a) 3 (b) 30 (c) 270 (d) 300

Solution:

(d) Let the number is x.

Then 30% of x = 90

$$\therefore \frac{30}{100} \times x = 90$$

$$\therefore x = 90 \times \frac{100}{30} = 300$$

Trick:

90 is 3 times of 30.

∴ The number is 3 × 100 = 300.

11. 20% of 30% of a number is 12, the number is:

(a) 24 (b) 40 (c) 100 (d) 200

Solution:

(d) Let the number is x.

Then 20% of 30% of x = 12

$$\therefore \frac{20}{100} \times \frac{30}{100} \times x = 12$$

$$\therefore x = 12 \times \frac{100}{20} \times \frac{100}{30} = 200$$

12. 15 litres of a mixture contains 20% alcohol and rest water. If 3 litres of water is added in it, percentage of alcohol in the new mixture will be?

(a) 16.67% (b) 20% (c) 25% (d) 33.33%

Solution:

(a) Alcohol in the mixture = $\frac{1}{5} \times 15 = 3$ litres

Mixture becomes (15 + 3) litres = 18 litres on adding 3 litres of water.

∴ Percentage of alcohol in the new mixture = $\frac{3}{18} = \frac{1}{6} = 16.67\%$

Alternative Method :-

$$20 \times \frac{15}{15+3} = 16.67\%$$

13. In an examination, a student has to secure 40% of the maximum marks to pass. One student got 40 marks and failed by 40 marks. The maximum marks in the examination are:

(a) 100 (b) 200 (c) 300 (d) 400

Solution:

(b) Minimum passing marks = (40 + 40) = 80

= 40% of the maximum marks (80 is 2 times of 40)

∴ Maximum marks = 2 × 100 = 200

14. A student has to secure 40% marks in an examination to qualify. He gets 120 marks and fails by 80 marks. The maximum marks are:

(a) 200 (b) 300 (c) 400 (d) 500

Solution:

(d) Minimum passing marks = 120 + 80 = 200

= 40% of the maximum marks (200 is 5 times of 40)

∴ Maximum marks = 5 × 100 = 500

15. A student gets 29% marks in an examination but fails by 24 marks. If the pass percentage is 35%, the maximum marks are.

(a) 200 (b) 300 (c) 400 (d) 500

Solution:

(c) Student failed by 35% – 29% = 6% = 24 marks (i.e. 4 times of 6)

∴ Maximum marks = 4 × 100 = 400 marks

16. Two students A and B appeared in an examination. If A secured 9 marks more than B and his marks are 56% of sum of their marks, find marks obtained by A.

(a) 33 (b) 42 (c) 48 (d) 56

Solution :

(b) Marks secured by B = 100 – 56 = 44%

∴ Difference in marks = (56 – 44)% = 12% = 9

$$\therefore \text{ Total marks } = \frac{9}{12} \times 100 = 75 \text{ marks}$$

$$\text{A's marks } = 75 \times \frac{56}{100} = 42 \text{ marks}$$

17. A student got 42% marks and has secured 12 marks more than the minimum passing marks. Another student who got 45% has obtained 30 marks more than the minimum passing marks. The maximum marks are:

(a) 250 (b) 300 (c) 450 (d) 600

Solution:

(d) Difference in percentage of marks = 45% – 42% = 3%

Difference in marks = 30 – 12 = 18 (i.e. 6 times of 3)

∴ Maximum marks = 6 × 100 = 600

18. A student gets 35% marks and fails by 10 marks and another student who gets 31% marks fails by 30 marks. The maximum marks are:

(a) 400 (b) 500 (c) 600 (d) 1000

Solution:

(b) Difference in percentage of marks = 35% – 31% = 4%

Difference in marks = 30 – 10 = 20 (i.e. 5 times of 4)

∴ Total marks = 5 × 100 = 500

19. A student gets 30% marks and fails by 20 marks and another student who gets 35% marks, gets 10 marks more than minimum passing marks. The maximum marks are:

(a) 200 (b) 350 (c) 500 (d) 600

Solution:

(d) Difference in percentage of marks = 35% – 30% = 5%

Difference in marks = 10 – (– 20) = 10 + 20 = 30 (i.e. 6 times of 5)

∴ Total marks = 6 × 100 = 600

20. In an examination, a student gets 38% of the maximum marks and fails by 6 marks. Another student who gets 45% of the maximum marks, gets 15 marks more than the required passing percentage. Find passing percentage of the marks.

(a) 39% (b) 40% (c) 44% (d) Can't say

Solution:

(b) Difference in percentage of marks = 45% – 38% = 7%

Difference in marks = 15 – (– 6) = 15 + 6 = 21

∴ 7% of maximum marks = 21

∴ 1% of maximum marks = 3

First student after obtaining 38% is failed by 6 marks.

6 marks = 2%

∴ Pass percentage = 38% + 2% = 40%

21. A student scores 37% marks and fails by 12 marks and another student who scores 42% marks and got 8 marks more than the minimum passing marks. The passing percentage is:

(a) 37% (b) 38% (c) 40% (d) 42%

Solution:

(c) Difference in percentage of marks = 42% – 37% = 5%

Difference in marks = 8 – (– 12) = 8 + 12 = 20 (i.e. 4 times of 5)

∴ Total marks = 4 × 100 = 400

First student is failed by 12 marks.

∴ He is failed by $\frac{12}{400} \times 100 = 3\%$

∴ Pass % = 37% + 3% = 40%

22. A person saves 10% of his income. If his income is increased by 20% and he saves 15% of the new income, by what percent his savings will increase?

(a) 5% (b) 35% (c) 75% (d) 80%

Solution:

(d) Let previous income = Rs. 100

Previous savings = 10% of Rs. 100 = Rs. 10

Increased income = Rs. 100 + 20% of Rs. 100 = Rs. 120

Increased savings = 15% of Rs. 120 = Rs. 18

∴ Increase in savings = Rs. 18 – Rs. 10 = Rs. 8

∴ Percent increase in savings = $\frac{8}{10} \times 100 = 80\%$

23. A fruit seller sells 30% apples and still has 630 apples in the stock. How many apples had he bought?

(a) 189 (b) 700 (c) 900 (d) 2100

Solution:

(c) Stock left unsold = (100 – 30)% = 70% = 630 apples (i.e. 9 times of 70)

∴ Apples bought = 9 × 100 = 900

24. A man spends 20% of his income on food and 70% on clothing. If he still has a balance of Rs. 200, what is his total income?

(a) Rs. 100 (b) Rs. 200 (c) Rs. 1000 (d) Rs. 2000

Solution:

(d) Total expenditure = 20% + 70% = 90% of income.

∴ Savings = (100 – 90)% = 10% of income = Rs. 200 (i.e. 20 times of 10)

∴ His income = Rs. 20 × 100 = Rs. 2000

25. If a man after spending 97% of his income, saves Rs. 300 per month, his annual income is:

(a) Rs. 10000 (b) Rs. 12000 (c) Rs. 36000 (d) Rs. 120000

Solution:

(d) Saving = (100 – 97)% = 3% of income = Rs. 300 (i.e. 100 times of 3)

∴ Income = 100 × Rs.100 = Rs.10000 p.m.

= Rs. 10000 × 12 p.a. = 120000 p.a.

26. In an election contest between A and B, A wins by the margin of 240 votes. If A gets 60% of the total votes, total votes are:

(a) 400 (b) 600 (c) 1200 (d) 2400

Solution:

(c) Votes casted in favour of A = 60%

Votes casted in favour of B = (100 – 60)% = 40%

∴ A wins by (60% – 40%) = 20% of the votes = 240 (i.e. 12 times of 20)

∴ Total votes = 12 × 100 = 1200

Hint: We assume that no vote was invalid.

27. If 20% of 250 + 25% of x = 100, then x = ?

(a) 100 (b) 200 (c) 250 (d) 25

Solution:

(b) Given 20% of 250 + 25% of x = 100

$\therefore 50 + \frac{1}{4}x = 100$

$\therefore \frac{x}{4} = 100 - 50 = 50$

$\therefore x = 50 \times 4 = 200$

28. 60% of 2300 = 30% of ?

(a) 6000 (b) 4600 (c) 2300 (d) 3000

Solution:

(b) Let 60% of 2300 = 30% of x

$\therefore x = \frac{60}{100} \times 2300 \times \frac{100}{30} = 4600$

Trick:

Here first part is halved i.e. from 60% to 30%.

∴ Second part is to be doubled to get the same product.

∴ The second number is 2300 × 2 = 4600.

29. If 20% of a number is equal to 25% of the other number, find ratio between the two numbers.

(a) 2 : 3 (b) 3 : 4 (c) 4 : 5 (d) 5 : 4

Solution:

(d) $x : y = \frac{1}{20} : \frac{1}{25} = 25 : 20 = 5 : 4$

30. In a town 40% of the male population and 45% of the female population are married. Find ratio between male and female population in the town assuming that one man marries one woman and vice-versa.

(a) 8 : 9 (b) 9 : 8 (c) 5 : 7 (d) 7 : 5

Solution:

(b) Number of married males = Number of married females

$\therefore$ 40% of the male population = 45% of the female population

$\therefore$ Male : Female = 45 : 40 = 9 : 8

31. Population of a village is 90000, $\frac{5}{9}$th of them are males and rest are females. If 40% of the males are married, what is the percentage of married females?

(a) 40% (b) 45% (c) 50% (d) 60%

Solution:

(c) Let male population = 5x

Then female population = 4x

Let a% of females are married.

Then 40% of 5x = a% of 4x

$\therefore a = 40 \times \frac{5x}{4x} = 50\%$

32. Total number of boys and girls in a school is 150. If the number of boys is x, the number of girls is *x*% of the total number of students. The number of boys is :

(a) 30 (b) 45 (c) 60 (d) 90

Solution :

(c) $x + \frac{x}{100} \times 150 = 150$

$\frac{5}{2}x = 150$

$x = \frac{150 \times 2}{5} = 60$

33. If present price of sugar after 10% reduction is Rs. 18 per kg., find the original price of sugar per kg.

(a) Rs. 1.80 (b) Rs. 10.80 (c) Rs. 16.20 (d) Rs. 20

Solution:

(d) Let original price of sugar = Rs. 100 per kg.

Then reduced price = Rs. 100 – Rs. 10 = Rs. 90

But reduced price = Rs. 18

$\therefore$ Original price = Rs. 18 × $\frac{100}{90}$ = Rs. 20

34. Sum of 5% of a number and 9% of other number is equal to sum of 8% of first number and 7% of the second number. Find ratio between the numbers.

(a) 2 : 3 (b) 3 : 2 (c) 5 : 7 (d) 8 : 9

Solution:

(a) Let the numbers are x and y.

Then 5% of x + 9% of y = 8% of x + 7% of y

$\therefore$ 3% of x = 2% of y

$\therefore$ x : y = 2 : 3

Trick:

First number is increased by 3% and second number is decreased by 2% but the result remains the same

∴ Ratio between numbers is 2 : 3 i.e. inverse ratio of 3% and 2%

35. If sum of 5% of a number and 9% of another number is equal to $\frac{3}{4}$ of sum of 8% of first number and 11% of the second number. Find ratio between the numbers.

(a) 1:4 (b) 3:4 (c) 4:3 (d) 5:9

Solution:

(b) Let the numbers are x and y.

Then 5% of x + 9% of y = $\frac{3}{4}$ × (8% of x + 11% of y)

∴ 20% of x + 36% of y = 24% of x + 33% of y

∴ 4% of x = 3% of y

∴ x : y = 3 : 4

36. A number when decreased by 20% gives 80. The number is:

(a) 60 (b) 64 (c) 96 (d) 100

Solution:

(d) Decrease a number by 20% means (100 – 20)% or 80% of the number.

∴ 80% of the number = 80 (i.e. 1 time of 80)

∴ The number is 1 × 100 = 100

37. A number when increased by 25% gives 80. The number is:

(a) 60 (b) 64 (c) 96 (d) 100

Solution:

(b) Increase a number by 25% means (100 + 25)% or 125% of the number.

∴ 125% of the number = 80

∴ The number is 80 × $\frac{100}{125}$ = 64

38. When 40 is deducted from 40% of a number, the result is 40. The number is:

(a) 40 (b) 80 (c) 200 (d) 300

Solution:

(c) Let the number is x.

Then (40% of x) – 40 = 40

∴ 40% of x = 80 (i.e. 2 times of 40)

∴ x = 2 × 100 = 200

39. When 60% of a number is added to 60, the result is same number. The number is:

(a) 40 (b) 60 (c) 100 (d) 150

Solution:

(d) Let the number is x.

Then (60% of x) + 60 = x

$\therefore$ (100 – 60)% of x = 60

$\therefore$ 40% of x = 60

$\therefore x = 60 \times \frac{100}{40} = 150$

Hint: A number is 100% of itself.

40. When 75% of a number is added to 75, the result is same number. The number is:

(a) 100 (b) 150 (c) 225 (d) 300

Solution:

(d) Let the number is x.

Then 75% of x + 75 = x

$\therefore$ (100 – 75)% of x = 75

$\therefore$ 25% of x = 75 (i.e. 3 times of 25)

$\therefore x = 3 \times 100 = 300$

41. Price of an article is increased by 3%. Had it been Rs. 16 less, price would have been decreased by 1% of the original price. Find original price of the article.

(a) Rs. 400 (b) Rs. 500 (c) Rs. 800 (d) Rs. 1600

Solution:

(a) Difference in two prices = 3% + 1% = 4% of original price

$\therefore$ 4% of the original price = 16 (i.e. 4 times of 4)

$\therefore$ Original price = 4 × Rs. 100 = Rs. 400

42. A man spends 10% of his income on food and 80% of the remaining income on clothing. If he still has a balance of Rs. 180, what is his total income?

(a) Rs. 2000 (b) Rs. 1800 (c) Rs. 1000 (d) Rs. 900

Solution:

(c) Total Income $= 180 \times \frac{100}{100-10} \times \frac{100}{100-80}$

$= 180 \times \frac{100}{90} \times \frac{100}{20}$ = Rs. 1000

43. A person gave 20% of his income to his elder son, 30% of the remaining to the younger son and 10% of the balance to a trust. If he is left with Rs. 10080, his income was :

(a) Rs. 20000 (b) Rs. 25000 (c) Rs. 30000 (d) Rs. 40000

Solution :

(a) Total Income $= 10080 \times \frac{100}{100-10} \times \frac{100}{100-30} \times \frac{100}{100-20}$

$= 10080 \times \frac{10}{9} \times \frac{10}{7} \times \frac{10}{8}$ = Rs. 20000

44. A man had some eggs. Out of them 10% had to be thrown away as they were broken. 80% of the remaining eggs were sold and now he is left with only 90 eggs. How many eggs had he in the beginning?

(a) 500 (b) 800 (c) 900 (d) 1000

Solution:

(a) Eggs in the beginning $= 90 \times \frac{100}{100-10} \times \frac{100}{100-80}$

$= 90 \times \frac{100}{90} \times \frac{100}{20} = 500$ eggs

45. If A's salary is 110% of B's, B's salary is 80% of C's. Find C's salary, if A's salary is Rs. 440.

(a) Rs. 450 (b) Rs. 500 (c) Rs. 550 (d) Rs. 600

Solution:

(b) If B's salary is Rs. 100, then A's salary = Rs. 110% of Rs.100

= Rs. 110.

$\therefore$ B's salary = Rs. 440 × $\frac{100}{110}$

If C's salary is Rs. 100, then B's salary = 80% of Rs. 100 = Rs. 80.

$\therefore$ C's salary = Rs. 440 × $\frac{100}{110} \times \frac{100}{80}$ = Rs. 500

46. A's salary is 20% more than B's, B's salary is 10% less than C's. If A's salary is Rs. 1080, find C's salary.

(a) Rs. 900 (b) Rs. 1000 (c) Rs. 1100 (d) Rs. 1200

Solution:

(b) If B's salary is Rs. 100, then A's salary = Rs. 100 + Rs. 20 = Rs. 120.

$\therefore$ B's salary = Rs. 1080 × $\frac{100}{120}$

If C's salary is Rs.100, then B's salary = Rs. 100 – Rs. 10 = Rs. 90.

$\therefore$ C's salary = Rs. 1080 × $\frac{100}{120} \times \frac{100}{90}$ = Rs. 1000

47. In an examination, A gets 20% less marks than B, B gets 30% marks less than C. If A gets 112 marks, find marks obtained by C.

(a) 150 (b) 190 (c) 200 (d) 250

Solution:

(c) If B's marks is 100, then A's marks = 100 – 20 = 80.

$\therefore$ Marks obtained by B = 112 × $\frac{100}{80}$

If C's marks is 100, then B's marks = 100 – 30 = 70.

$\therefore$ Marks obtained by C = 112 × $\frac{100}{80} \times \frac{100}{70}$ = 200

48. In an examination, 35% students failed in English and 30% in Math. If 10% students failed in both subjects, students passed in both subjects are:

(a) 65% (b) 55% (c) 45% (d) 25%

Solution:

(c) Students failed in English only = 35% – 10% = 25%

Students failed in Math only = 30% – 10% = 20%

Students failed in both subjects = 10%

∴ Students failed in either or both subjects = (25 + 20 + 10)% = 55%

∴ Students passed in both subjects = (100 – 55)% = 45%

49. In an examination, 50% students failed in English and 40% in Math and 15% students failed in both subjects. If 200 students passed in both the subjects, find the number of students appeared in the exam.

(a) 400 (b) 800 (c) 1000 (d) 2000

Solution:

(b) Students failed in English only = (50 – 15)% = 35%

Students failed in Math only = (40 – 15)% = 25%

Students failed in both subjects = 15%

∴ Students failed in either or both subjects = 35 + 25 + 15 = 75%

∴ Students passed in both subjects = (100 – 75)% = 25%

But number of students passed = 200 (i.e. 8 times of 25)

∴ Students appeared = 8 × 100 = 800

50. In an examination, 35% of the candidates failed in Math and 25% in English while 15% failed in both subjects. How many candidates passed in one subject but not in the other?

(a) 30% (b) 45% (c) 55% (d) 70%

Solution:

(a) Students failed in Math only = (35 – 15)% = 20%

∴ Students passed in English only = 20%

Students failed in English only = (25 – 15)% = 10%

∴ Students passed in Math only = 10%

∴ Total candidates passed in one subject but not in the other

= 10% + 20% = 30%

51. In an examination, 60% of the candidates passed in English, 50% in Math and 20% failed in both the subjects. How many students passed in both the subjects?

(a) 90% (b) 80% (c) 70% (d) 30%

Solution:

(d) Students failed in English = (100 – 60)% = 40%

Students failed in Math = (100 – 50)% = 50%

Students failed in both subjects = 20%

∴ Students failed in English only = 40% – 20% = 20%

∴ Students failed in Math only = 50% – 20% = 30%

∴ Students failed in either or both subjects = (20 + 30 + 20)% = 70%

∴ Students passed in both subjects = (100 – 70) = 30%

52. In an examination, 60% of the candidates passed in English, 70% in Math and 40% in both subjects. How many students failed in both subjects ?

(a) 10% (b) 20% (c) 30% (d) 40%

Solution:

(a) Students passed in English only = (60 – 40)% = 20%

Students passed in Math only = (70 – 40)% = 30%

Students passed in both subjects = 40%

∴ Students passed in either or both subjects = 20 + 30 + 40 = 90%

∴ Students failed in both subjects = 10%

53. In an election contest between two candidates, 10% of total voters did not take part. The candidate who was elected got 60% of the votes casted. If he got 180 votes more than his opponent, find total number of voters.

(a) 300 (b) 900 (c) 1000 (d) 1800

Solution:

(c) Votes casted = 100% – 10% = 90%

Votes in favour of winning candidates = 60% of 90% of total votes

Votes in favour of opponent = 40% of 90% of total votes

∴ (60 – 40)% of 90% of total votes = 180

∴ 20% of 90% of total votes = 180

∴ Total voters = $180 \times \frac{100}{90} \times \frac{100}{20} = 1000$

54. A reduction of 10% in the price of sugar enables a customer to buy 4 kg. more of sugar for Rs. 400. What is the reduced price per kg ?

(a) Rs. 10 (b) Rs. 15

(c) Rs. 20 (d) Data insufficient

Solution:

(a) Saving due to reduction in price = 10% of Rs. 400 = Rs. 40

With this amount he can buy 4 kg. more of sugar.

∴ Reduced price per kg. of sugar = $\frac{40}{4}$ = Rs. 10

55. A reduction of 20% in the price of sugar enables a purchaser to buy 4 kg. more of sugar for Rs. 80. The original price of the sugar per kg. is:

(a) Rs. 4 (b) Rs. 4.50 (c) Rs. 5 (d) Rs. 6

Solution:

(c) Saving due to reduction in price = 20% of Rs. 80 = Rs. 16

With this amount he can buy 4 kg. more of sugar.

∴ Reduced price per kg. of sugar = $\frac{16}{4}$ = Rs. 4

∴ Original price = $4 \times \frac{100}{80}$ = Rs. 5.

56. A salesman receives salary of Rs. 4000 per month and 10% commission on total sales. What is his earnings, if sales is Rs. 20000?

(a) Rs. 3000 (b) Rs. 5000 (c) Rs. 6000 (d) Rs. 8000

Solution:

(c) Salary = Rs. 4000

Commission = 10% of Rs. 20000 = Rs. 2000

∴ Total earnings = Rs. 4000 + Rs. 2000 = Rs. 6000

57. A salesman receives salary of Rs. 5000 per month and 20% commission on total sales. What is the amount of sales, if his total earnings in a month is Rs. 9000?

(a) Rs. 15000 (b) Rs. 20000 (c) Rs. 50000 (d) Rs. 60000

Solution:

(b) Total earnings = Rs. 9000

Salary = Rs. 5000

Commission = Total earnings – Salary = Rs. 9000 – Rs. 5000

= Rs. 4000

Commission = 20% of Sales = Rs. 4000

∴ Sales = Rs. $4000 \times \frac{100}{20}$ = Rs. 20000

58. A salesman receives a salary of Rs. 1000 per month and 10% commission on sales in excess of Rs. 5000. What is the amount of sales, if he earns Rs. 2000 in a particular month?

(a) Rs. 10000 (b) Rs. 15000 (c) Rs. 20000 (d) Rs. 25000

Solution:

(b) Total earnings = Rs. 2000

Salary = Rs. 1000

∴ Commission = Rs. 2000 – Rs. 1000 = Rs. 1000

Sales in excess of Rs. 5000 (to earn Rs. 1000 as commission)

= 1000 × 10 = Rs. 10000

∴ Total sales = Rs. 10000 + Rs. 5000 = Rs. 15000

59. Mohan's income and expenditure are in the ratio 5 : 3. His income increased by 12% and expenditure by 15%. By what per cent does his savings increase ?

(a) 3% (b) 5% (c) 6% (d) 7.5%

Solution :

(d) Percentage increase in savings $= \frac{ax - by}{x - y} = \frac{5 \times 12 - 3 \times 15}{5 - 3} = 7.5\%$

60. Sohan's expenditure and savings are in the ratio 3 : 2. His income increases by 12% and savings by 15%. By what per cent would his expenditure increase ?

(a) 3% (b) 5% (c) 8% (d) 10%

Solution :

(d) Ratio of his income and savings = (3 + 2) : 2 = 5 : 2

Percent increase in expenditure $= \frac{5 \times 12 - 2 \times 5}{5 - 2} = 10\%$

61. A mixture of 3 litres contains 20% alcohol and another mixture of 7 litres contains 30% alcohol. Find the strength of alcohol in the resulting mixture, if two mixtures are mixed together.

(a) 25% (b) 27% (c) 27.5% (d) 28%

Solution:

(b) Total mixture = 3 + 7 = 10 litres

Alcohol in the mixture = 3 × 0.2 + 7 × 0.3 = 0.6 + 2.1 = 2.7 litres

$\therefore$ Percentage of Alcohol = $\frac{2.7}{10} \times 100 = 27\%$

62. In a class of 500 students, $\frac{2}{5}$ are boys and the rest are girls. If 20% of the boys and 30% of the girls fail, find the percentage of students failed.

(a) 20% (b) 25% (c) 26% (d) 50%

Solution:

(c) Boys in class = $\frac{2}{5}$

$\therefore$ Girls in the class = $1 - \frac{2}{5} = \frac{3}{5}$

$\therefore$ Percentage of students failed = $\frac{2}{5} \times 20\% + \frac{3}{5} \times 30\%$

= 8% + 18% = 26%

63. The ratio of boys and girls in a class is 3 : 2. If 10% of boys and 15% of girls failed in an examination, the percentage of passed students is:

(a) 75% (b) 80% (c) 88% (d) 87.5%

Solution:

(c) Students failed = $\frac{3}{5} \times 10\% + \frac{2}{5} \times 15\% = 6\% + 6\% = 12\%$

$\therefore$ Students passed = 100% – 12% = 88%

64. 500 students of a school appeared in an examination, 40% of the students are girls and 5% of the girls failed in the exam. Find percentage of boys failed if the total pass percentage of the school is 92%.

(a) 3% (b) 6% (c) 8% (d) 10%

Solution:

(d) Total students failed = (100 – 92)% = 8%

Percent girls failed = 5% of 40% = 2% (of total students)

$\therefore$ Percentage of boys failed = 8% – 2% = 6% (of total students)

But boys in school = 100% – 40% = 60%

$\therefore$ Percent boys failed (of boys) = $6 \times \frac{100}{60} = 10\%$

65. A man usually buys goods from market A. In market B, he can get the same goods 10% cheaper. If he spends extra amount of Rs. 55 on transportation in market B and still saves Rs. 145 by buying in market B. What does he pay for the goods?

(a) Rs. 550 (b) Rs. 1450 (c) Rs. 1800 (d) Rs. 2000

Solution:

(c) His total savings in buying = Rs. 55 + Rs. 145 = Rs. 200.

He saves 10% of the bill amount by buying in market B.

∴ Cost of goods purchased to save Rs. 200 = $200 \times \frac{100}{10}$ = Rs. 2000.

∴ Amount paid by him = 2000 – 10% of 2000 = Rs. 1800

66. The Population of a town is 10000. In a particular year, male population is increased by 5% and female population by 8%. If the population at the end of year is 10590, what was the size of male population in the town in the beginning of year?

(a) 3000 (b) 4000 (c) 6000 (d) 7000

Solution:

(d) Actual increase in the population = 10590 – 10000 = 590

Let the population consists of male population only.

Then increase in population = 5% of 10000 = 500

Which is short from actual increase by 590 – 500 = 90

The difference is because female population with higher rate of increase also forms part of the population.

Higher rate of increase for female population = 8% – 5 % = 3%.

∴ 3% of female population = 90 (i.e. 30 times of 3)

∴ Female population = 30 × 100 = 3000

∴ Male population = 10000 – 3000 = 7000

Chapter 19

PERCENTAGE II – GOLDEN RULES

In this chapter, we are continuing 'Percentage' chapter. Rules stated in this chapter are very important and useful in solving the typical problems. These rules have wider application in other chapters also.

FORMULAE AND RULES

1. The product of two or more numbers is increased by x% if one of the numbers is increased by x%.
2. The product of two or more numbers is decreased by x% if one of the numbers is decreased by x%.

3. If price of a commodity is changed from Re.1 to Rs. $\frac{x}{y}$.

   Then to stabilize his expenditure, a consumer must buy $\frac{y}{x}$ units of the commodity instead of 1 unit.

Hint : $\frac{x}{y} \times \frac{y}{x} = 1$

IMPORTANT RULES

Now we are giving three very important rules. Read the rules very carefully. Once you understand these rules, you will find that a lot of typical problems can be solved easily in almost no time.

4. If we have two numbers – one of them is **increased by x%** and the other is **decreased by x%.**

   Then, **product of the numbers will be decreased** by $\frac{x^2}{100}$ %.

Note: This formula can be used if a number is increased by x% and then the resultant number is decreased by x% and vice-versa.

5. If out of two numbers **first number is increased by x%** and **second number is increased by y%.**

   Then **per cent increase in the product of two numbers** is:

   $x + y + \frac{xy}{100}$

Note: Decrease in a number can be written as negative increase e.g. if the first number is decreased by x% then we can write it as (–x)% increase.

Note: This formula **can be used in numerous situations**, as:

⇒ If a number is increased by x% and then the increased number is again increased by y%, then total increase in percentage = $x + y + \frac{xy}{100}$

⇒ If a number is increased by x% and the other number is decreased by y%, then percent increase in the product of two numbers

$= x + (-y) + \frac{x(-y)}{100} = x - y - \frac{xy}{100}$

⇒ If price of a commodity is increased by x% and its sales (in units) is decreased by y%, then net effect on sales $= x - y - \frac{xy}{100}$

Alternative Method:

1. We assume that the original numbers are 10 each.
2. Find new numbers after adding percent increase in the numbers.
3. Find product of the new numbers.
4. Deduct 100 from the product of the new numbers. Result obtained is net result of increase or decrease in the original numbers.

Example:

If out of two numbers, one number is increased by 10% and the other number by 20%, find per cent increase in the product of the two numbers.

Solution:

Let both the numbers are 10 each.

Then new numbers are 11 and 12.

$11 \times 12 - 100 = 32$

∴ Increase = 32%.

6. (a) If A's income is $\frac{1}{x}$ more than B's income, then B's income is less than A's income by $\frac{1}{x+1}$.

(b) If A's income is $\frac{1}{x}$ less than B's income, then B's income is more than A's income by $\frac{1}{x-1}$.

EASIER METHOD OF REMEMBERING THE RULE STATED ABOVE

1. If increase/decrease in the original number is $\frac{1}{x}$ then decrease or increase in the new number to get the original number is $\frac{1}{x+1}$ or $\frac{1}{x-1}$.
2. If we add something to a number, the resulting number (new base) becomes bigger than the original base and vice-versa.
3. If **new base is bigger** than the original base, then **smaller fraction** is required, i.e. $\frac{1}{x+1}$.
4. If **new base is smaller** than the original base, then **bigger fraction** is required, i.e. $\frac{1}{x-1}$.

What happens **if the increase/decrease is** $\frac{a}{x}$?

(a) If the increase is $\frac{a}{x}$ then the decrease is $\frac{a}{x+a}$, and

(b) If the decrease is $\frac{a}{x}$, then the increase is $\frac{a}{x-a}$.

Formula stated above can be used in numerous situations, for example:

1. If a number is increased and then how much the new number is to be decreased to get the original number and vice-versa.
2. Price of a commodity is first increased and then how much new price is to be reduced to restore to the original price and vice-versa.
3. If price of a commodity is increased then how much quantity purchased is to be reduced so that total expenditure remains constant and vice-versa.
4. If first number is more than the second number then how much the second number is less than the first number and vice-versa.

SOLVED EXERCISE

1. Price of a commodity is increased by 10% and then resultant price is decreased by 10%. Find net increase or decrease in the price.

(a) 1% decrease (b) 1% increase

(c) No effect (d) 5% decrease

Solution:

(a) Here increase and decrease in percent are same.

$\therefore$ Net decrease in the new price from original price = $\frac{10 \times 10}{100}$

= 1% decrease

2. Selling price of a commodity is increased by 15% in a year and then the new price is decreased by 15% in the next year. Find net increase or decrease in the selling price.

(a) No effect (b) 2% decrease

(c) 2.25% decrease (d) 2.25% increase

Solution:

(c) Here increase and decrease in per cent are same.

$\therefore$ Net decrease in the selling price of the commodity = $\frac{15 \times 15}{100}$

= 2.25% decrease

3. Price of sugar having been increased by 20%, a housewife reduced consumption of sugar by 20%. What was the effect of this on her expenditure on sugar?

(a) 4% decrease (b) 4% increase

(c) No effect (d) 1% decrease

Solution:

(a) Here increase and decrease in percent are same.

$\therefore$ Net decrease in her expenditure on sugar = $\frac{20 \times 20}{100}$ = 4% decrease

4. The salary of an employee is first increased by 20% and then again increased by 30%. Find per cent increase in his salary from the original salary.

(a) 6% (b) 44% (c) 50% (d) 56%

Solution:

(d) Net effect on his salary = $20 + 30 + \frac{20 \times 30}{100} = 20 + 30 + 6$

= 56% increase

Alternative Method:

$12 \times 13 - 100 = 56\%$

5. The salary of an employee is Rs. 9870. It is first increased by 20% and then is decreased by 10%. Find percent increase in his salary from the original salary.

(a) 8% (b) 10% (c) 12% (d) 13%

Solution:

(a) Net effect on salary = $20 - 10 - \frac{20 \times 10}{100} = 20 - 10 - 2 = 8\%$ increase

Note: Actual salary in the beginning does not make any difference while calculating percentage increase or decrease. What relevant is increase or decrease in terms of percentage.

Alternative Method:

$12 \times 9 - 100 = 8\%$

6. The salary of an employee is first decreased by 10% and then is increased by 20%. Find per cent increase in his salary from the original salary.

(a) 8% (b) 10% (c) 12% (d) 13%

Solution:

(a) Increase from original salary

$= -10 + 20 - \frac{10 \times 20}{100} = -10 + 20 - 2 = 8\%$ increase

Alternative Method:

$9 \times 12 - 100 = 8\%$

7. Selling price of a commodity is increased by 20%. As a result of it, quantity sold is decreased by 30%. What is effect of this on total sales?

(a) 16% decrease (b) 10% increase

(c) 10% decrease (d) 50% increase

Solution:

(a) Net effect on total sales

$= 20 - 30 - \frac{20 \times 30}{100} = 20 - 30 - 6 = (-)\ 16\%$ i.e. 16% decrease

Alternative Method:

$12 \times 7 - 100 = -16\%$

8. The population of a city is decreased by 20% in a year. In the next year the population is again decreased by 30%. Find per cent decrease in population of the city from the original population.

(a) 44% (b) 50% (c) 55% (d) 56%

Solution:

(a) Net decrease in population

$= -20 - 30 + \frac{20 \times 30}{100} = -20 - 30 + 6 = 44\%$ decrease

Alternative Method:

$8 \times 7 - 100 = -44\%$

9. The selling price of a commodity is reduced by 25%. As a result, sale of the commodity (in units) is increased by 20%. What will be the effect of this on the cash collection by the sale of the commodity?

(a) 5% decrease (b) 10% increase

(c) 45% increase (d) 10% decrease

Solution:

(d) Net effect on the sale

$$= -25 + 20 - \frac{25 \times 20}{100} = -25 + 20 - 5 = (-)10\% \text{ i.e. } 10\% \text{ decrease}$$

Alternative Method:

$7.5 \times 12 - 100 = -10\%$

10. The population of a city is decreased by 10% in a year. In the next year the population is again decreased by 30%. Find percent decrease in the population of the city from the original population.

(a) 35% (b) 37% (c) 40% (d) 43%

Solution:

(b) Decrease in population

$$= -10 - 30 + \frac{10 \times 30}{100} = -10 - 30 + 3 = (-)\ 37\% = 37\% \text{ decrease}$$

Alternative Method:

$9 \times 7 - 100 = -37\%$

11. B's income is 20% more than A's and C's income is 25% more than B's. How many per cent is C's income more than A's?

(a) 5% (b) 40% (c) 45% (d) 50%

Solution:

(d) C's income is more than A's by

$$= 20 + 25 + \frac{20 \times 25}{100} = 20 + 25 + 5 = 50\%$$

Alternative Method:

$12 \times 12.5 - 100 = 50\%$

12. If B's salary is 30% more than A's and C's salary is 20% less than B's. How many per cent C's salary is more than A's?

(a) 4% (b) 6% (c) 8% (d) 10%

Solution:

(a) C's salary is more than A's by

$$= 30 - 20 - \frac{30 \times 20}{100} = 30 - 20 - 6 = 4\%$$

Alternative Method:

$13 \times 8 - 100 = 4\%$

13. The price of a commodity was increased in the ratio 4 : 5. If a customer reduces its consumption in the ratio 4 : 3, how much per cent his expenditure on the commodity will increse or decrease ?

(a) 1 % increase (b) 1 % decrease

(c) 6.25 % increase (d) 6.25 % decrease

Solution :

(d) Expenditure in the first case = 4 × 4 = 16

Expenditure in the second case = 5 × 3 = 15

Decrease in expenditure = 16 – 15 = 1

∴ Percentage decrease in expenditure $= \frac{1}{16} \times 100 = \frac{25}{4} = 6.25\%$

14. Price of an article increases by 20%. As a result turnover increases by 12%. Find the decrease in quantity sold.

(a) 5 % (b) 6.67 % (c) 8 % (d) 16 %

Solution :

(b) Quantity sold after increase in price $= (100+12) \times \frac{100}{100+20} = 112 \times \frac{100}{120} = \frac{280}{3}$

∴ Decrease quantity sold $= 100 - \frac{280}{3} = \frac{20}{3} = 6.67\%$

15. A sells his goods 20% cheaper than B but 20% dearer than C. How much a customer of B saves, if he buys goods from C for Rs. 100?

(a) Rs. 4 (b) Rs. 40 (c) Rs. 44 (d) Rs. 50

Solution:

(d) If C sells goods for Rs.100, then A charges = Rs. 100 + Rs. 20 = Rs. 120.

∴ Amount charged by B (for goods worth Rs. 100 of C) $= 120 \times \frac{100}{80} =$ Rs. 150

∴ His savings = Rs. 150 – Rs. 100 = Rs. 50.

16. Price of a commodity is increased by 25%. By how much new price must be reduced to restore it to the original price?

(a) 20% (b) 25% (c) $33\frac{1}{3}\%$ (d) 50%

Solution:

(a) Increase in price = 25% $= \frac{1}{4}$

∴ Price is to be decreased by $\frac{1}{4+1} = \frac{1}{5} = 20\%$

17. If A's salary is 25% more than B's, then how much per cent is B's salary less than A's:

(a) 20% (b) 30% (c) 25% (d) $33\frac{1}{3}\%$

Solution:

(a) A's salary is more than B's salary by 25% $= \frac{1}{4}$

$\therefore$ B's salary is less than A's salary by $\frac{1}{4+1} = \frac{1}{5} = 20\%$

18. If price of oil is increased by $33\frac{1}{3}\%$, by how much a housewife must reduce consumption of oil so that her expenditure on oil remains the same?

(a) $33\frac{1}{3}\%$ (b) 25% (c) 40% (d) 50%

Solution:

(b) New price $= 1 + \frac{1}{3} = \frac{4}{3}$ of original price

$\therefore$ New quantity with the same amount $= \frac{3}{4}$ of the original quantity

$\therefore$ Decrease in quantity $= 1 - \frac{3}{4} = \frac{1}{4} = 25\%$

Alternative Method:

Price is increased by $33\frac{1}{3}\% = \frac{1}{3}$

$\therefore$ Consumption is to be decreased by $\frac{1}{3+1} = \frac{1}{4} = 25\%$.

19. If the price of coal is increased by 25%, by what percent the consumption of coal must be reduced so that the expenditure is not increased?

(a) 20% (b) 25% (c) $33\frac{1}{3}\%$ (d) 50%

Solution:

(a) New price $= 1 + \frac{1}{4} = \frac{5}{4}$ of original price

$\therefore$ New quantity with the same amount $= \frac{4}{5}$ of the original quantity

$\therefore$ Decrease in quantity $= 1 - \frac{4}{5} = \frac{1}{5} = 20\%$

Alternative Method:

Price is increased by 25% $= \frac{1}{4}$

$\therefore$ Consumption is to be decreased by $\frac{1}{4+1} = \frac{1}{5} = 20\%$

20. Price of a commodity is reduced by 10%. By how much, new price must be increased to restore it to the original price?

(a) 10% (b) 11% (c) $9\frac{1}{11}\%$ (d) $11\frac{1}{9}\%$

Solution:

(d) Price is reduced by 10% $= \frac{1}{10}$

$\therefore$ Reduced price is to be increased by $\frac{1}{10-1} = \frac{1}{9} = 11\frac{1}{9}\%$

21. The difference between two numbers is 20% of the larger number. If the smaller number is 40, the larger number is:

(a) 32 (b) 48 (c) 50 (d) 60

Solution:

(c) Difference between the numbers is 20% $= \frac{1}{5}$ of the larger number.

$\therefore$ Difference $= \frac{1}{5-1} = \frac{1}{4}$ of the smaller number $= \frac{1}{4} \times 40 = 10$

$\therefore$ Larger number = 40 + 10 = 50

22. A's income is 50% less than B's. By how many per cent, B's income is more than A's.

(a) 33.33% (b) 50% (c) 75% (d) 100%

Solution:

(d) A's income is less than B's by 50% $= \frac{1}{2}$

$\therefore$ B's income is more than A's by $\frac{1}{2-1} = \frac{1}{1} = 100\%$

23. Price of cloth having been raised by 75%, by how much per cent a householder must reduce his consumption of cloth so as not to increase his expenditure?

(a) $42\frac{6}{7}\%$ (b) $57\frac{1}{7}\%$ (c) 75% (d) 50%

Solution:

(a) Price is increased by 75% $= \frac{3}{4}$

$\therefore$ Consumption is to be decreased $= \frac{3}{4+3} = \frac{3}{7} = 42\frac{6}{7}\%$

24. Price of cloth having been reduced by 75%, how much per cent a householder can increase his consumption of cloth so that his expenditure on the cloth remains the same?

(a) 50% (b) 75% (c) 200% (d) 300%

Solution:

(d) Price is decreased by 75% $= \frac{3}{4}$

$\therefore$ Consumption is to be increased by $\frac{3}{4-3} = \frac{3}{1} = 300\%$

25. If the price of sugar is increased by 50%, by how much percent a housewife must reduce the consumption of sugar so that her expenditure on sugar is increased by 20% only.

(a) 10% (b) 12% (c) 20% (d) 25%

Solution:

(c) New Price = 150% $= \frac{3}{2}$ of original price

$\therefore$ Consumption with same amount $= \frac{2}{3}$ of original consumption

But total expenditure on sugar is increased by 20%.

$\therefore$ Expenditure = 100 + 20 = 120% of original expenditure

∴ New consumption = $\frac{2}{3} \times 120 = 80\%$

∴ Decrease in consumption = 100% – 80% = 20%

26. Price of admission ticket is reduced by 20%. As a result, total collection amount is increased by 60%. Find percent increase in the sale of tickets.

(a) 24% (b) 40% (c) 80% (d) 100%

Solution:

(d) New Price = $1 - \frac{1}{5} = \frac{4}{5}$ of original price

∴ Tickets sold for same collection = $\frac{5}{4}$ of original sale of tickets

But new collection = 100 + 60 = 160% of original collection

∴ New sale = $\frac{5}{4} \times 160 = 200\%$ of original sale of tickets

∴ Increase in sale of tickets = 200% – 100% = 100% of original

27. If the price of a commodity is reduced by 10%. How much quantity of the commodity can be bought with the same money which was sufficient to buy 90 kg. at the original price?

(a) 80 kg (b) 81 kg (c) 99 kg (d) 100 kg

Solution:

(d) Reduction in the price = $10\% = \frac{1}{10}$

∴ Increase in consumption with the same money

= $\frac{1}{9}$ of the original quantity = $\frac{1}{9} \times 90$ kg. = 10 kg.

∴ Increased quantity = 90 + 10 = 100 kg.

28. Price of sugar having been fallen by 10%, a consumer can buy 10 kg. more of sugar than what he was buying originally. Find the quantity he was buying originally.

(a) 80 kg (b) 90 kg (c) 100 kg (d) 110 kg

Solution:

(b) Fall in price = $10\% = \frac{1}{10}$

∴ Quantity bought is to be increased by $\frac{1}{9}$ of the original quantity.

∴ Original quantity is 9 times of the increase in quantity.

∴ Original quantity = 9 × 10 kg. = 90 kg.

29. Price of a commodity having been increased by 20%, a consumer can buy 10 kg. less than the quantity he was buying originally. Find the quantity he was buying originally.

(a) 60 kg (b) 80 kg (c) 90 kg (d) 100 kg

Solution:

(a) Increase in price = $20\% = \frac{1}{5}$

∴ Quantity bought is to be decreased by $\frac{1}{6}$ of the original quantity.

∴ Original quantity is 6 times of decrease in the quantity.

∴ Original quantity = 6 × 10 kg. = 60 kg.

30. If price of sugar is fallen by 5%, a man can buy 220 kg sugar. Find the quantity of sugar he was buying originally.

(a) 209 kg (b) 210 kg (c) 240 kg (d) 242 kg

Solution:

(a) Fall in price of sugar = 5% = $\frac{1}{20}$

∴ Increase in quantity = $\frac{1}{19}$ of the original quantity

= $\frac{1}{20}$ of the increased quantity = $\frac{1}{20} \times 220$ kg = 11 kg.

∴ Original quantity of sugar = 220 – 11 = 209 kg.

Alternative Method :

$$220 \times \frac{95}{100} = 209 \text{ kg.}$$

31. If price of rice is increased by 10%, a housewife can buy 110 kg. rice for a certain amount. Find quantity of rice she was buying at original price.

(a) 99 kg (b) 100 kg (c) 110 kg (d) 121 kg

Solution:

(d) Increase in price of rice = 10% = $\frac{1}{10}$

∴ Decrease in quantity = $\frac{1}{11}$ of the original quantity

= $\frac{1}{10}$ of the decreased quantity = $\frac{1}{10} \times 110$ kg = 11 kg.

∴ Original quantity of rice = 110 + 11 = 121 kg.

Alternative Method :

$$110 \times \frac{110}{100} = 121 \text{ kg.}$$

32. Price of sugar having been fallen by 10%, a consumer can buy 22 kg. more than before. Had the price been increased by 10%, how much quantity of sugar could he have bought for the same sum?

(a) 180 kg (b) 200 kg (c) 220 kg (d) 250 kg

Solution:

Situation I :

(a) Decrease in price = 10% = $\frac{1}{10}$

∴ Quantity bought is increased by $\frac{1}{9}$ of the original quantity

∴ Original quantity is 9 times of increase in quantity.

∴ Original quantity = (9 × 22) kg.

Situation II:

Increase in price = 10% = $\frac{1}{10}$ of original price.

∴ Quantity bought is to be decreased by $\frac{1}{11}$ of the original quantity

∴ Decreased quantity = $1 - \frac{1}{11} = \frac{10}{11}$ of the original quantity

$= (9 \times 22) \times \frac{10}{11} = 180$ kg.

33. Income-tax rate is 20% in a particular year. If in the next year, a person's income is increased by 25% but total tax liability remaining the same, find tax-rate during the next year.

(a) 15% (b) 16% (c) 20% (d) 25%

Solution:

(b) Let his income in first year = Rs. 100
Then income in the next year = Rs. 100 + 25% of Rs. 100 = Rs. 125
Let tax-rate in the second year is x%.
Then 20% of Rs. 100 = x% of Rs. 125

$\therefore x = 20\% \times \frac{100}{125} = 16\%$

Alternative Method:

Increase in income in second year = 25% = $\frac{1}{4}$

∴ Decrease in tax-rate so that tax liability remains same = $\frac{1}{4+1} = \frac{1}{5}$

∴ Decrease in tax-rate in second year = $\frac{1}{5} \times 20\% = 4\%$

∴ Tax-rate in second year = 20% – 4% = 16%

34. The income of a sales agent remains the same even though the rate of commission is increased from 4% to 5%. Find the percent decrease in the sales.

(a) 1% (b) 5% (c) 20% (d) 25%

Solution:

(c) Increase in the rate of commission = 5% – 4% = 1%

∴ Relative increase in the rate of commission = $\frac{5-4}{4} = \frac{1}{4}$

∴ Decrease in the sales = $\frac{1}{4+1} = \frac{1}{5} = 20\%$

35. The rate of income tax is increased from 4% to 5% as the result of it, tax-liability of a person is increased by 10%. Find percent increase or decrease in his income.

(a) 5% increase (b) 9% increase
(c) 9% decrease (d) 12% decrease

Solution:

(d) New tax rate = $1 + \frac{1}{4} = \frac{5}{4}$ of original tax-rate.

∴ Income must be $\frac{4}{5}$ of the original income so that the tax-liability remains same.

But tax-liability = 100 + 10 = 110% of original liability

∴ Income = $\frac{4}{5}$ × 110% = 88% of original income

∴ Income is reduced by 12%.

36. Rate of commission is increased from 9% to 10%, but due to decrease in sale amount by Rs. 1000, a selling agent got the same commission as in the last year. Find the amount of sale achieved by him in the last year.

(a) Rs. 8000 (b) Rs. 9000 (c) Rs. 10000 (d) Rs. 12500

Solution:

(c) Increase in rate of commission = 10% – 9% = 1%

∴ Relative increase in rate of commission = $\frac{10-9}{9} = \frac{1}{9}$

∴ Sale might have decreased by $\frac{1}{10}$, if total commission earned is same in the current year.

∴ Sales in last year = 10 times of decrease in sales

But actual decline in sale = Rs. 1000

∴ Sales during last year = 10 × Rs. 1000 = Rs. 10000

37. A man's income is increased by Rs. 2000 in a year. Rate of income-tax having reduced from 25% to 20%, his tax-liability remains the same as in the last year. Find his income in the current year.

(a) Rs. 4000 (b) Rs. 5000 (c) Rs. 8000 (d) Rs. 10000

Solution:

(d) Decrease in tax-rate = 25% – 20% = 5%

Relative decrease in tax-rate = $\frac{5}{25} = \frac{1}{5}$

∴ Increase in income = $\frac{1}{4}$ of last year's income = Rs. 2000

∴ Income during last year = Rs. 2000 × 4 = Rs. 8000

∴ Income during current year = Rs. 8000 + Rs. 2000 = Rs. 10000

38. The population of a village is increased by 5% in one year and in the next year it is decreased by 5%. If at the end of second year, the population is 39900, what was the population at the beginning of first year?

(a) 39000 (b) 39500 (c) 39800 (d) 40000

Solution:

(d) Population in the beginning of first year

$$= 39900 \times \frac{100}{100+5} \times \frac{100}{100-5} = 39900 \times \frac{20}{21} \times \frac{20}{19} = 40000$$

Alternative method:

Since, increase and decrease is same in both years.

∴ Decrease in population in 2 years = $\frac{5}{100} \times \frac{5}{100}$

= $\frac{1}{400}$ of original population

= $\frac{1}{400-1} = \frac{1}{399}$ of the population at the end of two years

∴ Decrease in population = $\frac{1}{399}$ × 39900 = 100

∴ Population at the beginning

= Population at the end of two years + Decrease in population

= 39900 + 100 = 40000

Chapter 20

DISCOUNT

Discount is a very common term in the business world. By discount, we mean selling the product below its Marked or List Price. Usually, manufacturer allows discount to the wholesaler and the wholesaler to the retailer. Sometimes even retailer gives discount to the customer.

If a shopkeeper allows x% discount on the marked price of an article,

Then selling price of the article = (100 – x)% of the marked price.

Series of discount: More than one discount may be allowed on the article. In such case, first discount is allowed on the total price of the article and then the second discount is allowed on the amount payable after first discount.

Amount payable after a series of two discounts

$$= \text{Marked Price} \times \frac{100 - \text{First Discount}}{100} \times \frac{100 - \text{Second Discount}}{100}$$

Note: This formula can be used for any number of discount series.

Finding Single discount rate equivalent to the series of two discounts

$$= \text{First Discount} + \text{Second Discount} - \frac{\text{First Discount} \times \text{Second Discount}}{100}$$

How to find discount rate equivalent to a series of more than two discounts?

Method:

1. Find discount rate equivalent to series of two discounts.
2. Find discount rate equivalent to third discount and discount equivalent to first two discounts and so on.

Example:

Find discount equivalent to the series of discount of 10%, 20% and 25%.

Solution:

Discount rate equivalent to 10% and 20%

$$= 10 + 20 - \frac{10 \times 20}{100} = 10 + 20 - 2 = 28\%$$

Discount equivalent to 25% and 28%

$$= 25 + 28 - \frac{25 \times 28}{100} = 25 + 28 - 7 = 46\%$$

∴ Discount equivalent to 10%, 20% and 25% is 46%.

Alternative Method:

Let Marked Price = Rs. 100

$$\text{Then Selling price} = 100 \times \frac{100 - 10}{100} \times \frac{100 - 20}{100} \times \frac{100 - 25}{100}$$

$= 100 \times \frac{90}{100} \times \frac{80}{100} \times \frac{75}{100} = $ Rs. 54

$\therefore$ Discount = 100% – 54% = 46%

SOLVED EXERCISE

1. Single discount rate equivalent to a series of discounts of 20% and 25% is:

(a) 5% (b) 40% (c) 45% (d) 50%

Solution:

(b) Equivalent Discount $= 20 + 25 - \frac{20 \times 25}{100} = 45 - 5 = 40\%$

2. A TV whose catalogue price is Rs. 15000 is available at two successive discounts of 10% and 20%, Find the single equivalent discount.

(a) 25% (b) 28% (c) 30% (d) 32%

Solution:

(b) Single equivalent discount rate

$= 10 + 20 - \frac{10 \times 20}{100} = 30 - 2 = 28\%$

Hint: Equivalent discount by this method is calculated without using the catalogue price.

3. If A man allows 20% discount on all his articles, the number of the articles sold is increased by 20%. What is the effect on total sale?

(a) 40% Increase (b) 4% Increase (c) 4% Decrease (d) No effect

Solution:

(c) Effect on sales $= \frac{20 \times 20}{100} = 4\%$ decrease

4. Catalogue price of a TV is Rs. 10000. How much a customer need to pay, if two successive discounts of 20% and 20% are available on the TV?

(a) Rs. 6000 (b) Rs. 6200 (c) Rs. 6400 (d) Rs. 6500

Solution:

(c) Amount paid for T.V. $= 10000 \times \frac{100 - 20}{100} \times \frac{100 - 20}{100}$

$= 10000 \times \frac{80}{100} \times \frac{80}{100} = $ Rs. 6400

5. A watch whose list price is Rs. 500, is available at two successive discounts of 10% and 20%. Amount paid by the customer is:

(a) Rs. 350 (b) Rs. 360 (c) Rs. 370 (d) None of these

Solution:

(b) Amount paid for watch $= 500 \times \frac{100 - 10}{100} \times \frac{100 - 20}{100}$

$= 500 \times \frac{90}{100} \times \frac{80}{100} = $ Rs. 360

6. A man bought an article for Rs. 21. What was the marked price of the article if the article was sold at 30% discount?

(a) Rs. 30 (b) Rs. 6.30 (c) Rs. 14.70 (d) Rs. 70

Solution:

(a) Marked price of the article $= 21 \times \frac{100}{100 - 30} = 21 \times \frac{100}{70} =$ Rs. 30

7. The price of an article is raised by 30% and then two successive discounts of 10% each are allowed. What is net increase in the price of article ?

(a) 5% (b) 5.3% (c) 10% (d) 13%

Solution :

(b) Selling Price $= 100 \times \frac{130}{100} \times \frac{90}{100} \times \frac{90}{100} = 105.30$

$\therefore$ Net Increase in price = 5.3%

8. A shopkeeper allows two successive discounts of 3% and 7% while another allows two successive discounts of 2% and 8%. From which shopkeeper should a customer buy if marked price is same at both shops ?

(a) First (b) Second (c) Both are same (d) Data Inadequate

Solution :

(b) Discount at first shop $= 3 + 7 - \frac{3 \times 7}{100} = 10 - 2.1 = 7.9$

Discount at second shop $= 2 + 8 - \frac{2 \times 8}{100} = 10 - 1.6 = 8.4$

Hence offer at second shop is beneficial to the customer.

Note : In both cases, sum of discounts rates is same and the customer saves more on buying from the shop where product of the rates is less. (3 × 7 > 2 × 8)

9. A shopkeeper allows two successive discounts on an article whose marked price is Rs. 150 and selling price is Rs. 105. What is first discount if second discount is 12.5% ?

(a) 16.67% (b) 17.5% (c) 20% (d) 25%

Solution :

(c) Price before second discount $= 105 \div \left(1 - \frac{1}{8}\right) = 105 \times \frac{8}{7} =$ Rs.120

$\therefore$ First Discount $= \frac{30}{150} \times 100 = 20\%$

Chapter 21

PROFIT AND LOSS

In the exams, many questions are asked on this topic. This chapter is largely based on the concepts of percentage. Therefore, you must learn the rules mentioned in Percentage chapters before starting this chapter.

BASIC TERMS

Cost Price: The price paid by the purchaser for buying the article is called cost price of the article.

Selling Price: The price at which the article is sold by the seller is called selling price of the article.

Profit: The article is said to be sold at profit, if its Selling Price is more than its Cost Price.

Profit = Selling Price – Cost Price

Selling Price = Cost Price + Profit

Loss: The article is said to be sold at loss, if its Selling Price is less than its Cost Price:

Loss = Cost Price – Selling Price

Selling Price = Cost Price – Loss

PROFIT AND LOSS PER CENT

Profit/Loss per cent is relation between Profit/Loss and Cost Price of an article. Therefore, Profit or Loss per cent is always calculated on Cost Price, unless mentioned otherwise.

$$\text{Profit (per cent)} = \frac{\text{Profit}}{\text{Cost price}} \times 100$$

$$\text{Loss (per cent)} = \frac{\text{Loss}}{\text{Cost price}} \times 100$$

Example:

An article costing Rs. 10 is sold for a profit of Rs. 2. Find profit percent.

Solution:

$$\text{Profit per cent} = \frac{2}{10} \times 100 = 20\%$$

RELATION BETWEEN SELLING PRICE AND COST PRICE

When the article is sold at Profit:

$$\text{Selling Price} = \text{Cost Price} \times \frac{100 + \text{Profit percentage}}{100}$$

$$\text{Cost Price} = \text{Selling Price} \times \frac{100}{100 + \text{Profit percentage}}$$

When the article is sold at Loss:

$$\text{Selling Price} = \text{Cost Price} \times \frac{100 - \text{Loss percentage}}{100}$$

$$\text{Cost Price} = \text{Selling Price} \times \frac{100}{100 - \text{Loss percentage}}$$

CALCULATING PROFIT PER CENT WHEN GAIN IN ARTICLES IS GIVEN

If on selling 'x' articles, a man gains equal to the cost price of 'y' articles

Then Profit = $\frac{y}{x}$

To convert the fraction into percentage, multiply the fraction by 100.

If on selling 'x' articles, a man gains equal to the selling price of 'y' articles,

Then Profit = $\frac{y}{x - y}$

To convert the fraction into percentage, multiply the fraction by 100.

Note: Loss is denoted by negative profit.

MORE FORMULAE

1. A shopkeeper **marks** his goods at **x% above** the cost price and then **allows y% discount** on the marked price, then,

   His profit percent = $x - y - \frac{xy}{100}$

2. If price of an article is **marked x% above** the cost price **and x% discount** is allowed on its marked price.

   Then **loss** per cent = $\frac{x^2}{100}$

3. If **two articles** are sold for the **same selling price**. **On selling first article, a man gains x%** and on selling the **other article he loses x%,** in such cases **there will be always loss**, and

   Loss % = $\frac{x^2}{100}$ i.e. $\frac{(\text{Common Gain and Loss \%})^2}{100}$

4. If **two articles** are sold for the **same selling price. On selling first, a man gains x%** and on selling the **other he gains y%**

   Then profit percentage in the transaction = $\frac{100\,(x + y) + 2xy}{200 + x + y}$

   Note: Negative value of 'numerator' denotes loss.

5. **RELATION BETWEEN PROFIT/LOSS AND COST PRICE/ SELLING PRICE**

WHEN THE ARTICLE IS SOLD AT PROFIT:

We know that Cost price + Profit = Selling price

∴ Cost Price < Selling Price

∴ If we shift from Cost Price to Selling price,(i.e. to bigger base), smaller fraction is required and vice-versa.

(a) If profit is $\frac{1}{x}$ of cost price, then

Profit is $\frac{1}{x+1}$ of the selling price.

Example:

If profit on selling a commodity is $\frac{1}{3}$ of cost price, then

Profit is $\frac{1}{3+1} = \frac{1}{4}$ of the selling price.

(b) If profit is $\frac{1}{x}$ of selling price, then

Profit is $\frac{1}{x-1}$ of the cost price.

Example:

If profit on selling a commodity is $\frac{1}{3}$ of selling price, then

Profit is $\frac{1}{3-1} = \frac{1}{2}$ of the cost price

WHEN THE ARTICLE IS SOLD AT LOSS:

We know that, Cost price – Loss = Selling price

∴ Cost Price > Selling Price

∴ If we shift from Cost Price to Selling price (i.e. to smaller base), bigger fraction is required and vice-versa.

(a) If Loss is $\frac{1}{x}$ of cost price,

Then Loss is $\frac{1}{x-1}$ of the selling price

Example:

If loss on selling a commodity is $\frac{1}{3}$ of cost price,

Then Loss is $\frac{1}{3-1} = \frac{1}{2}$ of the selling price

(b) If Loss is $\frac{1}{x}$ of selling price,

Then Loss is $\frac{1}{x+1}$ of the cost price

Example:

If loss on selling a commodity is $\frac{1}{3}$ of selling price, then

Loss is $\frac{1}{3+1} = \frac{1}{4}$ of the cost price

6. On **selling** an article for **Rs. x**, a person earns **a% profit**. In order **to earn b%** profit, he must **sell the article for** Rs. $x \times \frac{100+b}{100+a}$

$$\text{New Selling price} = \text{Old Selling Price} \times \frac{100 + \text{Desired Profit (in \%)}}{100 + \text{Initial Profit (in \%)}}$$

Example:

If on selling an article for Rs. 220, a shopkeeper earns 10% profit, what should be the selling price if desired profit is 20%?

Solution:

$$\text{New Selling Price} = 220 \times \frac{100+20}{100+10} = 220 \times \frac{120}{110} = \text{Rs. } 240$$

7. If **two articles are sold** – first article at **x% profit** and the second article at **y% loss**, thus **making no profit, no loss in the transaction,**
Then Cost Price of the two articles are in the ratio of y : x.

Proof:

Let cost price of two articles is Rs.a and Rs.b respectively.

Then profit on first article = $\frac{ax}{100}$

And loss on second article = $\frac{by}{100}$

Since there is no profit or no loss in the bargain.

$\therefore \frac{ax}{100} = \frac{by}{100}$

$\therefore ax = by$

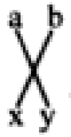
$\therefore \frac{a}{b} = \frac{y}{x}$

$\therefore$ Cost of first article : Cost of second article

= Loss % on second article : Profit % on first article

8. A man purchases some articles @ 'a' articles for Rs.b and sells them @ 'x' articles for Rs.y.
Then his profit % in the transaction is $\frac{ay - bx}{bx}$

Note: (ay – bx) is difference in cross multiplication of the four numbers as explained below:



Note: Negative value of (ay – bx) denotes loss.

SOLVED EXERCISE

1. If Cost Price and Selling Price of an article are Rs.12 and Rs.15 respectively, profit percentage is:

(a) 10% (b) 20% (c) 25% (d) 33.33%

Solution:

(c) Profit = Selling Price – Cost Price = Rs.15 – Rs. 12 = Rs. 3

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} = \frac{3}{12} \times 100 = 25\%$$

2. An article costing Rs.400 is sold at 20% profit. Find its selling price.

(a) Rs. 320 (b) Rs. 380 (c) Rs. 420 (d) Rs. 480

Solution:

(d) $\text{Selling price} = \text{Rs.}400 \times \frac{100+20}{100} = \text{Rs. } 400 \times \frac{120}{100} = \text{Rs. } 480$

Alternative method:

Selling Price = Cost price + Profit

= Rs. 400 + 20% of Rs. 400 = Rs. 400 + Rs. 80 = Rs. 480

3. A shopkeeper bought a chair whose marked price is Rs. 800, at two successive discounts of 10% and 15% respectively. He spent Rs. 28 on its transportation and then sold the chair for Rs. 800. What is his gain in percentage ?

(a) 10% (b) 20% (c) 25% (d) 40%

Solution :

(c) $\text{Cost Price} = \text{Rs.}800 \times \frac{90}{100} \times \frac{85}{100} + \text{Rs.}28 = \text{Rs. } 612 + \text{Rs. } 28 = \text{Rs. } 640$

Profit = Rs. 800 – Rs. 640 = Rs. 160

$$\therefore \text{ Profit percent } = \frac{160}{640} \times 100 = 25\%$$

4. An article costing Rs. 500 is sold at 10% loss. Find its selling price.

(a) Rs. 450 (b) Rs. 550 (c) Rs. 600 (d) Rs. 400

Solution:

(a) $\text{Selling price} = \text{Rs.}500 \times \frac{100-10}{100} = \text{Rs. } 500 \times \frac{90}{100} = \text{Rs. } 450$

Alternative method:

Selling Price = Cost price – Loss

= Rs. 500 – 10% of Rs. 500 = Rs. 500 – Rs. 50 =Rs. 450

5. An article is sold for Rs. 900 and hence gaining 10% of the selling price. Find its cost price.

(a) Rs. 800 (b) Rs. 810 (c) Rs. 990 (d) Rs. 1000

Solution:

(c) Difference in two selling prices = 25% – 20% = 5% of cost price

Actual difference in two selling prices = Rs. 35 (i.e. 7 times of 5)

∴ Cost Price = 7 × Rs. 100 = Rs. 700

17. A machine is sold at a loss of 10%. Had it been sold at a profit of 15%, it would have fetched Rs. 50 more. The cost price of the machine is:

(a) Rs. 200 (b) Rs. 500 (c) Rs. 750 (d) Rs. 1000

Solution:

(a) Difference in two selling prices

= 10% – (– 15%) = 10% + 15% = 25% of cost price

Actual difference in two selling prices = Rs. 50 (i.e. 2 times of 25)

∴ Cost Price = 2 × Rs. 100 = Rs. 200

18. Profit margin of a merchant is increased by 10%, if he sells an article for Rs. 650 instead of Rs. 600. What is the cost price of that article?

(a) Rs. 50 (b) Rs. 500 (c) Rs. 800 (d) Rs. 1000

Solution:

(b) Difference in two selling prices = 10% of cost price

Actual difference in two selling prices

= Rs. 650 – Rs. 600 = Rs. 50 (i.e. 5 times of 10)

∴ Cost Price = 5 × Rs. 100 = Rs. 500

19. A bicycle is sold at 10% profit. Had it been sold for Rs. 10 less, the profit would have been 5% only. What is the cost price of the bicycle?

(a) Rs. 100 (b) Rs. 180 (c) Rs. 190 (d) Rs. 200

Solution:

(d) Difference in two selling prices = 10% – 5% = 5% of cost price

Actual difference in two selling prices = Rs. 10 (i.e. 2 times of 5)

∴ Cost Price = 2 × Rs. 100 = Rs. 200

20. An article is sold at 10% loss. Had it been sold for Rs. 30 more, the loss would have been 5% only. What is the cost price of the article?

(a) Rs. 200 (b) Rs. 300 (c) Rs. 500 (d) Rs. 600

Solution:

(d) Difference in two selling prices = 10% – 5% = 5% of cost price

Actual difference in two selling prices = Rs. 30 (i.e. 6 times of 5)

∴ Cost Price = 6 × Rs. 100 = Rs. 600

21. On selling an article for Rs. 600, a man gains 20% of the selling price. Find profit percentage?

(a) 20% (b) 25% (c) 30% (d) 33.33%

Solution:

(b) Profit = $\frac{1}{5}$ of selling price

Solution:

(a) Cost price = Selling price + Loss = Rs. 250 + Rs. 50 = Rs. 300

$\therefore$ Loss percent = $\frac{50}{300} = \frac{1}{6}$ = 16.67%

Alternative Method:

Loss = $\frac{50}{250} = \frac{1}{5}$ of selling price = $\frac{1}{5+1}$ of Cost Price = $\frac{1}{6}$ of Cost Price = 16.67%

26. Selling price of an article is Rs. 330. Find cost price of the article, if gain is 10% on cost price.

(a) Rs. 280 (b) Rs. 290 (c) Rs. 300 (d) Rs. 310

Solution:

(c) Cost price = $330 \times \frac{100}{100+10}$ = Rs. 330 × $\frac{100}{110}$ = Rs. 300

Alternative Method:

Profit = 10% of cost price = $\frac{1}{10}$ of cost price

= $\frac{1}{10+1}$ of Selling price = $\frac{1}{11}$ × Rs. 330 = Rs. 30

$\therefore$ Cost price = Selling price – Profit = Rs. 330 – Rs. 30 = Rs. 300

27. On selling an article for Rs. 300, a shopkeeper gains 20% of the cost price. Find cost price of the article.

(a) Rs. 240 (b) Rs. 250 (c) Rs. 350 (d) Rs. 360

Solution:

(b) Cost price = 300 × $\frac{100}{100+20}$ = Rs. 300 × $\frac{100}{120}$ = Rs. 250

Alternative Method:

Profit = $\frac{1}{5}$ of cost price

= $\frac{1}{5+1}$ of Selling price = $\frac{1}{6}$ × 300 = Rs. 50

$\therefore$ Cost Price = Rs. 300 – Rs. 50 = Rs. 250

28. On selling an article for Rs. 900, a shopkeeper loses 10% of the cost price. Find cost price of the article.

(a) Rs.1000 (b) Rs. 810 (c) Rs. 990 (d) Rs. 800

Solution:

(a) Cost price = 900 × $\frac{100}{100-10}$ = Rs. 900 × $\frac{100}{90}$ = Rs. 1000

Alternative Method:

Loss = 10% of Cost Price = $\frac{1}{10}$ of Cost Price

= $\frac{1}{10-1}$ of Selling price = $\frac{1}{9}$ × 900 = Rs. 100

$\therefore$ Cost price = 900 + 100 = Rs. 1000

29. If an article costing Rs. 1000 is sold gaining 20% of its selling price. Find selling price of the article.

(a) Rs. 1250 (b) Rs. 1200 (c) Rs. 980 (d) Rs. 975

Solution:

(a) Profit = $\frac{1}{5}$ of selling price

$= \frac{1}{5-1}$ of Cost Price $= \frac{1}{4} \times 1000 =$ Rs. 250

$\therefore$ Selling Price = Rs. 1000 + Rs. 250 = Rs. 1250

30. A retailer buys goods at $\frac{9}{10}$ of its marked price and sells them at $\frac{12}{10}$ of the marked price, Find his profit per cent.

(a) 10% (b) 20% (c) 25% (d) 33.33%

Solution:

(d) Let the marked price of the goods = Rs. 10.

Then cost price = Rs. 10 × $\frac{9}{10}$ = Rs. 9

Selling price = 10 × $\frac{12}{10}$ = Rs. 12

Profit = Rs. 3 on Rs. 9 = $\frac{3}{9} = \frac{1}{3}$ = 33.33%

31. A shopkeeper purchases goods at $\frac{19}{20}$ of its marked price and sells them at 14% more than its marked price. Find his profit per cent.

(a) 10% (b) 20% (c) 25% (d) 33.33%

Solution:

(b) Let marked price of the goods = Rs. 100.

Then cost price = Rs.100 × $\frac{19}{20}$ = Rs. 95

Selling price = Rs. 100 + 14% of Rs. 100 = Rs. 114

$\therefore$ Profit = Rs. 114 – 95 = Rs. 19 on Rs. 95

$\therefore$ Profit = $\frac{19}{95} = \frac{1}{5}$ = 20%

32. A shopkeeper buys articles at 10% discount and sells them at 35% higher than the marked price. What is his profit percentage ?

(a) 25% (b) 35% (c) 45% (d) 50%

Solution :

(d) Let marked price = Rs. 100

Cost price = Rs. 100 – Rs. 10 = Rs. 90

Selling price = Rs. 100 + Rs. 35 = Rs. 135

Profit = Rs. 135 – Rs. 90 = Rs. 45

$\therefore$ Profit percent $= \frac{45}{90} \times 100 = 50\%$

33. A merchant sold 20 kg of rice for Rs. 240 and thus earned profit equal to the cost price of 4 kg. Find the cost price of rice per kg.

(a) Rs. 10 (b) Rs. 12 (c) Rs. 14 (d) Rs. 15

Solution:

(a) Selling price of 20 kg of rice = Rs. 240

$\therefore$ Cost price of 20 kg + Cost price of 4 kg = Rs. 240

$\therefore$ Cost price of 1 kg = Rs. $\frac{240}{24}$ = Rs. 10

34. A man bought 25 kg of rice for Rs. 750. On selling them for a certain price, he suffered loss equal to the selling price of 5 kg. Find the selling price of 1 kg of rice.

(a) Rs. 25 (b) Rs. 30 (c) Rs. 35 (d) Rs. 37.50

Solution :

(a) Selling price + Loss = Cost price

$\therefore$ Selling price of 25 kg + Selling price of 5 kg = Rs. 750

$\therefore$ Selling price of 1 kg = Rs. $\frac{750}{30}$ = Rs. 25

35. Some quantity of tea is sold at Rs. 22 per kg, making 10% profit. If total gain is Rs. 88, what is the quantity of tea sold?

(a) 4 kg (b) 40 kg (c) 44 kg (d) 50 kg

Solution:

(c) Profit = 10% of Cost price = $\frac{1}{10}$ of Cost Price

$= \frac{1}{11}$ of Selling Price $= \frac{1}{11} \times$ Rs. 22 = Rs. 2 per kg.

But actual gain = Rs. 88

$\therefore$ Quantity sold $= \frac{88}{2} = 44$ kg.

36. On selling an article for Rs. 550, a man gains 10%. What will be the selling price, if desired profit is 20%?

(a) Rs. 500 (b) Rs. 600 (c) Rs. 660 (d) Rs. 715

Solution:

(b) Selling price $= 550 \times \frac{100+20}{100+10}$ = Rs. $550 \times \frac{120}{110}$ = Rs. 600

37. On selling an article for Rs. 400, a man loses 20%. What will be the selling price, if desired profit is 10%?

(a) Rs. 200 (b) Rs. 500 (c) Rs. 550 (d) Rs. 600

Solution:

(c) Selling price $= 400 \times \frac{100+10}{100-20}$ = Rs. $400 \times \frac{110}{80}$ = Rs. 550

43. On selling 7 pens for Rs. 44, a man gains 10% of the cost price, what should be the selling price of 10 pens, if desired profit is 40%?

(a) Rs. 50 (b) Rs. 56 (c) Rs. 80 (d) Rs. 100

Solution:

(c) Selling price of 1 pen = $\frac{44}{7}$

$\therefore$ Selling price of 10 pens = $10 \times \frac{44}{7}$

$\therefore$ Selling price of 10 pens at new rate = $10 \times \frac{44}{7} \times \frac{140}{110}$ = Rs. 80

44. A man sells eggs @ 10 eggs for 7 rupee, gaining 30%. How many eggs did he buy for 7 rupee?

(a) 7 (b) 11 (c) 13 (d) 15

Solution:

(c) Selling price of 1 egg = $\frac{7}{10}$

Cost price of 1 egg = $\frac{7}{10} \times \frac{100}{130} = \frac{7}{13}$

$\therefore$ He bought 13 eggs for Rs. 7.

45. On selling an article for Rs. 600, a man gains 20%. What will be the profit if the article is sold for Rs. 800?

(a) 60% (b) 50% (c) 40% (d) 30%

Solution:

(a) Selling Price = Rs. 600 = 120% of cost price

$\therefore$ New selling price = 120% × $\frac{800}{600}$ = 160% of cost price

$\therefore$ Profit = 160% – 100% = 60% of the cost price

46. On selling an article for Rs. 630, a man loses 10%. What will be his profit if the article is sold for Rs. 749?

(a) 5% (b) 6% (c) 7% (d) 8%

Solution:

(c) Selling Price = Rs. 630 = 100% – 10% = 90% of the cost price

New selling price = 90% × $\frac{749}{630}$ = 107% of cost price

$\therefore$ Profit = 107% – 100% = 7% of the cost price

47. On selling an article for Rs. 50, a man loses $\frac{1}{6}$ of his outlay. Find his profit percentage, if the article is sold for Rs. 66.

(a) 5% (b) 10% (c) 20% (d) 25%

Solution:

(b) Loss = $\frac{1}{6}$ of Cost Price

$\therefore$ Selling price $= 1-\frac{1}{6}=\frac{5}{6}$ of Cost Price

$\therefore$ Rs. $50 = \frac{5}{6}$ of Cost Price

$\therefore$ Rs. $66 = \frac{5}{6}\times\frac{66}{50}$ of Cost Price $= \frac{11}{10}$ of Cost Price

$\therefore$ Profit $= \frac{11}{10}-1=\frac{1}{10}$ = 10% of cost price

48. On selling an article for Rs. 64, a man gains $\frac{1}{7}$ of his outlay. Find his profit percentage, if the article is sold for Rs. 63.

(a) 12.5% (b) 20% (c) 25% (d) 33.33%

Solution:

(a) Gain $= \frac{1}{7}$ of Cost Price

$\therefore$ Selling price $= 1+\frac{1}{7}=\frac{8}{7}$ of Cost Price

$\therefore$ Rs. $64 = \frac{8}{7}$ of Cost Price

$\therefore$ Rs. $63 = \frac{8}{7}\times\frac{63}{64}=\frac{9}{8}$ of Cost Price

$\therefore$ Profit $= \frac{9}{8}-1=\frac{1}{8}$ = 12.5% of cost price

49. A bought a pen for Rs. 200 and sold it to B at 10% gain. B sold it to C at 20% loss and C sold it to D at 25% gain. What was the price paid by D ?

(a) Rs. 180 (b) Rs. 200 (c) Rs. 220 (d) Rs. 225

Solution :

(c) Price Paid by D $= 200\times\frac{110}{100}\times\frac{80}{100}\times\frac{125}{100}$ = Rs.220

50. A manufacturer sells a TV at 10% profit to a wholesaler who in turn sells it to a retailer at 20% profit. If the price paid by the retailer is Rs. 13200, how much the TV costs to the manufacturer?

(a) Rs. 9000 (b) Rs. 9680 (c) Rs. 10000 (d) Rs. 11000

Solution:

(c) Cost to the manufacturer $= 13200 \times \frac{100}{120}\times\frac{100}{110}$ = Rs. 10000

51. A sells an article to B at a profit of 20%. B sells it to C at a profit of 30%. If C has paid Rs. 780, the cost price of the article to A is:

(a) Rs. 350 (b) Rs. 390 (c) Rs. 400 (d) Rs. 500

Solution:

(d) Cost price of the article to A $= 780 \times \frac{100}{130}\times\frac{100}{120}$ = Rs. 500

52. A watch is sold at 10% discount on its marked price of Rs. 480. If the retailer makes 20% profit on the cost price, find the cost price of the watch.

(a) Rs. 300 (b) Rs. 360 (c) Rs. 450 (d) Rs. 540

Solution:

(b) If marked price is Rs. 100, selling price = 100 – 10 = Rs. 90

If cost price is Rs. 100, selling price = 100 + 20 = Rs. 120

$$\therefore \text{Cost price} = 480 \times \frac{90}{100} \times \frac{100}{120} = \text{Rs. } 360$$

53. A shopkeeper allows 25% discount on the marked price of his articles and hence gains 25% of the Cost Price. What is the marked price of the article on selling which he gains Rs. 120?

(a) Rs. 400 (b) Rs. 600 (c) Rs. 800 (d) Rs. 1000

Solution:

(c) Marked price of the article = $\text{Rs. } 120 \times \frac{125}{25} \times \frac{100}{75} = \text{Rs. } 800$

Hint: If profit is Rs. 25, then Selling price = Rs. 100 + Rs. 25 = Rs.125.

If marked price is Rs.100, then Selling price = Rs.100 – Rs.25 = Rs.75.

54. A man bought a horse for Rs. 10000 and sold it to B at 10% profit and B sold it to C at 10% loss. Find the amount paid by C.

(a) Rs. 10000 (b) Rs. 9900 (c) Rs. 9999 (d) Rs. 11000

Solution:

(b) If A buys the horse for Rs. 100, B pays Rs. 110.

If B buys the horse for Rs. 100, C pays Rs. 90.

$$\therefore \text{Amount paid by C} = 10000 \times \frac{110}{100} \times \frac{90}{100} = 9900$$

55. A man purchased two articles for total cost of Rs. 9000. He sold the first article at 15% profit and the second at 12% loss. In the bargain, he neither gained nor lost anything. Find cost price of the first article.

(a) Rs. 4000 (b) Rs. 4500 (c) Rs. 5000 (d) Rs. 5500

Solution:

(a) Ratio of cost price of first and second articles = 12 : 15 = 4 : 5.

$$\therefore \text{Cost price of the first article} = \frac{4}{9} \times 9000 = \text{Rs. } 4000$$

56. A man purchased two articles for Rs. 10000 each. On selling first, he gains 20% and on the other, he loses 20%. What is profit/loss in the transaction?

(a) 4% loss (b) 4% gain (c) 2% loss (d) No profit/loss

Solution:

(d) Here, the cost price of both the articles are same.

∴ Profit made on one item is exactly equal to loss suffered on the other.

∴ No profit, no loss.

57. A man sold two articles for Rs. 10000 each. On selling first, he gains 10% and on the other, he loses 10%. What is profit/loss in the transaction?

(a) 1% loss (b) 1% gain (c) 2% loss (d) No profit/loss

Solution:

(a) Loss % $= \frac{(\text{Common Gain and Loss})^2}{100} = \frac{10^2}{100} = 1\%$

58. A shopkeeper marks price of his articles 12% above the cost price and then allows 12% discount on the marked price. What is his loss per cent?

(a) 0% (b) 1% (c) 1.44% (d) 2.4%

Solution:

(c) Loss % $= \frac{12^2}{100} = 1.44\%$

59. On increasing price of an article by 20%, number of articles sold fell by 20%. What is effect of this on total sales of the article?

(a) 4% decrease (b) 4% increase

(c) 20% decrease (d) No effect

Solution:

(a) Effect of Sales amount $= \frac{20 \times 20}{100} = 4\%$ Decrease

60. A merchant marks price of his articles 20% above cost price and allows 10% discount on the marked price. What per cent profit does he make?

(a) 8% (b) 10% (c) 12% (d) 30%

Solution:

(a) Profit $= 20 - 10 - \frac{20 \times 10}{100} = 10 - 2 = 8\%$

61. A merchant marks price of his articles 25% above the cost price and allows 12% discount to the customers. What per cent profit does he make?

(a) 8% (b) 10% (c) 13% (d) 16%

Solution:

(b) Profit $= 25 - 12 - \frac{25 \times 12}{100} = 13 - 3 = 10\%$

62. A shopkeeper marks his goods 20% higher than the cost price and sells them at 25% above the marked price. Find his profit percent.

(a) 20% (b) 25% (c) $33\frac{1}{3}\%$ (d) 50%

Solution :

(d) Let cost price = Rs. 100

Then marked price = 100 + 20 = Rs. 120

Selling price $= 120 \times \frac{125}{100} =$ Rs.150

$\therefore$ Profit = 150 – 100 = Rs. 50

$$\text{Profit percent } = \frac{50}{100} \times 100 = 50\%$$

63. A sells an article to B at a profit of 20%. B sells it to C at a profit of 10%. How many percent will C pay more than what A pays?

(a) 28% (b) 30% (c) 32% (d) 35%

Solution:

(c) Let Cost Price of the article = Rs. 100

Then amount paid by C = Rs. $100 \times \frac{120}{100} \times \frac{110}{100}$ = Rs. 132

$\therefore$ C pays more than A = Rs. 132 – Rs. 100 = Rs. 32 or 32%

Alternative Method:

$$20 + 10 + \frac{20 \times 10}{100} = 20 + 10 + 2 = 32\%$$

64. Two tables are purchased for the total cost of Rs.5000. First table is sold at 40% profit and second at 40% loss. If selling price is same for both the tables, what is the cost price of the table that was sold at profit?

(a) Rs. 1500 (b) Rs. 2000 (c) Rs. 3500 (d) Rs. 4000

Solution:

(a) 140% of cost price of first table = 60% of cost price of second table

$\therefore$ Cost price of first table : Cost price of second table

= 60 : 140 = 3 : 7

$\therefore$ Cost price of first table = $\frac{3}{10} \times 5000$ = Rs. 1500

65. A shopkeeper bought 2 pens for Rs. 480. He sold the first at a loss of 15% and the second at a profit of 19%. Find cost price of the second pen, if selling price of both the pens are same.

(a) Rs. 195 (b) Rs. 200 (c) Rs. 250 (d) Rs. 280

Solution:

(b) 85% of cost price of first pen = 119% of cost price of second pen

$\therefore$ Cost price of first pen : Cost price of second pen = 119 : 85 = 7 : 5

$\therefore$ Cost price of the second pen = $\frac{5}{12} \times 480$ = Rs. 200

66. A man sold two articles for Rs. 600 each. On selling first, he gains 20% and on the other 30%. What is profit percent in the transaction?

(a) 24.8% (b) 25% (c) 50% (d) 56%

Solution:

(a) Profit percent in the transaction

$$= \frac{100 \times (20 + 30) + 2 \times 20 \times 30}{200 + 20 + 30} = \frac{5000 + 1200}{250} = \frac{6200}{250} = 24.8\%$$

67. A man sold two articles for Rs. 2500 each. On selling first, he gains 20% and on the other he loses 10%. What is his percent profit in the transaction?

(a) 2.18% (b) 2.86% (c) 5% (d) 10%

Solution:

(b) Profit percent in the transaction

$$= \frac{100 \times (20-10) + 2 \times 20 \times (-10)}{200 + 20 - 10} = \frac{1000-400}{210} = \frac{600}{210} = 2.86\% \text{ (app.)}$$

68. A fruit seller buys oranges @ 3 for Rs. 2 and sells them @ 4 for Rs. 3. His profit per cent is:

(a) 10% (b) 12.5% (c) 25% (d) 50%

Solution:

(b) Let he buys and sells 12 oranges i.e. LCM of 3 and 4.

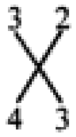
Cost price of 12 oranges $= 12 \times \frac{2}{3} =$ Rs. 8

And, Selling price of 12 oranges $= 12 \times \frac{3}{4} =$ Rs. 9

Profit = Rs. 9 – Rs. 8 = Re.1 on Rs. 8

$\therefore$ Profit per cent $= \frac{1}{8} = 12.5\%$

Short-cut Method:



$$\text{Profit} = \frac{3 \times 3 - 2 \times 4}{2 \times 4} = \frac{1}{8} = 12.5\%$$

Note: In short-cut method:

Quantity bought = 3 × 4 = 12 oranges

Selling price of 12 oranges = 3 × 3 = Rs. 9

Cost price of 12 oranges = 2 × 4 = Rs. 8

$\therefore$ Profit = Rs. 9 – Rs. 8 = Re.1

69. A fruit seller buys oranges @ 3 for Rs. 2 and sells them @ 2 for Rs. 3. His profit percentage is:

(a) 125% (b) 100% (c) 50% (d) 25%

Solution:

(a) Let he buys and sells 6 oranges i.e. LCM of 3 and 2.

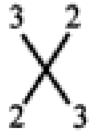
Cost price of 6 oranges $= 6 \times \frac{2}{3} =$ Rs. 4

Selling price of 6 oranges $= 6 \times \frac{3}{2} =$ Rs. 9

Profit = Rs. 9 – Rs. 4 = Rs. 5 on Rs. 4

$\therefore$ Profit percent $= \frac{5}{4} = 125\%$

Short-cut Method:



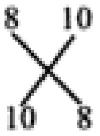
$$\therefore \text{ Profit percent} = \frac{3\times 3 - 2\times 2}{2\times 2} = \frac{9-4}{4} = \frac{5}{4} = 125\%$$

70. A fruit seller buys oranges @ 8 for Rs. 10 and sells them @ 10 for Rs. 8. His profit/loss percentage is:

(a) 36% loss (b) 36% gain (c) 45% gain (d) 45% loss

Solution:

(a)



$$\therefore \text{ Profit percent} = \frac{8\times 8 - 10\times 10}{10\times 10} = \frac{64-100}{100} = \frac{-36}{100} = (-)36\%$$

∴ Loss = 36%

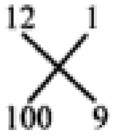
71. A merchant buys oranges at a certain price per dozen and sells them charging 9 times that price for 100 oranges. Find his profit per cent.

(a) 5% (b) 8% (c) 10% (d) 12.5%

Solution:

(b) Let cost price of one dozen of oranges = Re. 1

Then selling price of 100 oranges = 9 × Re. 1 = Rs. 9



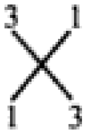
$$\therefore \text{ Profit \%} = \frac{12\times 9 - 1\times 100}{1\times 100} = 8\%$$

72. A fruit seller buys oranges @ 3 oranges for Re.1 and sells them @ Rs. 3 each. His percentage profit is:

(a) 50% (b) 100% (c) 200% (d) 800%

Solution:

(d)



$$\therefore \text{ Profit percent} = \frac{9-1}{1} = \frac{8}{1} = 800\%$$

73. A retailer bought some apples @ 7 apples for Rs. 4 and sold them @ 8 apples for Rs. 5. If he gains Rs. 30 on that day, find the quantity of apples sold by him on that day.

(a) 56 (b) 70 (c) 80 (d) 560

Solution:

(d) LCM of quantities purchased and sold i.e. LCM of 7 and 8 is 56.

Cost price of 56 apples = $56 \times \frac{4}{7}$ = Rs. 32

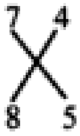
Selling price of 56 apples = $56 \times \frac{5}{8}$ = Rs. 35

Profit on selling 56 apples = Rs. 35 – Rs. 32 = Rs. 3

But actual profit is Rs. 30 (i.e. 10 times of 3)

∴ Apples sold = 56 × 10 = 560 apples

Short-cut Method:



∴ Profit is Rs. (7 × 5) – Rs. (4 × 8) on (7 × 8) apples.

∴ Profit is Rs. 3 on 56 apples.

But actual profit is Rs. 30 (i.e. 10 times of 3)

∴ Apples sold = 56 × 10 = 560 apples

74. A man bought some oranges at the rate of 3 oranges for one rupee and equal number of oranges at the rate of 2 oranges for one rupee. What is his profit, if he sells 2 oranges for one rupee.

(a) 20% (b) 33.33% (c) 50% (d) No profit/loss

Solution:

(a) LCM of quantities purchased and sold i.e. LCM of 3, 2 and 2 is 6.

Cost price of 6 oranges @ 3 oranges for a rupee = $\frac{1}{3} \times 6$ = Rs.2

Cost price of 6 oranges @ 2 oranges for a rupee = $\frac{1}{2} \times 6$ = Rs.3

∴ Cost price of 12 (i.e. 6 + 6) oranges = Rs. 2 + Rs. 3 = Rs. 5

Selling price of 12 oranges = $\frac{1}{2} \times 12$ = Rs. 6

Profit = Rs. 6 – Rs. 5 = Re.1 on Rs. 5

∴ Profit percent = $\frac{1}{5}$ = 20%

75. A shopkeeper sold one-third of his articles at 16% profit and the remaining at 10% profit. Find his overall profit per cent.

(a) 12% (b) 15% (c) 26% (d) 36%

Solution :

(a) Let total articles sold are 3 of Rs. 100 (cost price) each.

Then total gain = 1 × 16 + 2 × 10 = Rs. 36.

∴ Profit percent $= \frac{36}{3} \times 100 = 12\%$

76. A shopkeeper sold one-third of his articles at 5% profit. At what profit percent should he sell the remaining articles so that his overall gain is 15%.

(a) 10% (b) 20% (c) 25% (d) 30%

Solution :

(b) Let total articles sold are 3 of Rs. 100 (cost price) each.

Then total desired gain = 3 × 15 = Rs. 45.

Gain on the first item = Rs. 5

Required gain = 45 – 5 = Rs. 40 on the remaining articles.

$\therefore$ Profit percent on remaining stock $= \frac{40}{2} = 20\%$

77. A shopkeeper sold one-fourth of his articles at 15% loss. At what profit percent should he sell the remaining articles so that his overall gain is 6% ?

(a) 13% (b) 16% (c) 17% (d) 21 %

Solution :

(a) Let total articles sold are 4 of Rs. 100 (cost price) each.

Then total desired gain = 4 × 6 = Rs. 24.

Loss on the first item = Rs. 15

Required gain = 24 + 15 = Rs. 39 on the remaining articles.

$\therefore$ Profit percent on remaining stock $= \frac{39}{3} = 13\%$

78. By selling 4 articles, a shopkeeper gains equal to the cost price of 1 article. Find his gain per cent.

(a) 10% (b) 20% (c) 25% (d) 30%

Solution:

(c) Profit % $= \frac{1}{4} = 25\%$

79. If the cost price of 11 oranges is equal to selling price of 10 oranges. Find profit per cent.

(a) 9% (b) 9.99% (c) 10% (d) 11.11%

Solution:

(c) Profit on selling 10 oranges
= Cost Price of 11 oranges – Cost Price of 10 oranges
= Cost price of 1 orange

$\therefore$ Profit % $= \frac{1}{10} = 10\%$

80. By selling 10 articles, a shopkeeper gains equal to the selling price of 2 articles. Find his gain per cent.

(a) 6.33% (b) 20% (c) 25% (d) 30%

Solution:

(c) Profit % $= \frac{2}{10-2} = \frac{1}{4} = 25\%$

81. On selling 10 articles, a merchant loses equal to cost price of 2 articles. Find his loss per cent.

(a) 12.5% (b) 20% (c) 25% (d) 33.33%

Solution:

(b) Loss % = $\frac{2}{10}=\frac{1}{5}=20\%$

82. If the cost price of 15 tables is equal to selling price of 20 tables. Find profit/loss per cent.

(a) 5% loss (b) 5% gain (c) 25% gain (d) 25% loss

Solution:

(d) Profit on selling 20 tables

= Cost Price of 15 tables – Cost Price of 20 tables

∴ Loss = Cost Price of 5 tables

∴ Loss % = $\frac{5}{20}=\frac{1}{4}=25\%$

83. By selling 18 articles, a shopkeeper loses the selling price of 2 articles. Find his loss per cent.

(a) 10% (b) 11.11% (c) 12.5% (d) 25%

Solution:

(a) Profit % = $\frac{-2}{18-(-2)}=\frac{-2}{20}=\frac{-1}{10}=(-)\ 10\%$

∴ Loss = 10%

84. A trader marked his goods at 20% above the cost price. He sold half the stock at marked price, one quarter at a discount of 20% on marked price and rest at a discount of 40% on the marked price. What is his total gain ?

(a) 2% (b) 3% (c) 4% (d) 6%

Solution :

(a) Let cost price = Rs. 100

Then marked price = Rs. 120

Total selling price $=\frac{1}{2}\times120+\frac{1}{4}\times120\times\frac{80}{100}+\frac{1}{4}\times120\times\frac{60}{100}$

= Rs. (60 + 24 + 18) = Rs. 102

∴ Profit = 102 – 100 = Rs. 2 on Rs. 100 = 2%

85. A milk seller purchased milk at the rate of Rs.10 per litre and adds one fifth of water to it. What per cent profit does he make by selling the mixture at Rs.12 per litre?

(a) 20% (b) 25% (c) 30% (d) 50%

Solution:

(d) The mixture contains $\frac{1}{5}$ water in it.

∴ Quantity of milk in 1 litre of mixture = $1-\frac{1}{5}=\frac{4}{5}$ litre

Actual cost price per litre = Rs. 10 × $\frac{4}{5}$ = Rs. 8 per litre

Profit per litre = Rs. 12 – Rs. 8 = Rs. 4 on Rs. 8

∴ Profit = $\frac{4}{8}=\frac{1}{2}=50\%$

86. The profit earned on selling an article for Rs. 575 is equal to loss suffered when the article is sold for Rs. 325. Find the cost price of the article.

(a) Rs. 650 (b) Rs. 550 (c) Rs. 500 (d) Rs. 450

Solution:

(d) Let the loss and gain is Rs. x in each case.

Then Cost Price = 575 – x = 325 + x

∴ 2x = 575 – 325 = 250

∴ x = 125

∴ Cost Price = Selling Price (second) + Loss

= Rs. 325 + Rs. 125 = Rs. 450

Or Cost Price = Selling Price (first) – Profit

= Rs. 575 – Rs. 125 = Rs. 450

Alternative Method:

Let the loss and gain is Rs.x in each case.

Then Difference in two selling prices = 2x = Rs. 575 – Rs. 325

∴ 2x = Rs. 250

∴ $x = \frac{250}{2}$ = Rs. 125

∴ Cost Price = Selling Price (second) + Loss

= Rs. 325 + Rs. 125 = Rs. 450

Or Cost Price = Selling Price (first) – Profit

= Rs. 575 – Rs. 125 = Rs. 450

87. An article is sold for Rs. 500 and hence lost something. Had the article is sold for Rs. 700, the merchant would have gained three times the former loss. Find the cost price of the article.

(a) Rs. 525 (b) Rs. 550 (c) Rs. 600 (d) Rs. 650

Solution:

(b) Let loss in the first case = Rs. x.

Then profit in second case = Rs. 3x.

∴ Rs. 500 = Cost price – Rs. x (1)

And Rs. 700 = Cost price + Rs. 3x (2)

Solving equations (1) and (2), we get:

4x = Rs. 200, or x = Rs. 50

∴ Cost Price = Selling Price (first) + Loss

= Rs. 500 + Rs. 50 = Rs. 550

Or Cost Price = Selling Price (second) – Profit

= Rs. 700 – 3 × Rs. 50 = Rs. 700 – Rs. 150 = Rs. 550

Alternative Method:

Let loss in the first case = Rs. x.

Then profit in the second case = Rs. 3x

Difference in two selling prices = x + 3x = Rs. 700 – Rs. 500

91. If selling price of an article is $\frac{4}{3}$ of cost price, then profit percent is:

(a) $\frac{1}{3}\%$ (b) 25% (c) 33.33 % (d) 50%

Solution:

(c) Let cost price of the article = Re.1.

Then its selling price = Rs. $\frac{4}{3}$

Profit = Rs. $\frac{4}{3}$ – Re.1 = Re. $\frac{1}{3}$ on Re.1

$\therefore$ Profit = $\frac{1}{3}$ = 33.33%

Alternative Method:

Let cost price of the article = Rs. 3.

Then selling price = Rs. 3 × $\frac{4}{3}$ = Rs. 4

Profit = Rs. 4 – Rs. 3 = Re.1 on Rs. 3

$\therefore$ % Profit = $\frac{1}{3}$ = 33.33%

92. If cost price of an article is $\frac{3}{2}$ of its selling price, the profit or loss per cent is:

(a) 33.33 % loss (b) 33.33 % profit

(c) 50% loss (d) 50% profit

Solution:

(a) Let selling price of the article is Rs. 2.

Then cost price = Rs. 3

$\therefore$ Loss = Rs. 3 – Rs. 2 = Re.1 on Rs. 3

$\therefore$ Loss = $\frac{1}{3}$ = 33.33%

Alternative Method:

Cost price is $\frac{3}{2}$ of the selling price

$\therefore$ Selling price is $\frac{2}{3}$ of the cost price

$\therefore$ Loss = Cost Price – Selling price = $1-\frac{2}{3}=\frac{1}{3}$ = 33.33%

93. If selling price of $\frac{4}{5}$ of the stock of oranges is equal to cost price of the whole stock , find profit per cent.

(a) 20% (b) 25% (c) 30% (d) 33.33%

Solution:

(b) Let Cost price of 1 orange = Re.1

$\therefore$ Cost price of 4 oranges = Rs. 4

Selling price of 4 oranges = Cost price of 5 oranges = Rs. 5

$\therefore$ Profit = Rs. 5 – Rs. 4 = Re.1 on Rs. 4

$\therefore$ Profit % = $\frac{1}{4}$ = 25%

94. Find the profit per cent made on selling an article for a certain amount, if on selling the article for $\frac{3}{4}$ of the amount causes 10% loss?

(a) 2.5% (b) 7.5% (c) 15% (d) 20%

Solution:

(d) 10% loss means 100% – 10% = 90% of the cost price

Which is equal to $\frac{3}{4}$ of the Selling Price

$\therefore$ Selling Price = $\frac{4}{3}$ × 90% = 120% of Cost Price

$\therefore$ Profit = 120% – 100% = 20%

95. A man sold two articles for Rs. 990 each. On selling first, he gains 10% and on the other, he loses 10%. Find his total gain or loss.

(a) Rs. 9.90 (b) Rs. 10 (c) Rs. 19.80 (d) Rs. 20

Solution:

(d) Loss % = $\frac{10^2}{100}$ = 1% of the cost price = $\frac{1}{100}$ of the cost price

$= \frac{1}{100-1} = \frac{1}{99}$ of Selling Price

$\therefore$ Total loss on two articles = 2 × 990 × $\frac{1}{99}$ = Rs. 20

96. A man sold two articles for Rs.120 each. On selling first, he gains 20% and on the other, he loses 20%. Find his total gain or loss.

(a) Rs. 4.80 (b) Rs. 5 (c) Rs. 9.60 (d) Rs. 10

Solution:

(d) Loss % = $\frac{20^2}{100}$ = 4% of the cost price = $\frac{1}{25}$ of the cost price

$= \frac{1}{25-1} = \frac{1}{24}$ of Selling Price

$\therefore$ Total loss on two articles = 2 × 120 × $\frac{1}{24}$ = Rs. 10

97. A shopkeeper buys articles at 20% discount on marked price and sells them at 50% profit. How much percent the selling price is above the Marked Price?

(a) 20% (b) 30% (c) 60% (d) 70%

Solution :

(*a*) Let marked price = Rs. 100

Cost price = Rs. 100 – Rs. 20 = Rs. 80

Selling price $= \text{Rs.}80 \times \frac{150}{100} = \text{Rs.}120$

∴ Selling price is 20% above the marked price.

98. A merchant marks price of his articles 20% above the cost price and allows some discount on the marked price and hence makes 8% profit. Find the rate of discount.

(a) 10% (b) 12% (c) 28% (d) 30%

Solution:

(a) Let cost price = Rs. 100

Marked price = Rs. 100 + Rs. 20 = Rs. 120

Selling price = Rs. 100 + Rs. 8 = Rs. 108

Discount = Rs. 120 – Rs. 108 = Rs. 12

∴ Rate of discount $= \frac{12}{120} = \frac{1}{10} = 10\%$

99. A merchant marks price of the articles 25% above the cost price and allows some discount on the marked price and makes a profit of 10%. Find the rate of discount.

(a) 10% (b) 12% (c) 12.5% (d) 15%

Solution:

(b) Let cost price = Rs. 100

Marked price = Rs. 100 + Rs. 25 = Rs. 125

Selling price = Rs. 100 + Rs. 10 = Rs. 110

Discount = Rs. 125 – Rs. 110 = Rs. 15

∴ Rate of discount $= \frac{15}{125} \times 100 = 12\%$

100. A merchant earns 20% profit, even after allowing 20% discount on the marked price of an article. How much the marked price is above the cost price?

(a) 40% (b) 36% (c) 44% (d) 50%

Solution:

(d) Let cost price = Rs. 100

Then selling price = Rs. 100 + Rs. 20 = Rs. 120

Discount = 20% of Marked price $= \frac{1}{5}$ of Marked price

$= \frac{1}{5-1}$ of Selling price $= \frac{1}{4}$ of Rs. 120 = Rs. 30

∴ Marked price = 120 + 30 = Rs. 150

∴ Marked price is 50% higher than the cost price.

Alternative Method:

Selling Price = Rs. 120

$\therefore$ Marked Price = Rs. 120 × $\frac{100}{80}$ = Rs. 150

$\therefore$ Marked price is 50% higher than the cost price.

101. A merchant makes 60% profit, after allowing 20% discount on the marked price of an article. How much the marked price is above the cost price?

(a) 60% (b) 80% (c) 92% (d) 100%

Solution:

(d) Let cost price = Rs. 100

Then Selling price = Cost price + Profit = Rs. 100 + Rs. 60 = Rs. 160

$\therefore$ Discount = 20% of Marked price = $\frac{1}{5}$ of Marked price

= $\frac{1}{5-1}$ of Selling price = $\frac{1}{4}$ of Rs. 160 = Rs. 40

$\therefore$ Marked price = Selling price + Discount = 160 + 40 = Rs. 200

$\therefore$ Marked price is 100% higher than the cost price.

Alternative Method:

Selling Price = Rs. 160

$\therefore$ Marked Price = Rs. 160 × $\frac{100}{80}$ = Rs. 200

$\therefore$ Marked price is 100% higher than the cost price.

102. A shopkeeper allows 4% discount on the marked price of an article and earns 20% profit. Find his profit percent, if he allows 10% discount.

(a) 12% (b) 12.5% (c) 14% (d) 16%

Solution :

(b) Let the marked price of the article = Rs. 100

Selling price after 4% discount = Rs. 100 – Rs. 4 = Rs. 96

Cost price when profit is 20% $= 96 \times \frac{100}{120} = \text{Rs.}80$

Selling price after 10% discount = 100 – 10 = Rs. 90

$\therefore$ Profit percent $= \frac{10}{80} \times 100 = 12.5\%$

103. A shopkeeper earns a profit of 25%, after allowing 10% discount on the list price. What is the cost price of an article whose list price is Rs. 500 ?

(a) Rs. 360 (b) Rs. 400 (c) Rs. 450 (d) Rs. 540

Solution :

(a) Listed price (marked price) of the article = Rs. 500

Selling Price after 10% discount $= \text{Rs.}500 \times \frac{90}{100}$

Selling Price is 125% or $\frac{5}{4}$ of the Cost Price.

$\therefore$ Cost Price $= \text{Rs.}500 \times \frac{90}{100} \times \frac{4}{5} = \text{Rs.}360$

104. A shopkeeper marks his articles 20% above the cost price and gives a discount of 10% on the marked price. Find the cost price of the article, if the selling price is Rs. 540.

(a) Rs. 450 (b) Rs. 500 (c) Rs. 550 (d) Rs. 600

Solution :

(b) Selling price of the article = Rs. 540

Marked price $= \text{Rs.}540 \times \frac{100}{90}$

Cost price = marked price × $\frac{100}{120}$ $= \text{Rs.}540 \times \frac{100}{90} \times \frac{100}{120} = \text{Rs.}500$

105. A profit of 20% is earned even after allowing 4% discount on the marked price. How much the marked price is above the cost price?

(a) 16% (b) 24% (c) 25% (d) 30%

Solution:

(c) Let cost price = Rs. 100

Then Selling price = Cost price + Profit = Rs. 100 + Rs. 20 = Rs. 120

Discount = 4% of Marked price = $\frac{1}{25}$ of Marked price

$= \frac{1}{25-1}$ of Selling price = $\frac{1}{24}$ of Rs. 120 = Rs. 5

$\therefore$ Marked price = Selling price + Discount = 120 + 5 = Rs. 125

$\therefore$ Marked price is 25% higher than the cost price.

Alternative Method:

Selling Price = Rs. 120

$\therefore$ Marked Price = Rs. 120 × $\frac{100}{96}$ = Rs. 125

$\therefore$ Marked price is 25% higher than the cost price.

106. A merchant makes a profit of 12%, after allowing 30% discount on the marked price. How much the marked price is above the cost price?

(a) 42% (b) 45% (c) 60% (d) 80%

Solution:

(c) Let cost price of the article = Rs. 100

Then Selling price = Cost price + Profit = Rs. 100 + Rs. 12 = Rs. 112

Discount = 30% of marked price = $\frac{3}{10}$ of Marked price

$= \frac{3}{10-3}$ of Selling price = $\frac{3}{7}$ × Rs. 112 = Rs. 48

$\therefore$ Marked price = Selling price + Discount = Rs. 112 + Rs.48 = Rs.160

$\therefore$ Marked price is 60% higher than the cost price.

Alternative Method:

Selling Price = Rs. 112

$\therefore$ Marked Price = Rs. 112 × $\frac{100}{70}$ = Rs. 160

$\therefore$ Marked price is 60% higher than the cost price.

107. On allowing 20% discount, 12% loss is suffered. How much the marked price is above the cost price?

(a) 8% (b) 10% (c) 12% (d) 15%

Solution:

(b) Let cost price of the article = Rs. 100

Then Selling price = Cost price – Loss = Rs. 100 – Rs. 12 = Rs. 88

Discount = 20% of marked price = $\frac{1}{5}$ of Marked price

= $\frac{1}{5-1}$ of Selling price = $\frac{1}{4}$ × Rs. 88 = Rs. 22

$\therefore$ Marked price = Rs. 88 + Rs. 22 = Rs. 110 i.e. 10% above the cost price

Alternative Method:

Selling Price = Rs. 88

$\therefore$ Marked Price = Rs. 88 × $\frac{100}{80}$ = Rs. 110

$\therefore$ Marked price is 10% higher than the cost price.

108. A shopkeeper gains 17% after allowing a discount of 10% on the marked price of an article. Find his profit per cent if the articles are sold at marked price allowing no discount.

(a) 23% (b) 27% (c) 30% (d) 37%

Solution:

(c) Let cost price of the article = Rs. 100

Then Selling price = Cost price + Profit = Rs. 100 + Rs. 17 = Rs. 117

Discount = 10% of marked price = $\frac{1}{10}$ of Marked price

= $\frac{1}{10-1}$ of Selling price = $\frac{1}{9}$ × Rs. 117 = Rs. 13

$\therefore$ Marked price = Selling Price + Discount = Rs. 117 + Rs. 13 = Rs. 130

$\therefore$ Profit on selling at marked price

= Rs. 130 – Rs. 100 = Rs. 30 on Rs. 100 i.e. 30%.

Alternative Method:

Selling Price = Rs. 117

$\therefore$ Marked Price = Rs. 117 × $\frac{100}{90}$ = Rs. 130

$\therefore$ Profit = Rs. 130 – Rs. 100 = Rs. 30 on Rs. 100 i.e. 30%.

109. A man sold one watch and one pen for Rs. 492 and Rs. 168 respectively, gaining 10% of the total cost price of the two articles. Had he sold the watch for Rs. 435 and the pen at its cost price, he would have lost 5% on the total cost price. Find the cost price of the watch.

(a) Rs. 135 (b) Rs. 225 (c) Rs. 375 (d) Rs. 465

Solution:

(d) In the first case:

Total sales = Rs. 492 + Rs. 168 = Rs. 660

Profit earned = 10%

∴ Cost price of both the articles = Rs. 660 × $\frac{100}{110}$ = Rs. 600

In the second case, total loss = 5%

Total selling price at 5% loss = Rs. 600 – 5% of Rs. 600 = Rs. 570

∴ Selling price of watch + Cost price of pen = Rs. 570

∴ Rs. 435 + Cost of pen = Rs. 570

∴ Cost of pen = Rs. 570 – Rs. 435 = Rs. 135

∴ Cost of watch = Total Cost Price – Cost Price of pen

= Rs. 600 – Rs. 135 = Rs. 465

110. On selling an article for Rs. 144, a man's gain in percentage is equal to the cost price of the article. Find cost price of the article.

(a) Rs. 8 (b) Rs. 18 (c) Rs. 80 (d) Rs. 180

Solution:

(c) Let cost price of the article = Rs. x

Then profit earned = x%

∴ Selling price = Cost price × $\frac{100 + \text{Profit}\,\%}{100} = \frac{x(100 + x)}{100}$

∴ If profit is 10%, then cost price is Rs. 10 and selling price will be

$= \frac{10 \times 110}{100} = 1 \times 11 = 11$

If profit is 20%, then cost price is Rs.20 and selling price will be

$= \frac{20 \times 120}{100} = 2 \times 12 = 24$

Similarly for 30% profit, selling price = 3 × 13 = 39

And for 40% profit, selling price = 4 × 14 = 56

On checking we find that

8 × 18 =144

∴ Cost price = Rs. 80

111. On selling an article for Rs. 75, a shopkeeper's gain in percentage is equal to the cost price. Find percent profit on sale of the article.

(a) 5% (b) 25% (c) 50% (d) 150%

Solution:

(c) Using the method explained in the last question, we get

5 × 15 = 75

∴ Profit = 50%

112. A shopkeeper bought 200 articles, each costing the same. He sold 30% of the articles at 20% profit and the remaining at 10% profit. If total profit made by him is Rs. 2600, find cost price of one article.

(a) Rs. 100 (b) Rs. 200 (c) Rs. 1300 (d) Rs. 2600

Solution:

(a) Profit in the transaction = 30% × 20% + 70% × 10% = 6% + 7% = 13%

But actual profit = Rs. 2600.

$\therefore$ Cost Price of 200 articles = 2600 ÷ 13% = 2600 × $\frac{100}{13}$

$\therefore$ Cost price of 1 article = 2600 × $\frac{100}{13} \times \frac{1}{200}$ = Rs. 100

113. A machine is sold at a profit of 10%. Had it been bought for 10% less and sold for Rs. 10 less, the profit would have been 20%. What is the cost price of the machine?

(a) Rs. 110 (b) Rs. 120 (c) Rs. 200 (d) Rs. 500

Solution:

(d) Let cost price of the machine = Rs. 100.

Then its selling price = Rs. 100 + Rs. 10 = Rs. 110

In second situation:

Cost price = Rs. 100 – Rs. 10 = Rs. 90

Selling price = Rs. 90 × $\frac{120}{100}$ = Rs. 108

Difference between two selling prices = Rs. 110 – Rs. 108 = Rs. 2

But actual difference is Rs. 10 (i.e. 5 times of Rs. 2)

$\therefore$ Cost price = 5 × Rs. 100 = Rs. 500

Alternative Method:

In the first situation, Selling price = 110% of Cost Price

In the second situation, Cost Price = 90% of original Cost Price

$\therefore$ New Selling price = 120% of 90% of original Cost Price

$\therefore$ 110% of Cost Price – Rs. 10 = 120% of 90% of Cost Price

$\therefore$ 110% of Cost Price – Rs. 10 = 108% of Cost Price

$\therefore$ 2% of Cost Price = Rs. 10 (i.e. 5 times of 2)

$\therefore$ Cost price = 5 × Rs. 100 = Rs. 500

114. A man sells an article at 10% profit. Had he bought it for 10% less and sold it for Rs. 121 less, he would still have gained 10%. Original Cost Price of the article is:

(a) Rs. 1000 (b) Rs. 1100 (c) Rs. 1200 (d) Rs. 1210

Solution:

(b) Let cost price of the article = Rs. 100.

Then its selling price = Rs. 100 + Rs. 10 = Rs. 110

In second situation:

Cost price = Rs. 100 – Rs. 10 = Rs. 90

Selling price = Rs. 90 × $\frac{110}{100}$ = Rs. 99

Difference between two selling prices = Rs. 110 – Rs. 99 = Rs. 11

But actual difference = Rs. 121 (i.e. 11 times of Rs. 11)

$\therefore$ Cost price = 11 × Rs. 100 = Rs. 1100

Alternative Method:

110% of Cost Price – Rs. 121 = 110% of 90% of Cost Price

∴ 110% of Cost Price – Rs. 121 = 99% of Cost Price

∴ 11% of Cost Price = Rs. 121 (i.e. 11 times of 11)

∴ Cost price = 11 × Rs. 100 = Rs. 1100

115. A man sold an article at a loss of 10%. Had he bought it for 20% less and sold it for Rs. 55 more, he would have made a profit of 40%. Original cost price of the article is:

(a) Rs. 225 (b) Rs. 250 (c) Rs. 300 (d) Rs. 500

Solution:

(b) Let cost price of the article = Rs. 100.

Then its selling price = Rs. 100 – Rs. 10 = Rs. 90

In second situation:

Cost price = Rs. 100 – Rs. 20 = Rs. 80

Selling price = Rs. 80 × $\frac{140}{100}$ = Rs. 112

Difference between two selling prices = Rs. 112 – Rs. 90 = Rs. 22

But actual difference = Rs. 55

∴ Cost price = $\frac{55}{22}$ × Rs. 100 = Rs. 250

Alternative Method:

90% of Cost Price + Rs. 55 = 140% of 80% of Cost Price

∴ 90% of Cost Price + Rs. 55 = 112% of Cost Price

∴ 22% of Cost Price = Rs. 55

∴ Cost price = 55 × $\frac{100}{22}$ = Rs. 250

116. A man sold an article at 10% loss. Had he bought it for 10% more and sold it for Rs. 20 less, he would have lost 20%. Original Cost Price of the article is:

(a) Rs. 200 (b) Rs. 500 (c) Rs. 1000 (d) Rs. 2000

Solution:

(c) Let Cost price of the article = Rs. 100

Then its selling price = Rs. 90

In second situation:

Cost price = Rs. 100 + Rs. 10 = Rs. 110

Selling price = Rs. 110 × $\frac{80}{100}$ = Rs. 88

Difference between two selling prices = Rs. 90 – Rs. 88 = Rs. 2

But actual difference = Rs. 20 (i.e. 10 times of Rs. 2)

∴ Cost price = 10 × Rs. 100 = Rs. 1000

Alternative Method:

90% of Cost Price – Rs. 20 = 80% of 110% of Cost Price

∴ 90% of Cost Price – Rs. 20 = 88% of Cost Price

∴ 2% of Cost Price = Rs. 20 (i.e. 10 times of 2)

∴ Cost price = 10 × Rs. 100 = Rs. 1000

117. A man sold a horse at 10% profit. Had he bought it at 20% less and sold for Rs. 10 more, he would have made a profit of 40%. What is the cost price of the horse?

(a) Rs. 400 (b) Rs. 500 (c) Rs. 600 (d) Rs. 1000

Solution:

(b) Let cost price of the horse = Rs. 100

Then selling price = Rs. 100 + 10% of Rs. 100 = Rs. 110

In the second situation:

Cost price = Rs. 100 – 20% of Rs. 100 = Rs. 80.

Now selling price = Rs. 80 + 40% of Rs. 80 = Rs. 112

Difference between two selling prices = Rs. 112 – Rs. 110 = Rs. 2

But actual difference = Rs. 10 (i.e. 5 times of 2)

∴ Cost price = 5 × Rs. 100 = Rs. 500

Alternative Method:

110% of Cost Price + Rs. 10 = 140% of 80% of Cost Price

∴ 110% of Cost Price + Rs. 10 = 112% of Cost Price

2% of Cost Price = Rs. 10 (i.e. 5 times of 2)

∴ Cost price = 5 × Rs. 100 = Rs. 500

Chapter 22

AVERAGE

Average is calculated by adding all the observations and dividing the sum by number of observations. Average is also known as arithmetic mean.

$\therefore$ Average of 'n' terms = $\frac{\text{sum of n terms}}{n}$

$\therefore$ If average of 'n' terms is 'x', then sum of 'n' terms = nx.

FORMULAE

1. If average of some numbers is 'x' and a constant number **'a' is added to each term** of the series,

   Then new average = x + a.
2. If average of some numbers is 'x' and a constant number **'a' is subtracted from each term** of the series,

   Then new average = x – a.
3. If average of some numbers is 'x' and **each term** of the series is **multiplied by a constant number** 'a',

   Then new average = 'ax'.
4. If average of some numbers is 'x' and **each term** of the series **is divided by a constant number** 'a',

   Then new average = $\frac{x}{a}$.
5. **Average of 'n' terms in an Arithmetic Progression** is:

(a) Mid-term i.e. $\left(\frac{n+1}{2}\right)^{th}$ term (if 'n' is an odd number).

(b) Average of two mid-terms i.e.

Average of $\frac{n}{2}$ and $\left(\frac{n}{2}+1\right)^{th}$ term (if 'n' is an even number).

Example 1:

Find average of 3, 5, 7.

Solution:

Number of terms is 3, an odd number.

$\therefore$ Average = $\left(\frac{3+1}{2}\right)^{th}$ or 2^{nd} term = 5

Example 2:

Find average of 12, 14, 16, 18, 20.

Solution:

Number of terms is 5, an odd number.

$\therefore$ Average = $\left(\frac{5+1}{2}\right)^{th}$ or 3rd term = 16

Example 3:

Find average of 4, 6, 8, 10.

Solution:

Number of terms is 4, an even number.

$\therefore$ Average = Average of two mid-terms i.e.

Average of $\left(\frac{4}{2}\right)^{th}$ and $\left(\frac{4}{2}+1\right)^{th}$ term

= Average of 2nd and 3rd term i.e. $\frac{6+8}{2} = 7$

6. **If a number is increased while the other numbers remain constant in a series, the average of the series will increase and vice-versa.**
   This implies if one or more numbers are increased, then one or more of the remaining numbers must be decreased to keep the same average for the series and vice-versa.

Note: This rule is the basic rule for solving almost all problems based on average.

Example 1:

Average of 12 and 18 is $\frac{30}{2} = 15$.

Average of 14 and 16 is $\frac{30}{2} = 15$.

We note that in the second series, first term is increased by 2 and therefore second term is to be decreased by 2 to get the same average in both the cases.

Example 2:

Average of 4, 5 and 6 is $\frac{15}{3} = 5$.

If we decrease first and second numbers by 3 each, then we have to increase the third term by $2 \times 3 = 6$ to get the same average.

$\therefore$ New series is 1, 2, 12 and average is $\frac{15}{3} = 5$.

7. Questions based on average can broadly be divided into two categories:

(a) Addition Case:

Let average of 'n' items is 'x'.

On including one more item to the series, average of (n + 1) items becomes 'y'.

Then value of new item included = $y + n \times (y - x)$

i.e. New average + Number of items (previous) × Increase in average

Or, $x + (n + 1)(y - x)$

Note : What happens if more than one item have been included ?

Solution : Let m items are included

Then average of new items $= y + \frac{n}{m} \times (y - x)$

$$= \text{New Average} + \frac{\text{Number of items (previous)}}{\text{Number of items (new)}} \times \text{Increase in Average}$$

(b) Replacement Case:

When one item is excluded and one new item is included in a series of 'n' items and the average of the series is increased by 'a' by doing so.

Then New Item = Replaced item + a × n

i.e. New Item = Replaced item + (Increase in Average × Number of items)

Note: Total items remain 'n' in both the cases.

Note: Decrease means negative increase.

8. When items are divided into two parts and **mid-term is excluded from both the parts,** then:

Mid-term
= Average of all items
+ (Combined average – Average of first part) × No. of items in the first part
+ (Combined average – Average of second part) × No. of items in the second part

Note: This rule can also be used when a term other than the middle-term is excluded from both the parts.

9. When items are divided into two parts and **mid-term is included in both the parts**, then:

Mid-term
= Average of all items
+ (Average of first part – Combined Average) × No.of items in the first part
+ (Average of second part – Combined Average) × No. of items in the second part

Note: This rule can also be used when a term other than the middle-term is included in both the parts.

10. If average of 'a' items is 'b' and that of other 'b' items is 'a'.

Then combined average of all the items $= \frac{2ab}{a + b}$

SOLVED EXERCISE

1. The average of 99 items is 27. If 3 is added to each number of the series, the new average is:

(a) 24 (b) 27 (c) 30 (d) 33

Solution:

(c) New average = 27 + 3 = 30

2. The average of 18 numbers is 24. If each item of the series is multiplied by 3, the new average is:

(a) 8 (b) 24 (c) 27 (d) 72

$= 6 \times (1+2+3+.........+8) = 6 \times \frac{8 \times 9}{2}$

$\therefore$ Average of first eight multiples of 6 = $6 \times \frac{8 \times 9}{2 \times 8} = 27$.

Direct Method:

Average of first eight multiples of 6

$= \text{Average of } \left(\frac{8}{2}\right)^{\text{th}} \text{ term and } \left(\frac{8}{2}+1\right)^{\text{th}} \text{ term}$

= Average of 4th and 5th term

$= \frac{(6 \times 4) + (6 \times 5)}{2} = \frac{6 \times (4 + 5)}{2} = \frac{6 \times 9}{2} = 27$

8. Average of first five odd multiples of 3 is:

(a) 12 (b) 15 (c) 18 (d) 21

Solution:

(b) Sum of first 5 odd multiples of 3

$= (3 \times 1) + (3 \times 3) + (3 \times 5) + (3 \times 7) + (3 \times 9)$

$= 3 \times (1 + 3 + 5 + 7 + 9) = 3 \times 5^2$

$\therefore$ Average of first 5 odd multiples of 3 = $3 \times 5^2 \times \frac{1}{5} = 15$.

Direct Method:

$\text{Average} = \text{mid-term} = \left(\frac{5+1}{2}\right)^{\text{th}}$ i.e. 3rd term = $3 \times 5 = 15$

9. Out of three numbers, first number is twice of the second and thrice of the third number. If average of the three numbers is 550, what is the smallest number ?

(a) 50 (b) 100 (c) 150 (d) 300

Solution :

(d) Let the numbers are 6x, 3x and 2x

Sum of the numbers = $11x = 550 \times 3$

$\therefore$ Smallest number $= \frac{2}{11} \times 550 \times 3 = 300$

10. If average of 5 observations is 46 and that of 5 other observations is 50, find the average of all the 10 observations.

(a) 96 (b) 50 (c) 48 (d) 46

Solution:

(c) Since number of items are equal in both the cases.

Average of all the items is equal to average of two averages.

$\therefore$ Average of 10 observations = $\frac{46 + 50}{2} = 48$.

11. Average of 'n' observations is 35, average of 'n' other observations is 37 and average of remaining 'n' observations is 42. Average of all the observations is:

(a) 36 (b) 38 (c) 40 (d) 57

Solution:

(b) Since number of items are equal in all the three cases.

Average of all the items is equal to average of three averages.

$\therefore$ Average of all observations = $\frac{35+37+42}{3} = \frac{114}{3} = 38.$

12. One year ago, the population of a town was 12000. In one year, male population is increased by 8% and female population by 6% but the total population is increased by 7%. What was the size of the male population one year ago.

(a) 5000 (b) 5500 (c) 6000 (d) 7000

Solution:

(c) Total increase in population is 7% which is equal to average of 6% and 8%.

$\therefore$ Male population = Female population

$\therefore$ Male population = $\frac{1}{2} \times 12000 = 6000$

13. Sum of three consecutive even numbers is 32 more than the average of the numbers. Find middle number in the series.

(a) 14 (b) 16 (c) 18 (d) 20

Solution:

(b) Let Average = Middle number = x

And sum of three numbers = 3 × x = 3x

According to the question, $3x - x = 32$

$\therefore x = 16$

14. The average monthly income of A and B is Rs. 5500, average monthly income of B and C is Rs. 4500 and the average monthly income of A and C is Rs. 4000. What is the monthly income of B ?

(a) Rs. 3000 (b) Rs. 5000 (c) Rs. 6000 (d) Rs. 12000

Solution :

(c) B's income = Average of A & B + Average of B & C – Average of A & C

= Rs. (5500 + 4500 – 4000) = Rs. 6000

QUESTIONS ON ADDITION CASE

15. The average age of 40 students in a class is 15 years. If the age of teacher is also included, the average becomes 16 years, find the age of the teacher.

(a) 15 years (b) 16 years (c) 55 years (d) 56 years

Solution:

(d) Age of teacher = Average age of all + Total increase in age

= 16 + (1 × 40) = 56 years

16. The average of 11 numbers is 70 and that of first 10 numbers is 69. The 11th number is:

(a) 59 (b) 79 (c) 80 (d) 81

Solution:

(c) 11th number = Average of all + Total increase

= 70 + (1 × 10) = 80

17. The average marks obtained by ten students is 45 and that of eleven students is 48. How many marks did the eleventh student obtain?

(a) 76 (b) 78 (c) 80 (d) 86

Solution:

(b) Marks of 11th student

= Average marks of all + Total increase in marks

= 48 + (3 × 10) = 78 marks.

18. A cricketer has an average of 28 runs in 24 innings. How many runs must he score in the 25th inning to make his average 29?

(a) 1 (b) 4 (c) 5 (d) 53

Solution:

(d) Runs scored in 25th inning

= Average of all innings + Total increase in runs

= 29 + (1 × 24) = 53 runs

19. The average age of 25 students of a class is 13 years. If the age of teacher is also included, the average age is increased by 2 years. Find the age of the teacher.

(a) 65 years (b) 75 years (c) 85 years (d) 95 years

Solution:

(a) New average = 13 + 2 = 15 years

Age of teacher = Average of all + Total increase in age

= 15 + (2 × 25) = 65 years

20. The average age of 4 brothers is 6 years. If age of their father is included, the average is increased by 5 years. Find the age of the father.

(a) 24 years (b) 31 years (c) 35 years (d) 38 years

Solution:

(b) New Average = 6 + 5 = 11

∴ Father's age = Average age of all + Total increase in age

= 11 + (5 × 4) = 31 years

21. Average age of 6 persons is decreased by 1 year when one new person is included in the group. Find the age of new man, if average age of 6 persons was 39 years.

(a) 32 years (b) 33 years (c) 38 years (d) 40 years

Solution:

(a) New Average = 39 – 1 = 38 years

Age of new man = Average age of 7 persons – Total decrease in age

= 38 – (1 × 6) = 32 years

22. Out of 10 terms, the average of first 5 terms is 34 and that of last 5 terms is 38. If the average of first 9 terms is 35, find the 10th term.

(a) 26 (b) 35 (c) 44 (d) 45

Solution:

(d) Average of 10 terms = $\frac{34 + 38}{2} = 36$

Average of 9 terms = 35

$\therefore$ 10th term = Average of all + Total increase

$= 36 + (1 \times 9) = 36 + 9 = 45$

23. The average marks obtained by a group of 10 students is 41 marks. Find the new average if a new student who scored 63 marks is also included in the group.

(a) 39 (b) 40 (c) 43 (d) 45

Solution:

(c) Average of 10 students = 41 marks

Excess marks obtained by 11th student over the average of 10 students = 63 – 41 = 22

Increase in Average of 11 students = $\frac{22}{11}$ = 2 marks

$\therefore$ Average marks of 11 students = 41 + 2 = 43 marks.

24. The average of 9 items is 79. If one new item whose value is 59 is also included in the series, find average of all the 10 items.

(a) 77 (b) 80 (c) 81 (d) 83

Solution:

(a) Average of 9 items = 79

10th item is short from the average of 9 items by 79 – 59 = 20

$\therefore$ Decrease in Average of 10 items due to its inclusion = $\frac{20}{10} = 2$

$\therefore$ Average of 10 items = 79 – 2 = 77

25. Average age of 9 members of a club is 29 years. If 2 more persons with the average age of 40 years have become the members of the club, find average age of all the 11 members.

(a) 30 years (b) 31 years (c) 32 years (d) 35 years

Solution:

(b) Average age of 9 members = 29 years

Excess age of 2 new members = $(40 - 29) \times 2 = 22$ years

$\therefore$ Increase in the average age on inclusion of 2 new members

$= \frac{22}{11} = 2$ years

$\therefore$ Average age of 11 members = 29 + 2 = 31 years

26. A batsman has a certain average of runs for 16 innings. In the 17th inning, he scores 85 runs, thereby increases his average by 3 runs. What was his average before 17th inning?

(a) 88 runs (b) 82 runs (c) 34 runs (d) 37 runs

Solution:

(c) Increase in average = 3 runs

Total increase in runs = 3×17

$\therefore$ Average of 16 innings = $85 - 3 \times 17 = 34$ runs.

27. The batting average of a cricketer for 78 innings is 38 runs. His highest score exceeds the lowest score by 226 runs. If these innings are excluded, the average of the remaining innings is 36 runs. What is his highest score ?

(a) 156 runs (b) 158 runs (c) 227 runs (d) 228 runs

Solution:

(d) Difference between the age of old and new member

= Total members × Time Difference

= 10 × 2 = 20 years

33. Average age of 6 persons in a group is same as it was 5 years ago, when an old man in the group is replaced by a young man, what is the age of the new person if the man replaced is of 70 years.

(a) 30 years (b) 35 years (c) 40 years (d) 45 years

Solution:

(c) Difference between the age of two persons = 6 × 5 years = 30 years

∴ Age of new man = 70 – 30 = 40 years

34. Average weight of 30 students in a class is increased by 200 grams when one student whose weight is 20 kg. is replaced by a new student. Find the weight of the new student admitted.

(a) 10 kg (b) 25 kg (c) 26 kg (d) 30 kg

Solution:

(c) 200 grams = $\frac{1}{5}$ kg.

Total increase in weight = $\frac{1}{5} \times 30 = 6$ kg.

Weight of the new student

= Weight of student replaced + Total increase in weight

= 20 + 6 = 26 kg

35. The average age of 12 men in a group is increased by 2 years when two men whose ages are 20 years and 22 years, are replaced by new members. What is the average age of the new men included?

(a) 32 years (b) 33 years (c) 35 years (d) 66 years

Solution:

(b) Total age of two men replaced = 20 + 22 = 42 years

Total increase in age on replacement = 2 × 12 = 24 years

Total age of two new persons included = 42 + 24 = 66 years

∴ Average age of new persons = $\frac{66}{2}$ = 33 years

36. The average marks obtained by 9 students was calculated to be 65. Later on it was found that the marks of one student was wrongly read as 76 instead of 67. The correct average is:

(a) 56 (b) 64 (c) 66 (d) 74

Solution:

(b) Total marks were wrongly increased by = 76 – 67 = 9

∴ Average was wrongly increased by $\frac{9}{9}$ = 1 mark

∴ Correct average = 65 – 1 = 64 marks

37. Average marks of 100 students is 58. Later on it is found that marks of one student is misread as 47 instead of 87. Find the correct average.

(a) 57.4 (b) 57.6 (c) 58.4 (d) 58.6

Solution:

(c) Total marks were wrongly decreased by = 87 – 47 = 40

∴ Average was wrongly decreased by $\frac{40}{100}$ = 0.4 mark

∴ Correct average = 58 + 0.4 = 58.4 marks

38. Average age of 15 men is increased by 1 year when one man in the group is replaced with other man whose age is 49 years. Find the age of the man replaced.

(a) 34 years (b) 48 years (c) 50 years (d) 64 years

Solution:

(a) Age of man replaced = Age of new man – Total increase in age
= 49 – 1 × 15 = 34 years

TYPICAL QUESTIONS ON ADDITION CASE

39. Four years ago, the average age of 4 persons A, B, C and D was 38 years. With joining of E, now the average age of 5 persons becomes 36 years. Find E's present age.

(a) 26 years (b) 12 years (c) 30 years (d) 32 years

Solution:

(b) Average age (Present) of A, B, C and D = 38 + 4 = 42 years.
Average age (Present) of A, B, C, D and E = 36 years.
Total decrease in age on including E = 4 × (42 – 36) = 4 × 6 = 24 years
∴ E's age = 36 – 24 = 12 years

40. Average age of 7 members of a family is 29 years. If present age of the youngest member is 5 years, find average age of the remaining members at the time of birth of the youngest member.

(a) 22 years (b) 24 years (c) 26 years (d) 28 years

Solution:

(d) Average age (present) of 7 members = 29 years
5 years ago, average age of 7 members was 29 – 5 = 24 years.
Since the youngest member was not born 5 years ago.

∴ Average age of remaining 6 members is increased by $\frac{24}{6}$ = 4 years.

∴ 5 years ago, average age of 6 members was 24 + 4 = 28 years.

41. Average weight of 8 persons is 48 kg. If one man weighing 34 kg. is died, what is average age of the remaining 7 persons.

(a) 46 kg (b) 50 kg (c) 55 kg (d) 60 kg

Solution:

(b) Average weight of 8 persons = 48 kg.
∴ Excess of average weight than the weight of man died
= 48 kg. – 34 kg. = 14 kg.

of 46 kg. having joined the class, the average weight of the class is increased by 2 kg. Find the number of students in the class originally?

(a) 10 (b) 15 (c) 20 (d) 25

Solution:

(a) Total increase in weight on including 5 more students

$= 5 \times (46 - 40) = 30$ kg.

Actual increase in average weight = 2 kg.

$\therefore$ Total students (at present) $= \frac{30}{2} = 15$ students

$\therefore$ Students (originally) $= 15 - 5 = 10$ students

47. 4 years ago, the average age of 5 members of a family was 22 years, A baby having been born, the average age of the family is the same today. Find the age of the baby.

(a) 2 years (b) 3 years (c) 5 years (d) 17 years

Solution:

(a) Present average age of 5 members = 22 + 4 = 26 years

Present average age of 6 members = 22 years

Total decrease in age of 5 members $= 5 \times (26 - 22) = 20$ years

$\therefore$ Age of baby = Average age of all – Total decrease $= 22 - 20 = 2$ years

TYPICAL QUESTIONS ON REPLACEMENT CASE:

48. Average marks obtained by first 10 students out of 11 students in a class is 189. If the first student has obtained 163 marks and the 11th student 193 marks, find the average marks of the last 10 students.

(a) 190 (b) 192 (c) 199 (d) 219

Solution:

(b) More marks obtained by 11th student than 1st student

$= 193 - 163 = 30$ marks

$\therefore$ Average of last 10 students is more than first 10 students by

$\frac{30}{10} = 3$ marks

$\therefore$ Average marks of last 10 students $= 189 + 3 = 192$ marks

49. Average temperature for Monday, Tuesday and Wednesday is 35°C and that for Tuesday, Wednesday and Thursday is 36°C. What is the temperature on Thursday, if temperature on Monday is 31°C?

(a) 31°C (b) 34°C (c) 35°C (d) 36°C

Solution:

(b) Difference between temperature on Monday and Thursday

$= 3 \times (36°C - 35°C) = 3°C$

Since, temperature for 3 days including Thursday is more than that of 3 days including Monday.

$\therefore$ Temperature on Thursday is more than the temperature on Monday.

Solution:

(b) 6th observation = $50 + 6 \times (49 - 50) + 6 \times (52 - 50)$

$= 50 - 6 \times 1 + 6 \times 2 = 56$

59. The average of 15 numbers is 8. If average of first 8 numbers is 10 and that of last 8 numbers is 9, find the 8th number.

(a) 8 (b) 16 (c) 24 (d) 32

Solution:

(d) 8th term = $8 + 8 \times (10 - 8) + 8 \times (9 - 8)$

$= 8 + 8 \times 2 + 8 \times 1 = 8 + 16 + 8 = 32$

60. The average of 11 observations is 72. If average of first 6 observations is 70 and that of last 6 observations is 71, then the 6th observation is:

(a) 54 (b) 75 (c) 78 (d) 87

Solution:

(a) 6th observation = $72 + 6 \times (70 - 72) + 6 \times (71 - 72)$

$= 72 - 6 \times 2 - 6 \times 1 = 54$

61. The average of 9 readings is 9. The average of first 5 readings is 10 and that of last 5 readings is 8. What is the 5th reading?

(a) 9 (b) 10 (c) 9.5 (d) 8

Solution:

(a) 5th reading = $9 + 5 \times (10 - 9) + 5 \times (8 - 9)$

$= 9 + 5 \times 1 - 5 \times 1 = 9 + 5 - 5 = 9$

62. Average age of 13 students of a class is 12 years. If average age of first four students is 11 years and that of last 10 students is 13 years, what is the age of fourth student?

(a) 12 years (b) 18 years (c) 28 years (d) 36 years

Solution:

(b) Age of 4th student = $12 + 4 \times (11 - 12) + 10 \times (13 - 12)$

$= 12 - 4 + 10 = 18$ years

MISCELLANEOUS QUESTIONS

63. The average expenditure of 40 persons is Rs. 60 per day and that of 60 other persons is Rs. 40 per day. Find the average expenditure of all the 100 persons.

(a) Rs. 40 (b) Rs. 48 (c) Rs. 50 (d) Rs. 100

Solution:

(b) Average expenditure of 100 persons = $\frac{2 \times 40 \times 60}{100}$ = Rs. 48

64. The average age of 30 persons is 20 years and that of 20 other persons is 30 years. Find the average age of all the 50 persons.

(a) 24 years (b) 25 years (c) 26 years (d) 50 years

Solution:

(a) Average age of 50 persons = $\frac{2 \times 30 \times 20}{50}$ = 24 years

65. Average students from class I to X in a school is 54. What is the average number of students in class I to III, if the average number of students in the remaining seven classes is 57?

(a) 47 (b) 50 (c) 51 (d) 60

Solution:

(a) Extra students (than average) in 7 classes i.e. from class IV to X

$= 7 \times (57 - 54) = 7 \times 3 = 21$

∴ In first 3 classes, students must be less than the average number of students.

∴ Average of students in these classes is less by $\frac{21}{3} = 7$

∴ Average students in class I to III = 54 – 7 = 47

66. Average expenditure of a person for the first 3 days of a week is Rs. 350 and for the next 4 days is Rs. 420. Average expenditure of the man for the whole week is:

(a) Rs. 410 (b) Rs. 390 (c) Rs. 385 (d) Rs. 375

Solution:

(b) Assumed mean = Rs. 350

Total excess than assumed mean = 4 × (Rs. 420 – Rs. 350) = Rs. 280

∴ Increase in average expenditure = Rs. $\frac{280}{7}$ = Rs. 40

∴ Average expenditure for 7 days = Rs. 350 + Rs. 40 = Rs. 390

67. Average expenditure of a person for the first 5 months of a year is Rs. 2500 per month and that for the next 7 months is Rs. 2740. Average expenditure of the man for the year is:

(a) Rs. 2600 (b) Rs. 2620 (c) Rs. 2640 (d) Rs. 2660

Solution:

(c) Assumed mean = Rs. 2500

Total excess than assumed mean = 7 × (Rs. 2740 – Rs. 2500) = Rs. 1680

∴ Increase in average expenditure = Rs. $\frac{1680}{12}$ = Rs. 140

∴ Average expenditure = Rs. 2500 + Rs. 140 = Rs. 2640

68. A man's average expenditure for first 11 days of a month was Rs. 104 per day and for the next 19 days, it was Rs. 98 per day. If he could save Rs. 54 during that month, find his income for the month.

(a) Rs. 2940 (b) Rs. 3000 (c) Rs. 3060 (d) Rs. 3120

Solution:

(c) Assumed mean = Rs.100

Total excess of income = Saving + Deviation from the assumed mean

= Rs. 54 + 11 × (Rs. 104 – Rs. 100) + 19 × (Rs. 98 – Rs. 100)

= Rs. 54 + Rs. 44 – Rs. 38 = Rs. 60

∴ Average income = Rs. 100 + Rs. $\frac{60}{30}$ = Rs. 102

∴ Monthly income = Rs. 102 × 30 = Rs. 3060

69. A shopkeeper purchased 25 kg. of sugar at Rs. 5 per kg. and 75 kg. of sugar at Rs. 9 per kg. He mixed the two varieties and sells the mixture at Rs. 10 per kg., find his profit percentage.

(a) 11.11% (b) 25% (c) 33.33% (d) 100%

Solution:

(b) Let cost price of the mixture = Rs. 5 per kg.

Extra price paid for 75 kg. = Rs. (9 – 5) = Rs. 4 per kg.

∴ Total extra amount paid = 75 × Rs. 4 = Rs. 300

This will increase the assumed average price by $\frac{300}{100}$ = Rs. 3 per kg.

∴ Average cost price per kg. of mixture = Rs. 5 + Rs. 3 = Rs. 8

Selling price per kg. of mixture = Rs. 10

∴ Profit = Rs. 10 – Rs. 8 = Rs. 2 per kg.

∴ Profit % = $\frac{2}{8} \times 100 = 25\%$

70. The batting average of a batsman in 42 innings is 45 runs. He was out for a duck in 7 innings. His batting average for the remaining innings is:

(a) 50 (b) 54 (c) 55 (d) 60.25

Solution:

(b) Total runs of 42 innings = 42 × 45

∴ Average of remaining (42 – 7) or 35 innings = $\frac{42 \times 45}{35} = 54$

Alternative Method:

Average runs of 7 innings = 45

∴ Total runs of 7 innings = 45 × 7

∴ Average of remaining 35 innings is increased by $\frac{45 \times 7}{35}$ = 9 runs

∴ Average of remaining 35 innings = 45 + 9 = 54 runs

71. The batting average of a batsman in 57 innings is 58 runs. He was out for a duck in 7 innings. His batting average for the remaining innings is:

(a) 60 (b) 65 (c) 66.12 (d) 70.28

Solution:

(c) Total runs of 57 innings = 57 × 58

∴ Average of remaining (57 – 7) or 50 innings = $\frac{57 \times 58}{50} = 66.12$

Alternative Method:

Average runs of 7 innings = 58

Total runs of 7 innings = 58 × 7

∴ Average of remaining 50 innings is increased by $\frac{58 \times 7}{50}$ = 8.12 runs

∴ Average of remaining 50 innings = 58 + 8.12 = 66.12 runs

72. 11 friends went to a hotel and decided to pay the bill amount equally. But 10 of them could pay Rs. 60 each as a result 11th has to pay Rs. 50 extra than his share. Find the amount paid by him.

(a) Rs. 105 (b) Rs. 110 (c) Rs. 115 (d) Rs. 120

Solution:

(c) Average amount paid by 10 persons = Rs. 60

Increase in average due to Rs. 50 paid extra by the 11th men

$= \text{Rs. } \frac{50}{10} = \text{Rs. } 5$

∴ Average expenditure of 11 friends = Rs. 60 + Rs. 5 = Rs. 65

∴ Amount paid by the 11th men = Rs. 65 + Rs. 50 = Rs. 115

73. 9 men started a partnership business contributing Rs. 10000 each. Later on 10th men joined the partnership contributing Rs. 3600 more than the average contribution of all the 10 men. How much did the 10th men contribute?

(a) Rs. 3600 (b) Rs. 10400 (c) Rs. 14000 (d) Rs. 15000

Solution:

(c) Average contribution of 9 partners = Rs. 10000

Increase in average due to Rs. 3600 paid extra by the 10th partner

$= \text{Rs. } \frac{3600}{9} = \text{Rs. } 400$

∴ Average contribution of 10 partners = Rs. 10000 + Rs. 400 = Rs. 10400

∴ Amount paid by the 10th partner = Rs. 10400 + Rs. 3600 = Rs. 14000

74. 5 men started a business contributing Rs. 20000 each. Later on 6th men joined the partnership contributing Rs. 2000 less than the average contribution of all the 6 men. Find the amount contributed by the 6th partner.

(a) Rs. 2000 (b) Rs. 17600 (c) Rs. 19600 (d) Rs. 20500

Solution:

(b) Average contribution of 5 partners = Rs. 20000

Decrease in average due to Rs. 2000 paid less by the 6th partner

$= \text{Rs. } \frac{2000}{5} = \text{Rs. } 400$

∴ Average contribution of 6 partners = Rs.20000 – Rs.400 = Rs.19600

∴ Amount paid by the 6th partner = Rs.19600 – Rs.2000 = Rs.17600

75. The average marks obtained by some students in an examination is 54. If 20% of the students got a mean score of 90 marks and the 30% of the students got a mean score of 20. Find the average marks of the remaining students.

(a) 42 (b) 45 (c) 50 (d) 60

Solution:

(d) Remaining students = 100% – 20% – 30% = 50%

Let remaining students got a mean score of x marks.

Then 20% of 90 + 30% of 20 + 50% of x = 54

∴ 18 + 6 + 50% of x = 54

∴ 50% of x = 54 – 18 – 6 = 30

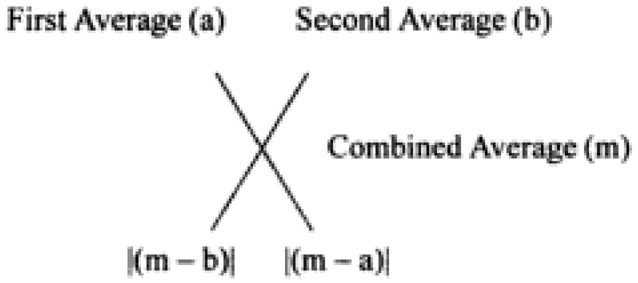
∴ x = 30 × 2 = 60 marks

Chapter 23

ALLIGATION

Alligation Rule: It is also called Rule of Mixture. Alligation Rule is used to find:

1. The combined average of some items which are divided into two groups and both the groups have different averages, and
2. The ratio in which items from two groups should be taken to get a certain combined average for all the items.



If averages of two different groups are 'a' and 'b'. Then the items from these groups should be mixed in the ratio of |(m – b)| and |(m – a)| to get the combined average 'm' and vice-versa.

Note: |(m – b)| and |(m – a)| are absolute values of (m – b) and (m – a) respectively. Absolute value of a number doesn't carry negative sign.

Example:

Absolute value of |3 – 6| is 3 (not – 3)

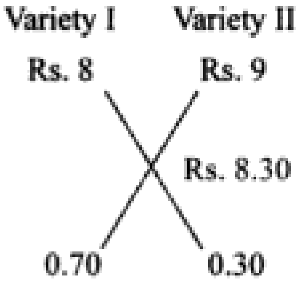
SOLVED EXERCISE

1. In what ratio two varieties of milk costing Rs. 8 and Rs. 9 per litre respectively mixed, so that the mixture costs Rs. 8.30 per litre?

(a) 9 : 8 (b) 7 : 3 (c) 3 : 7 (d) 8 : 9

Solution:

(b)



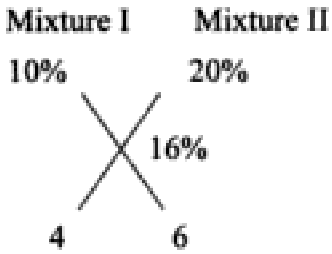
∴ Ratio in which two mixtures are mixed = 0.70 : 0.30 = 7 : 3.

2. Two mixtures of milk and water contain 10% and 20% water. In what ratio two mixtures should be mixed so that the resulting mixture contains 16% water?

(a) 1 : 2 (b) 2 : 1 (c) 2 : 3 (d) 3 : 2

Solution:

(c)



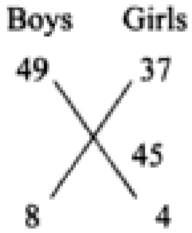
∴ Ratio in which two mixtures are mixed = 4 : 6 = 2 : 3

3. The average weight of 30 students of a class is 45 kg. The average weight of girls is 37 kg. and that of boys is 49 kg. Find the number of boys in the class.

(a) 10 (b) 15 (c) 20 (d) 22

Solution:

(c)



∴ Ratio of Boys and Girls in the class = 8 : 4 = 2 : 1.

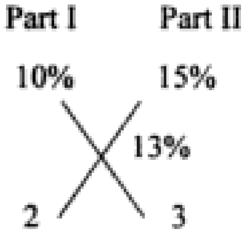
∴ Boys in the class = $\frac{2}{2+1} \times 30$ = 20 boys

4. A merchant sells 50 kg. of a commodity – partly at 10% profit and the rest at 15% profit. If net profit in the deal is 13%, then the quantity sold at 10% profit is:

(a) 20 kg (b) 25 kg (c) 30 kg (d) 40 kg

Solution:

(a)



∴ Ratio of two quantities sold = 2 : 3.

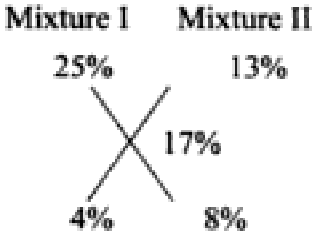
∴ Quantity sold at 10% = $\frac{2}{2+3} \times 50$ kg. = 20 kg

5. A merchant mixes two varieties of wine containing 25% and 13% alcohol. The resultant mixture contains 17% alcohol. Find the quantity of second mixture, if 8 litre of first mixture is taken.

(a) 4 litres (b) 16 litres (c) 24 litres (d) 32 litres

Solution:

(b)



∴ Ratio of two varieties = 4 : 8 = 1 : 2.

Given, quantity of first mixture = 8 litres.

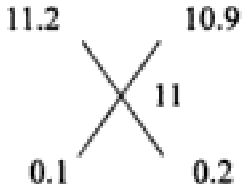
∴ Quantity of second mixture = 8 × 2 = 16 litres.

6. The average age of students of a class is 11 years. If the average age of boys is 11.2 years and that of girls is 10.9 years. Find number of total students, if there are 50 boys in the class.

(a) 100 (b) 150 (c) 160 (d) 200

Solution:

(b) Boys Girls



∴ Ratio of boys and girls = 0.1 : 0.2 = 1 : 2

Boys in the class = 50

∴ Girls in the class = 50 × 2 = 100

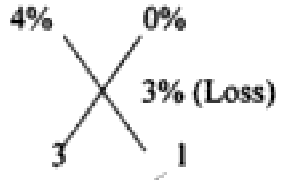
∴ Total students = 50 + 100 = 150

7. A merchant sold 80 dozen oranges. He sold some at 4% loss and rest at the cost price and thus losing 3% on the whole. What is the quantity sold at loss?

(a) 20 dozen (b) 40 dozen (c) 50 dozen (d) 60 dozen

Solution:

(d) At Loss At Cost price



∴ Ratio of oranges sold at loss and at cost price = 3 : 1.

Total oranges sold = 80 dozen.

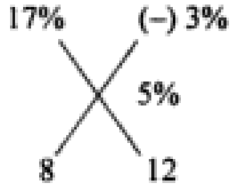
∴ Quantity sold at loss = $\frac{3}{4} \times 80 = 60$ dozen.

8. A shopkeeper sold 45 kg. of goods. If he sells some quantity at a loss of 3% and rest at 17% profit, making 5% profit on the whole, find the quantity sold at profit.

(a) 9 kg (b) 15 kg (c) 18 kg (d) 30 kg

Solution:

(c) At profit At loss



Note: 17 – 5 = 12, and 5 – (– 3) = 5 + 3 = 8

∴ Ratio of quantities sold at profit and at loss = 8 : 12 = 2 : 3

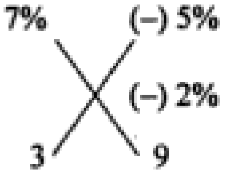
∴ Quantity sold at profit = $\frac{2}{5} \times 45 = 18$ kg.

9. A shopkeeper sold 40 kg. of goods – partly at 7% profit and rest at 5% loss, suffering 2% loss on the whole, what is the quantity sold at profit?

(a) 10 kg (b) 15 kg (c) 25 kg (d) 30 kg

Solution:

(a) At profit At loss



Note: 7 – (– 2) = 7 + 2 = 9 and – 2 – (– 5) = 3.

Ratio of quantities sold at profit and at loss = 3 : 9 = 1 : 3.

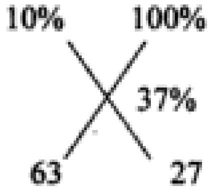
∴ Quantity sold at profit = $\frac{1}{4} \times 40 = 10$ kg.

10. A mixture of 70 litres of wine and water contains 10% water. How much more water should be added to the mixture to make 37% water in the resulting mixture?

(a) 37 litres (b) 30 litres (c) 27 litres (d) 18.9 litres

Solution:

(b) Mixture Water



∴ Ratio = 63 : 27 = 7 : 3

∴ For every 7 litres of mixture, 3 litres of water is to be added.

∴ For 70 litres of mixture, water to be added = $\frac{3}{7} \times 70 = 30$ litres

Alternative Method:

Water = 10% of 70 litres = 7 litres

Wine = 70 – 7 = 63 litres

In the new mixture, water is 37% and wine is 63%.

If wine is 63 litres, then water = 37 litres.

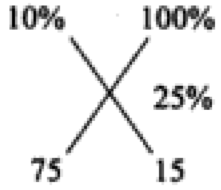
∴ Water to be added = 37 – 7 = 30 litres

11. A mixture of 70 litres of wine and water contains 10% water. How much water should be added to make 25% water in the resulting mixture?

(a) 7 litres (b) 10.5 litres (c) 14 litres (d) 21 litres

Solution:

(c) Mixture Water



∴ The ratio is 75 : 15 = 5 : 1.

∴ For every 5 litres of mixture, 1 litre of water is added.

∴ For 70 litres of mixture, water to be added = $\frac{1}{5} \times 70$ = 14 litres.

Alternative Method:

Water = 10% of 70 litres = 7 litres

Wine = 70 – 7 = 63 litres

In the new mixture, water is 25% and wine is 75%.

In new mixture wine is 3 times of water.

∴ Water in new mixture = $\frac{1}{3} \times 63$ = 21 litres

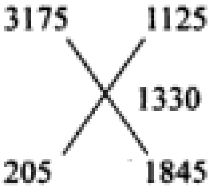
∴ Water to be added = 21 – 7 = 14 litres

12. The average salary per labour in a factory is Rs. 1125 and the average salary of 12 officers is 3175. Find the total number of employees working in the factory if average salary per employee is Rs. 1330.

(a) 30 (b) 60 (c) 108 (d) 120

Solution:

(d) Officers Labour



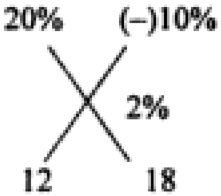
∴ Ratio of number of officers and labour = 205 : 1845 = 1 : 9.
Officers in the factory = 12
∴ Labour in the factory = 12 × 9 =108
∴ Total employees = 12 + 108 = 120

13. A man purchased a horse and a cow for Rs. 5000. He sells the horse at 20% profit and the cow at 10% loss. If he gains 2% on the whole transaction, the cost price of the horse is :

(a) Rs. 2000 (b) Rs. 2500 (c) Rs. 2800 (d) Rs. 3000

Solution:

(a) Horse Cow



∴ Ratio between cost price of a horse and that of a cow = 12 : 18 = 2 : 3.

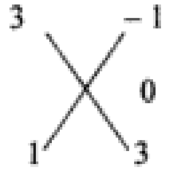
∴ Cost price of the horse = $\frac{2}{5} \times 5000$ = Rs. 2000

14. In an examination, a student gets 3 marks for every correct answer and loses 1 mark for every wrong answer. If he scores '0' marks in a paper of 100 questions, how many of his answers were correct?

(a) 25 (b) 50 (c) 60 (d) 75

Solution:

(a) Correct Incorrect



∴ Ratio of correct and wrong answers = 1 : 3

∴ Correct answers = $\frac{1}{4}$ × 100 = 25

15. In an examination, a student gets 3 marks for every correct answer and loses 1 mark for every wrong answer. If he scores 120 marks in a paper of 100 questions, how many of his answers were correct?

(a) 15 (b) 40 (c) 55 (d) 60

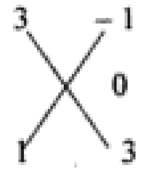
Solution:

(c) Let he scores 120 marks in the first ‘x’ questions.

∴ Correct answer are $\frac{120}{3}$ = 40 i.e. first 40 questions.

In the remaining 60 (i.e. 100 – 40) questions, he scores ‘0’ marks.

Correct Incorrect



∴ Ratio of correct and wrong answers = 1 : 3

∴ Correct answers = $\frac{1}{4}$ × 60 = 15

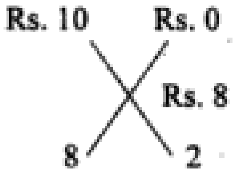
∴ Total correct answers = 40 + 15 = 55 answers.

16. Pure milk costs Rs.10 per litre. A milkman adds some water to 20 litres of pure milk and sells the mixture at Rs. 8 per litre and hence making no profit or loss. How many litres of water does he add?

(a) 2 litres (b) 5 litres (c) 8 litres (d) 10 litres

Solution:

(b) Cost of Milk Cost of Water



∴ Milk : Water = 8 : 2 = 4 : 1

Hence for every 4 litres of pure milk, he adds 1 litre of water.

∴ For 20 litres of pure milk, he adds 5 litres of water.

Alternative Method:

Cost price of 20 litres = Rs. 200

$\therefore$ Quantity sold = $\frac{200}{8}$ = 25 litres

$\therefore$ Water added = 25 – 20 = 5 litres

17. In what ratio a shopkeeper must add water to milk so that he gains 25% on selling it at cost price ?

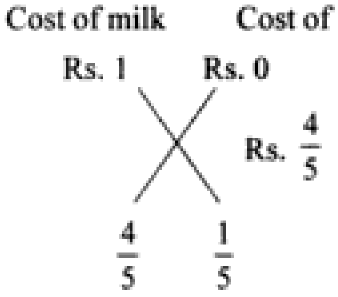
(a) 2 : 3 (b) 3 : 2 (c) 3 : 1 (d) 4 : 1

Solution :

(d) Let cost price of pure milk is Re. 1 per litre.

Then selling price of mixture = Rs. 1 per litre.

Cost price of 1 litre of mixture = $\frac{100}{125}$ = Re. $\frac{4}{5}$



$\therefore$ Ratio of milk and water = $\frac{4}{5}:\frac{1}{5} = 4:1$.

18. Pure milk costs Rs. 12 per litre. A milkman adds some water to 16 litres of pure milk and sells the mixture at Rs. 10 per litre making 25% profit. How many litres of water does he add?

(a) 5 litres (b) 6 litres (c) 8 litres (d) 10 litres

Solution :

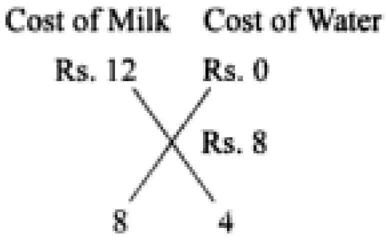
(c) Profit = $\frac{1}{4}$ of the cost price

= $\frac{1}{5}$ of the selling price = $\frac{1}{5}$ × Rs. 10 = Rs. 2 per litre

$\therefore$ Cost price of the mixture = Rs. 10 – Rs.2 = Rs. 8 per litre

Cost price of pure milk = Rs. 12

By Alligation Method:



$\therefore$ Milk : Water = 8 : 4 = 2 : 1

Hence for every 2 litres of pure milk, he adds 1 litre of water.

$\therefore$ For 16 litres of pure milk, he adds 8 litres of water.

19. A shopkeeper mixes two varieties of rice purchased @ Rs. 35 per kg and @ Rs. 28 per kg respectively. If he sells the mixture @ Rs. 36 per kg. and thus gains 20%, find the ratio in which the two varieties are mixed.

(a) 2 : 5 (b) 4 : 7 (c) 5 : 2 (d) 7 : 4

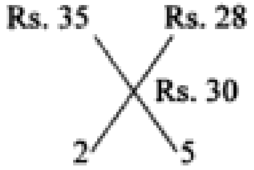
Solution:

(a) Profit = $\frac{1}{5}$ of cost price

$= \frac{1}{6}$ of selling price $= \frac{1}{6} \times$ Rs. 36 = Rs. 6

$\therefore$ Cost of the mixture = Rs. 36 – Rs. 6 = Rs. 30

Mixture I Mixture II



$\therefore$ Ratio in which the two varieties are mixed = 2 : 5

20. Two vessels A and B contain milk and water in the ratio 7 : 1 and 9 : 7 respectively. Find the ratio in which the mixtures must be taken from the two vessels to make the resultant mixture in the ratio 2 : 1.

(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 3 : 1

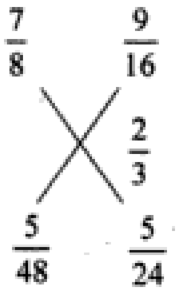
Solution :

(b) Milk in vessel $A = \frac{7}{8}$

Milk in vessel $B = \frac{9}{16}$

Resultant Mixture $= \frac{2}{3}$

Mixture I Mixture II



$\therefore$ Ratio of the mixture from vessels A and B $= \frac{5}{48} : \frac{5}{24} = 1 : 2$

21. A batsman has batting average of 27 runs per inning. In the next series of 3 innings he scores only 27 runs, thereby decreases his batting average by 2 runs. Find the total number of innings played by him.

(a) 25 (b) 26 (c) 27 (d) 28

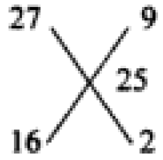
Solution:

(c) Average of 3 innings = $\frac{27}{3}$ = 9 runs

Average of innings played earlier = 27 runs

And combined average = 27 – 2 = 25 runs

Average I Average II



∴ Ratio of innings played (before and after) = 16 : 2 = 8 : 1

Innings played (after) = 3

∴ Innings played (before) = 8 × 3 = 24

∴ Total innings played = 24 + 3 = 27

22. The batting average of a cricket player is 72 runs per inning. In the next 4 innings, he could score only 80 runs and thereby decreases his batting average by 2 runs. What is total number of innings played by him till last match?

(a) 72 (b) 88 (c) 92 (d) 104

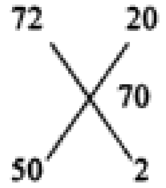
Solution:

(d) Average of last 4 innings = $\frac{80}{4}$ = 20 runs

Average of innings played earlier = 72 runs

New average = 72 – 2 = 70 runs

Earlier last 4 innings



Ratio of innings played (before and after) = 50 : 2 = 25 : 1

Innings played (after) = 4

∴ Innings played (before) = 25 × 4 = 100

∴ Total innings played = 100 + 4 = 104

Alternative Method:

Average runs in the last 4 innings = $\frac{80}{4}$ = 20 runs

Short from previous average = 72 – 20 = 52

Short (total) = 52 × 4

Short runs (per inning) = 2 runs

∴ Total innings played = $\frac{52 \times 4}{2}$ = 104 innings

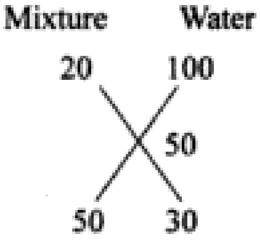
25. A mixture 40 litres contains milk and water in ratio 4 : 1. How many litres of mixture must be replaced with water so that the new ratio is 1 : 1 ?

(a) 12 litres (b) 15 litres (c) 16 liters (d) 18 litres

Solution :

(b) Water in mixture $= \frac{1}{5} = 20\%$

Water in resultant mixture $= \frac{1}{2} = 50\%$



$\therefore$ Ratio = 50 : 30 = 5 : 3

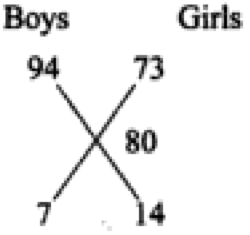
$\therefore$ Mixture replaced $= \frac{3}{8} \times 40 = 15$ litres

26. A class of 60 students contributed Rs. 4800 for a charity. If each boy has contributed Rs. 94 and each girl contributed Rs. 73. Find the number of boys in the class.

(a) 20 (b) 30 (c) 40 (d) 45

Solution :

(a) Average contribution per student $= \frac{4800}{60} = \text{Rs.}80$



$\therefore$ Boys : Girls = 7 : 14 = 1 : 2

$\therefore$ Boys $= \frac{1}{3} \times 60 = 20$

Chapter 24

SIMPLE INTEREST

Simple interest is calculated on the Principal uniformly for each year. In other words, Principal remains same throughout the period.

BASIC TERMS USED

Principal: The money borrowed by the borrower from the lender is called Principal.

Rate of Interest: Rate at which the Principal is charged.

Time: The period at the expiry of which the amount is to be returned.

Amount: The money returned by the borrower at the end of the certain period is called Amount.

Amount = Principal + Interest

Interest: The extra money paid by the borrower to the lender is called Interest.

Interest = Amount – Principal

FORMULAE

1. $\text{Simple interest} = \frac{\text{Principal} \times \text{Rate} \times \text{Time}}{100}$
2. $\text{Principal} = \frac{\text{Simple Interest} \times 100}{\text{Rate} \times \text{Time}}$
3. $\text{Rate of Interest} = \frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Time}}$
4. $\text{Time} = \frac{\text{Simple Interest} \times 100}{\text{Principal} \times \text{Rate}}$
5. If rate of Simple interest differs from year to year, then

   $\text{Simple interest} = \text{Principal} \times \frac{(R_1 + R_2 + R_3 + \ldots\ldots\ldots)}{100}$
6. $\text{Amount} = \text{Principal} \times \frac{100 + (\text{Rate} \times \text{Time})}{100}$
7. $\text{Principal} = \text{Amount} \times \frac{100}{100 + (\text{Rate} \times \text{Time})}$
8. **If a sum of money becomes double in ‘n’ years. It will become 3 times in ‘2n’ years and fourfold in ‘3n’ years.**

Formula:

If a sum of money becomes double in ‘ n’ years.

Then it will become ‘x’ times in (x – 1) × n years.

9. If ratio of Simple interest on two equal sums = x : y.

   Then ratio of sums to earn equal interest = y : x.

10. A sum of money is divided into 'n' parts in such a way that the interest on the first part at r_1% for t_1 years, on second part at r_2% for t_2 years, on third part at r_3% for t_3 years and so on, are equal.

    Then, the ratio in which the sum is divided in 'n' parts is:

$$\frac{1}{r_1 \times t_1} : \frac{1}{r_2 \times t_2} : \frac{1}{r_3 \times t_3} \ldots\ldots\ldots\ldots : \frac{1}{r_n \times t_n}$$

11. If annual payment of Rs.x discharges a debt of Rs. A due in 'n' years. Find the amount of each instalment, if rate of simple interest is r % p.a.

Solution:

First instalment of Rs. x paid after 1 year, in (n – 1) years becomes

$$= x \times \frac{100+(n-1)r}{100}$$

Second instalment of Rs. x paid after 2 years, in (n – 2) years becomes

$$= x \times \frac{100+(n-2)r}{100}$$

Third instalment of Rs. x paid after 3 years, in (n – 3) years becomes

$$= x \times \frac{100+(n-3)r}{100}$$

Similarly second last instalment of Rs. x paid after (n – 1) year, in 1 year becomes

$$= x \times \frac{100+(1)r}{100}$$

Final instalment of Rs. x paid after n years becomes in (n – n) years equal to

$$x \times \frac{100+(n-n)r}{100} = \frac{100}{100}x$$

$$\therefore A = x \times \left(\frac{100+(n-1)r}{100} + \frac{100+(n-2)r}{100} + \frac{100+(n-3)r}{100} + \ldots + \frac{100+(1)r}{100} + \frac{100}{100}\right)$$

$$\therefore 100\,A = x\,[100 \times n + r\{(n-1)\,(n-2)\,(n-3) \ldots\ldots\ldots (2)\,(1)\}]$$

$$\therefore 100\,A = x\left(100n + r\frac{(n-1)(n)}{2}\right)$$

$$\boxed{\therefore x = \frac{100\,A}{100n + \frac{r(n)(n-1)}{2}}}$$

SOLVED EXERCISE

1. Simple interest on Rs. 5000 at 10% per annum for 4 years is:

    (a) Rs. 1500 (b) Rs. 2000 (c) Rs. 2500 (d) Rs. 5000

Solution:

(b) Simple interest = $\frac{5000 \times 10 \times 4}{100}$ = Rs. 2000

2. Find Simple interest on Rs. 1500 for 2 years at the rate of 1 paise per rupee per month.

    (a) Rs. 30 (b) Rs. 36 (c) Rs. 300 (d) Rs. 360

∴ Simple interest = Rs. 2 – Re.1 = Re.1

∴ Rate of interest = $\frac{1 \times 100}{1 \times 8}$ = 12.5%

14. At what rate per cent of Simple interest per annum will a sum of money become three times in 10 years?

(a) 5% (b) 10% (c) 20% (d) 25%

Solution:

(c) Let Principal = Re. 1

Then Amount after 10 years = 3 × Re. 1 = Rs. 3

∴ Simple interest = Rs. 3 – Re. 1 = Rs. 2

∴ Rate of interest = $\frac{2 \times 100}{1 \times 10}$ = 20%

15. If difference between simple interest on a certain sum at 4% for 6 years and at 5% for 4 years is Rs. 28, find the sum.

(a) Rs. 200 (b) Rs. 400 (c) Rs. 500 (d) Rs. 700

Solution:

(d) Let the sum = Rs. 100

Then Simple interest in first case = 100 × 4% × 6 = Rs. 24

And Simple interest in second case = 100 × 5% × 4 = Rs. 20

Difference in Simple interest = Rs. 24 – Rs. 20 = Rs. 4

Actual difference = Rs. 28 (i.e. 7 times of Rs. 4)

∴ Principal = 7 × Rs. 100 = Rs. 700

16. At what rate per cent of Simple interest per annum will a sum of money earn interest equal to $\frac{3}{5}$ of the principal in 4 years?

(a) 15% (b) 20% (c) 25% (d) 12.5%

Solution:

(a) Let Principal = Rs. 5

Then Simple interest = $\frac{3}{5}$ × Rs. 5 = Rs. 3

∴ Rate of interest = $\frac{3 \times 100}{5 \times 4}$ = 15%

17. A certain sum of money at a certain rate of Simple interest becomes double in 10 years. It will become three times in:

(a) 20 years (b) 15 years (c) 10 years (d) $7\frac{1}{2}$ years

Solution:

(a) Principal will become 3 times in (3 – 1) × 10 = 20 years.

18. A certain sum of money at a certain rate of Simple interest becomes double in 5 years. It will become four times in:

(a) $7\frac{1}{2}$ years (b) 10 years (c) 15 years (d) 20 years

me, i.e. inverse proportion

i.e. inverse proportion

$\times \frac{10}{18} = 4$ years

n Rs. 300 for 4 years at certain rate of interest is Rs. 100. What will be 600 for 8 years at the same rate of interest?

(b) Rs. 200 (c) Rs. 50 (d) Rs. 25

Time	Simple interest
4	100
8	x

erest on Rs. 600 for 8 years = Rs. 100 × $\frac{600}{300} \times \frac{8}{4}$ = Rs. 400

Principal = Double, Time = Double

∴ Interest becomes 2 × 2 = 4 times of original interest

∴ Interest = 4 × Rs. 100 = Rs. 400

24. What sum of money lent at 4% p.a. for 3 years will earn same interest as Rs. 1200 earns in 4 years at 5% p.a. of Simple interest?

(a) Rs. 1000 (b) Rs. 1500 (c) Rs. 2000 (d) Rs. 2500

Solution:

(c)

Principal	Time	Rate
1200	4	5
x	3	4

Less time, more principal, i.e. inverse proportion

Less rate, more principal, i.e. inverse proportion

∴ Principal = 1200 × $\frac{4}{3} \times \frac{5}{4}$ = Rs. 2000

Alternative method:

Interest on Rs. 100 in two cases are Rs. 12 and Rs. 20 respectively.

∴ Amount must be invested in the ratio of 20 : 12 or 5 : 3 to earn the same interest in both the cases.

∴ Amount invested at 4% = $\frac{5}{3}$ × 1200 = Rs. 2000

25. If a sum of Rs. 500 amounts to Rs. 575 in 3 years. How much will Rs. 600 amount to in 4 years at the same rate of simple interest.

(a) Rs. 120 (b) Rs. 620 (c) Rs. 650 (d) Rs. 720

Solution:

(d) Simple interest on Rs. 500 for 3 years = Rs. 575 – Rs. 500 = Rs. 75

Rate of interest = $\frac{75 \times 100}{500 \times 3}$ = 5% p.a.

Interest on Rs. 600 for 4 years = Rs. 600 × 5% × 4 = Rs. 120

∴ Amount = Rs. 600 + Rs. 120 = Rs. 720

Alternative method:

Simple interest on Rs. 500 for 3 years = Rs. 575 – Rs. 500 = Rs. 75

∴ Simple interest on Rs. 600 for 4 years = $75 \times \frac{600}{500} \times \frac{4}{3}$ = Rs. 120

∴ Amount = Rs. 600 + Rs. 120 = Rs. 720

26. A sum of Rs. 29000 is divided into two parts and invested at simple interest in such a first part invested at 5% p.a. for 6 years earns the same interest as the second part for 7 y 4% p.a. Find the amount invested in first part.

(a) Rs. 14000 (b) Rs. 14500 (c) Rs. 15000 (d) Rs. 16500

Solution:

(a) Let the amount invested in each part = Rs. 100.

Then Simple interest on first part = 100 × 5% × 6 = Rs. 30

And Simple interest on second part = 100 × 4% × 7 = Rs. 28

∴ The ratio of interest earned on equal amount = 30 : 28 = 15 : 14

∴ Ratio of amount invested to get equal interest = 14 : 15

∴ Amount invested in first part = $\frac{14}{29}$ × Rs. 29000 = Rs. 14000

27. A man invested Rs. 4000 into two parts at simple interest – one at 4% and other at 5% p.a. If interest on first part for 5 years is equal to interest on second part for 6 years, find amount invested at 4%.

(a) Rs. 1600 (b) Rs. 1800 (c) Rs. 2000 (d) Rs. 2400

Solution:

(d) Interest on Rs. 100 at 4% for 5 years and at 5% for 6 years will be

$\frac{100 \times 4 \times 5}{100}$ and $\frac{100 \times 5 \times 6}{100}$ i.e. Rs. 20 and Rs. 30 respectively.

But Simple interests on two parts are equal.

∴ Amounts invested are in the ratio of 30 : 20 or 3 : 2

∴ Amount invested at 4% = $\frac{3}{5}$ × Rs. 4000 = Rs. 2400

28. A man invested Rs. 600 into three parts at simple interest – first part at 3% for 4 years, second part at 4% for 5 years and the third part at 5% for 3 years. If interest received on each part is equal, find amount invested at 5%.

(a) Rs. 150 (b) Rs. 200 (c) Rs. 250 (d) Rs. 300

Solution:

(b) Ratio of principals of three parts = $\frac{1}{3 \times 4} : \frac{1}{4 \times 5} : \frac{1}{5 \times 3}$

= 5 : 3 : 4 (On multiplying each ratio by 3 × 4 × 5)

Sum of ratios = 5 + 3 + 4 = 12

∴ Amount invested at 5% = $\frac{4}{12}$ × Rs. 600 = Rs. 200

29. A sum of Rs.5500 is divided into two parts and invested at 4% p.a. for 3 years and at 5% p.a. for 6 years respectively. If interest earned on the second part is three times of that on the first part, find the amount invested in second part.

(a) Rs. 2000 (b) Rs. 2250 (c) Rs. 2500 (d) Rs. 3000

Solution:

(d) Let the amount invested in each part = Rs. 100

Then Simple interest on first part = 100 × 4% × 3 = Rs. 12

And Simple interest on second part = 100 × 5% × 6 = Rs. 30

∴ Ratio of interest earned on equal amount = 12 : 30 = 2 : 5

∴ Ratio of amount to be invested to get equal interest = 5 : 2

∴ To get 3 times interest on the second part, the amount must be divided in the ratio of (5 × 1) : (2 × 3) = 5 : 6

∴ Amount invested in second part = $\frac{6}{11}$ × Rs. 5500 = Rs. 3000

30. A man invested a total sum of Rs. 8500 in names of his sons at 4% and 5% respectively in such a way that they get equal amount on attaining age of 20 years. If their present ages are 15 years and 13 years respectively, what is the sum invested in the name of elder son ?

(a) Rs. 3750 (b) Rs. 4000 (c) Rs. 4250 (d) Rs. 4500

Solution :

(d) Let Rs. 100 is invested in each case.

Then amount received by elder son = 100 + 4 × 5 = Rs. 120

And amount received by younger son = 100 + 5 × 7 = Rs. 350

∴ Ratio of sum invested $= \frac{1}{120} : \frac{1}{135} = 135 : 120 = 9 : 8$

∴ Sum invested in name of elder son $= \frac{9}{17} \times 8500 = 4500$

31. Rs. 4310 is invested into three parts at simple interest so that the amounts received after 1, 2 and 3 years respectively in each part are equal. Find the amount invested for 3 years, if rate of interest is 10% p.a.

(a) Rs. 1320 (b) Rs. 1430 (c) Rs. 1560 (d) Rs. 1650

Solution:

(a) Let Rs. 100 is invested in each part.

Then amount received in three parts on maturity are Rs. 110, Rs. 120 and Rs. 130 respectively.

∴ Ratio of amount invested in each part (to get equal amounts)

$= \frac{1}{110} : \frac{1}{120} : \frac{1}{130} = \frac{1}{11} : \frac{1}{12} : \frac{1}{13}$

Solution:

(c) Simple interest for 2 years = Rs. 700 – Rs. 600 = Rs. 100

$\therefore$ Simple interest for 1 year = $\frac{100}{2}$ = Rs. 50

$\therefore$ Principal = Amount after 2 years – Simple interest for 2 years

= Rs. 600 – 2 × Rs. 50 = Rs. 500

Hence Simple interest for 1 year is Rs. 50 and Principal is Rs. 500.

$\therefore$ Rate of interest = $\frac{50 \times 100}{500 \times 1}$ = 10% p.a.

37. If simple interest on a certain sum is $\frac{1}{4}$th of the principal and the number of years are equal to the rate per cent per annum, the rate of interest per annum is:

(a) 5% (b) 10% (c) 20% (d) 25%

Solution:

(a) Let Principal = Rs. 4

Then Simple interest = Re. 1

$$\text{Rate} \times \text{Time} = \frac{1 \times 100}{4}$$

$(\text{Rate})^2 = 25$

$\therefore$ Rate of interest = $\sqrt{25}$ = 5% p.a.

38. If Simple interest on a certain sum of money is $\frac{1}{9}$th of the Principal and the number of years are equal to the rate per cent per annum, the rate of interest per annum is:

(a) 3% (b) $\frac{1}{3}$% (c) $\frac{10}{3}$% (d) $\frac{3}{10}$%

Solution:

(c) Let Principal = Rs. 9

Then Simple interest = Re.1

$\therefore$ Rate × Time = $(\text{Rate})^2 = \frac{1 \times 100}{9} = \frac{100}{9}$

$\therefore$ Rate = $\sqrt{\frac{100}{9}} = \frac{10}{3}$% p.a.

39. A man invested $\frac{1}{3}$ of his income at 5% p.a., $\frac{1}{4}$ of the income at 4% and the remaining $\frac{5}{12}$ income at 3%. If his annual income is Rs. 470, find total money invested by him.

(a) Rs. 3000 (b) Rs. 6000 (c) Rs. 12000 (d) Rs. 15000

Solution:

(c) LCM of 3, 4 and 12 is 12.

We assume that the total sum = Rs. 1200

Money invested in three parts be Rs. 400, Rs. 300 and Rs. 500 respectively.

∴ Simple interest on first part = 400 × 5% = Rs. 20

Simple interest on second part = 300 × 4% = Rs. 12

∴ Simple interest on third part = 500 × 3% = Rs. 15

Total Simple interest = Rs. (20 + 12 + 15) = Rs. 47

Actual Simple interest = Rs. 470 (i.e. 10 times of 47)

∴ Amount invested = 10 × Rs. 1200 = Rs. 12000

40. A man lent Rs. 2000 – partly at 5% and the balance at 4%. If he receives Rs. 92 towards annual interest, find the amount lent at 5%.

(a) Rs. 800 (b) Rs. 900 (c) Rs. 1000 (d) Rs. 1200

Solution:

(d) Let the whole amount is invested at 4% p.a.

Then Simple interest = $\frac{2000 \times 4 \times 1}{100}$ = Rs. 80

This interest is short from actual interest by Rs. 92 – Rs. 80 = Rs. 12

The difference is because the amount is also invested at 5% p.a.

Difference in two rates of interest = 5% – 4% = 1% p.a.

Difference in rate is 1%, and difference in interest = Rs. 12

∴ Amount invested at 5% = 12 × $\frac{100}{1}$ = Rs. 1200

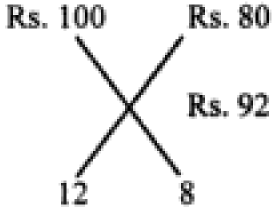
Alternative Method:

Simple interest on Rs. 2000 at 5% p.a. = $\frac{2000 \times 5 \times 1}{100}$ = Rs. 100

Simple interest on Rs. 2000 at 4% p.a. = $\frac{2000 \times 4 \times 1}{100}$ = Rs. 80

By Alligation Method:

At 5% At 4%



∴ Ratio between amounts lent at 5% and 4% = 12 : 8 = 3 : 2

∴ Sum lent at 5% = $\frac{3}{5}$ × 2000 = Rs. 1200

41. A man invested Rs. 3500 into two parts at Simple interest – one at 4% and other at 6% p.a. If his yearly income from investment is Rs. 170, how much did he invest at 6%?

(a) Rs. 1500 (b) Rs. 1750 (c) Rs. 2000 (d) Rs. 2500

Solution:

(a) Let the money was invested at 4% p.a.

Then interest for one year = 4% of Rs. 3500 = Rs. 140

Short from actual interest = Rs. 170 – Rs. 140 = Rs. 30

Chapter 25

COMPOUND INTEREST

In case of compound interest, amount for the first year becomes the principal for the second year, amount for the second year becomes the principal for the third year and so on.

$$\text{Compound interest} = \text{Principal}\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}} - \text{Principal}$$

Note: If interest is compounded half-yearly, rate of interest is halved i.e. rate becomes $\frac{r}{2}$ and time is doubled i.e. time becomes 2t, and

If interest is compounded quarterly, rate of interest is divided by 4 i.e. rate becomes $\frac{r}{4}$ and time is multiplied by 4 i.e. time becomes 4t and so on.

It is recommended to convert r % into fraction for solving problems on compound interest.

Note: In solving problems based on compound interest, we are frequently required to solve $\left(1+\frac{r}{100}\right)$.

∴ It is better to remember the following values instead of solving them each time.

Rate = 2.5% p.a. $\Rightarrow (1+r) = 1+\frac{1}{40} = \frac{41}{40}$

Rate = 4% p.a. $\Rightarrow (1+r) = 1+\frac{1}{25} = \frac{26}{25}$

Rate = 5% p.a. $\Rightarrow (1+r) = 1+\frac{1}{20} = \frac{21}{20}$

Rate = 10% p.a. $\Rightarrow (1+r) = 1+\frac{1}{10} = \frac{11}{10}$

Rate = 20% p.a. $\Rightarrow (1+r) = 1+\frac{1}{5} = \frac{6}{5}$

Rate = 25% p.a. $\Rightarrow (1+r) = 1+\frac{1}{4} = \frac{5}{4}$

Rate = 50% p.a. $\Rightarrow (1+r) = 1+\frac{1}{2} = \frac{3}{2}$

OTHER FORMULAE

1. Amount = Principal $\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}}$

2. Principal = $\dfrac{\text{Amount}}{\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}}}$

3. $\dfrac{\text{Amount}}{\text{Principal}} = \left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}}$

4. If a sum of money becomes 'x' times in 'n' years.
   Then it will become x^a times in 'an' years.

5. If **rate of compound interest differs from year to year**, then

$$\text{Amount} = \text{Principal} \times \frac{100 + R_1}{100} \times \frac{100 + R_2}{100} \times \frac{100 + R_3}{100} \ldots\ldots\ldots\ldots$$

6. If present value of a machine is Rs. P and **annual depreciation** is r%, then,

$$\text{Value of machine after 'n' years} = P \times \left(1 - \frac{r}{100}\right)^n$$

$$\text{Value of machine 'n' years before} = \frac{P}{\left(1 - \frac{r}{100}\right)^n}$$

7. A sum is divided into two parts and invested at a certain rate of compound interest in such a way that amount received from first part after 'm' years and amount received from second part after 'n' years are equal. What is the ratio of amounts invested into two parts?

Solution:

Let money divided into two parts is Rs. x and Rs. y.

After 'm' and 'n' years the amount will become $x(1 + r)^m$ and $y(1 + r)^n$ respectively.

But both the amounts are equal.

$$\therefore\ x(1 + r)^m = y(1 + r)^n$$

$$\therefore\ \frac{x}{y} = (1 + r)^{n-m}$$

8. A man borrows Rs. A, which is to be discharged with compound interest @ r% p.a. in 'n' equal yearly instalments, payable at the end of each year. Find the amount of each instalment.

Solution:

Let amount of each instalment = Rs. x

$$\text{Then principal of first instalment} = x \div \left(1 + \frac{r}{100}\right)$$

$$= x \div \frac{a}{b} = x\left(\frac{b}{a}\right)$$

Note : We substituted $\left(1 + \frac{r}{100}\right)$ with $\frac{a}{b}$

$$\text{Similarly principal of second instalment} = x\left(\frac{b}{a}\right)^2$$

$$\text{Principal of third instalment} = x\left(\frac{b}{a}\right)^3$$

$$\text{Principal of } n^{th} \text{ instalment} = x\left(\frac{b}{a}\right)^n$$

$\therefore$ Ratio between the principals

$$= x\left(\frac{b}{a}\right) : x\left(\frac{b}{a}\right)^2 : x\left(\frac{b}{a}\right)^3 : \ldots\ldots\ldots\ldots : x\left(\frac{b}{a}\right)^n$$

$$= 1 : \frac{b}{a} : \left(\frac{b}{a}\right)^2 : \left(\frac{b}{a}\right)^3 : \ldots\ldots\ldots\ldots : \left(\frac{b}{a}\right)^{n-1}$$

Note : We have divided each ratio by $x\left(\frac{b}{a}\right)$

9. Difference between compound interest on a sum for $1\frac{1}{2}$ years at r% p.a. when interest is compounded yearly and half-yearly is $P\left(1+\frac{r}{2}\right)\left(\frac{r}{2}\right)^2$

SOLVED EXERCISE

1. Find compound interest on Rs. 1600 at 2.5% p.a. for 2 years.

(a) Rs. 80 (b) Rs. 81 (c) Rs. 82 (d) Rs. 1681

Solution:

(b) $(1+r) = 1 + \frac{1}{40} = \frac{41}{40}$

$\therefore$ Amount $= 1600 \times \frac{41}{40} \times \frac{41}{40} = 1681$

$\therefore$ Compound interest = Rs. 1681 – Rs. 1600 = Rs. 81

Trick:

$(1+r) = 1 + \frac{1}{40} = \frac{41}{40}$

$\therefore$ Amount $= \left(\frac{41}{40}\right)^2$

$\therefore$ Compound interest $= (41^2 - 40^2)$ = Rs. $(41 + 40)$ = Rs. 81 on Rs. 40^2 = Rs.1600

Hint: If question is silent, we assume that interest is compounded annually.

2. Find compound interest on Rs. 8000 at 5% p.a. for 3 years.

(a) Rs. 1200 (b) Rs. 1250 (c) Rs. 1261 (d) Rs. 9261

Solution:

(c) $(1+r) = 1 + \frac{1}{20} = \frac{21}{20}$

$\therefore$ Amount $= 8000 \times \left(\frac{21}{20}\right)^3 = 9261$

Compound interest = Amount – principal = Rs. 9261 – Rs. 8000 = Rs. 1261

3. Rs. 10000 is borrowed at 20% p.a., interest compounded half-yearly. Find the amount repayable after one year.

(a) Rs. 11000 (b) Rs. 12000 (c) Rs. 12100 (d) Rs. 14400

Solution:

(c) Principal = Rs. 10000

Time = 1 year = 2 half years

Rate = 20% p.a. = 10% per half-yearly

$\therefore$ Amount = $10000 \times \frac{11}{10} \times \frac{11}{10} = 12100$

4. Find compound interest on Rs. 8000 at 20% p.a. for 9 months, interest being compounded quarterly.

(a) Rs. 1000 (b) Rs. 1200 (c) Rs. 1250 (d) Rs. 1261

Solution:

(d) Principal = Rs. 8000

Time = 9 months = 3 quarters

Rate = 20% p.a. = 5% per quarter (since, 1 year = 4 quarters)

$\therefore$ Amount = $8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} = 9261$

$\therefore$ Compound interest = 9261 – 8000 = Rs. 1261

5. Find compound interest on Rs. 25000 at 20% p.a. for $2\frac{1}{2}$ years, if interest is compounded annually.

(a) Rs. 39600 (b) Rs. 14600 (c) Rs. 37500 (d) Rs. 12500

Solution:

(b) Amount = $25000 \times \left(1 + \frac{20}{100}\right)^2 \times \left(1 + \frac{10}{100}\right)^1$

$= 25000 \times \left(\frac{6}{5}\right)^2 \times \frac{11}{10} = 39600$

$\therefore$ Compound interest = 39600 – 25000 = 14600

Hint: Rate is halved for 3rd year as time is half of one year.

6. Find compound interest on Rs. 10000 at 10% p.a. for 4 years, if interest is compounded annually.

(a) Rs. 4000 (b) Rs. 4400 (c) Rs. 4641 (d) Rs. 5200

Solution:

(c) Amount = Rs. $10000 \times \left(\frac{11}{10}\right)^4 = 14641$

$\therefore$ Compound interest = Rs. 14641 – Rs. 10000 = Rs. 4641

Approximate Method:

We know that,

Compound interest = Simple interest + Interest on simple interest

Simple interest = $10000 \times \frac{1}{10} \times 4$ = Rs. 4000

$\therefore$ Simple interest for one year = Rs. 4000 ÷ 4 = Rs. 1000.

$\therefore$ Interest on interest

= 0 + 10% of Rs. 1000 + 10% of Rs. 2000 + 10% of Rs. 3000 + interest on interest

= 0 + 100 + 200 + 300 + interest on this amount

= 600 + interest on this amount

∴ Compound interest = Rs. 4000 + Rs. 600 + interest on Rs. 600

= Rs. 4600 + Interest on Rs. 600

Out of given options, amount nearest to it is Rs. 4641.

7. Find Compound interest on Rs. 10000 for 3 years if rate of interest is 5%, 10% and 20% for first, second and third years respectively.

(a) Rs. 3310 (b) Rs. 3500 (c) Rs. 3860 (d) Rs. 3980

Solution:

(c) Amount after 3 years

$$= \text{Principal} \times \frac{100 + R_1}{100} \times \frac{100 + R_2}{100} \times \frac{100 + R_3}{100}$$

$$= \text{Rs.}10000 \times \frac{21}{20} \times \frac{11}{10} \times \frac{6}{5} = \text{Rs. } 13860$$

∴ Compound interest = Rs. 13860 – Rs. 10000 = Rs. 3860

Hint : To find 10 × 21 × 11 × 6, we first find 21 × 6 = 126;

Now 126 × 11 = 1386; 1386 × 10 = 13860

8. If compound interest for second year on a certain sum at 10% p.a. is Rs. 132, find the principal.

(a) Rs. 600 (b) Rs. 1000 (c) Rs. 1100 (d) Rs. 1200

Solution:

(d) Let principal = Rs. 100

$$\text{Amount after two years} = 100 \times \left(\frac{11}{10}\right)^2 = \text{Rs. } 121$$

∴ Compound interest for second year = Rs. 121 – Rs. 110 = Rs. 11

But actual compound interest for second year

= Rs. 132 (i.e. 12 times of Rs. 11)

∴ Principal = 12 × Rs. 100 = Rs. 1200

9. If compound interest on a certain sum for 2 years at 10% p.a. is Rs. 315, what is the sum lent ?

(a) Rs. 1500 (b) Rs. 1800 (c) Rs. 2000 (d) Rs. 2250

Solution :

$$\text{(a) Compound Interest } = \text{Principal}\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}} - \text{Principal}$$

$$\therefore \quad 315 = \text{Principal} \times \left(\frac{121}{100} - 1\right)$$

$$\therefore \quad 315 = \text{Principal} \times \frac{21}{100}$$

$\therefore$ Principal $= 315 \times \frac{100}{21} =$ Rs.1500

10. A certain sum of money invested at a certain rate of compound interest doubles in 5 years. In how many years will it become 4 times?

(a) 10 years (b) 12 years (c) 15 years (d) 20 years

Solution:

(a) $2^2 = 4$.

$\therefore$ The amount will become 4 times in 2 × 5 = 10 years.

11. A certain sum of money invested at a certain rate of compound interest doubles in 6 years. In how many years will it become 8 times?

(a) 12 years (b) 18 years (c) 24 years (d) 30 years

Solution:

(b) $2^3 = 8$.

$\therefore$ The amount will become 8 times in 3 × 6 = 18 years.

12. An amount is lent at 15% p.a. compound interest for 2 years. Find the percent increase in the amount at the end of 2 years.

(a) 22.5% (b) 30% (c) 32.25% (d) 35.5%

Solution:

(c) In 2 years, Re.1 will become $\left(1+\frac{15}{100}\right)^2$ times of itself

$= \left(\frac{115}{100}\right)^2$ times of itself $= \frac{13225}{10000}$ times of itself

$\therefore$ Increase $= \frac{13225}{10000} - 1 = \frac{3225}{10000} = 32.25\%$

Hint: $115^2 = 11 \times 12 \mid 25 = 13225$

Note: If denominator of a fraction is a number which have zeros at the end and the answer is required to be calculated in decimals, then it is advisable not to reduce the fraction to the lowest terms.

$\therefore$ The fractions are not reduced to the lowest terms in the solution.

But the fraction can be reduced if there is some common factor other than zeros in the denominator.

13. A person returns Rs. 1331 after 3 years from the date of borrowing. If the rate of interest is 10% p.a., interest compounded annually, find the amount borrowed.

(a) Rs. 900 (b) Rs. 1000 (c) Rs. 1100 (d) Rs. 1200

Solution:

(b) Principal $= \frac{\text{Amount}}{(1 + \text{Rate})^{\text{Time}}} = \frac{1331}{\left(\frac{11}{10}\right)^3}$

$= 1331 \times \left(\frac{10}{11}\right)^3 =$ Rs. 1000

14. In how many years will Rs. 400 amount to Rs. 441 at 5% compound interest?

(a) 1 year (b) $1\frac{1}{2}$ years (c) 2 years (d) $2\frac{1}{2}$ years

Solution:

(c) $\frac{\text{Amount}}{\text{Principal}} = \left(1 + \frac{\text{Rate}}{100}\right)^{\text{Time}}$

$\therefore \frac{441}{400} = \left(\frac{21}{20}\right)^x$

$\therefore \left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^x$

$\therefore$ Time = 2 years

15. The population of a village increases @ 5% p.a. If present population is 8000, after how many years the population will be 9261?

(a) 2 years (b) 3 years (c) $3\frac{1}{2}$ years (d) 4 years

Solution:

(b) Let population become 9261 in 'x' years.

$\frac{\text{Amount}}{\text{Principal}} = \left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}}$

$\therefore \frac{9261}{8000} = \left(\frac{21}{20}\right)^x$

$\therefore \left(\frac{21}{20}\right)^3 = \left(\frac{21}{20}\right)^x$

$\therefore$ Time = 3 years

16. If Rs. 8000 amounts to Rs. 8820 in 2 years at compound interest, find rate of interest p.a.

(a) 5% (b) 5.125% (c) 10% (d) 10.5%

Solution:

(a) $\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}} = \frac{\text{Amount}}{\text{Principal}}$

$\therefore (1 + r)^2 = \frac{8820}{8000} = \frac{441}{400} = \left(\frac{21}{20}\right)^2$

$\therefore 1 + r = \frac{21}{20}$

$\therefore r = \frac{21}{20} - 1 = \frac{1}{20} = 5\%$

17. At what rate percent of compound interest will a sum be 16 times of itself in 4 years ?

(a) 20% (b) 25% (c) 50% (d) 100%

Solution :

(d) Let Principal = Re.1, then Amount = Rs. 16

$\therefore\ (1+r)^4 = \frac{16}{1}$.

$\therefore\ 1+r=2$

$\therefore\ r = 1 = 100\%$

18. At what rate per cent of compound interest, a sum of Rs. 2000 will amount to Rs. 2662 in 3 years?

(a) 5% (b) 10% (c) 11% (d) None of these

Solution:

(b) $\left(1+\frac{\text{Rate}}{100}\right)^{\text{Time}} = \frac{\text{Amount}}{\text{Principal}}$

$\therefore\ (1+r)^3 = \frac{2662}{2000} = \frac{1331}{1000} = \left(\frac{11}{10}\right)^3$

$\therefore\ 1+r = \frac{11}{10}$

$\therefore\ r = \frac{11}{10} - 1 = \frac{1}{10} = 10\%$

19. A man invested Rs. 16000 at compound interest for 3 years, interest compounded annually. If he got Rs. 18522 at the end of 3 years, what is rate of interest?

(a) 4% (b) 5% (c) 6% (d) 7%

Solution:

(b) $(1+r)^3 = \frac{18522}{16000} = \frac{9261}{8000} = \left(\frac{21}{20}\right)^3 = \left(1+\frac{1}{20}\right)^3$

$\therefore$ Rate of interest = $\frac{1}{20}$ = 5%

Trick :

Compound interest = Rs. 18522 – Rs. 16000 = Rs. 2522

Let the amount is invested at 1% p.a. simple interest.

Then simple interest of 3 years = 16000 × 1% × 3 = Rs. 480

$\therefore$ Rate of interest = $\frac{2522}{480}$ = 5 + (Remainder is Rs. 122)

Note: If Rate is 6%, then simple interest = 480 × 6 = 2880, which is more than the given compound interest which is not possible.

$\therefore$ Rate of interest $\geq$ 6% is not possible.

$\therefore$ Rate of interest is 5% p.a.

20. Height of a tree is 1210 cm. What was its height two years ago, if it increases at the rate of 10% p.a.?

(a) 1331 cm (b) 1000 cm (c) 900 cm (d) None of these

25. Compound interest on a certain sum for two successive years is Rs. 150 and Rs. 153 respectively. Find rate of interest.

(a) 2% (b) 3% (c) 5% (d) 6%

Solution:

(a) Difference in compound interest of two successive years

= Rs. 153 – Rs. 150 = Rs. 3

The difference is equal to simple interest on Rs. 150 for one year.

$$\therefore \text{Rate of interest} = \frac{3 \times 100}{150} = 2\% \text{ p.a.}$$

26. If a certain sum on compound interest becomes Rs. 2000 in 2 years and Rs. 2420 in 4 years. Rate of interest p.a. is:

(a) 6.25% (b) 10% (c) 12.5% (d) 25%

Solution:

(b) Amount after 2 years = Rs. 2000

Amount after 4 years = Rs. 2420

∴ Compound interest on Rs. 2000 = Rs. 2420 – Rs. 2000 = Rs. 240

Time = 4 years – 2 years = 2 years

$$\left(1 + \frac{\text{Rate}}{100}\right)^{\text{Time}} = \frac{\text{Amount}}{\text{Principal}}$$

$$\therefore (1 + r)^2 = \frac{2420}{2000} = \frac{121}{100} = \left(\frac{11}{10}\right)^2$$

$$\therefore 1 + r = \frac{11}{10}$$

$$\therefore r = \frac{11}{10} - 1 = \frac{1}{10} = 10\%$$

27. A certain sum of money amounts to Rs. 640 after 2 years and Rs. 720 after 3 years. How much will it amount to in 4 years ?

(a) Rs. 760 (b) Rs. 800 (c) Rs. 810 (d) Rs. 825

Solution :

(c) In 1 year (i.e. 3 – 2), the amount becomes $\frac{720}{640}$ of the principal

∴ In 4 years, Rs. 720 will amount to $720 \times \frac{720}{640} = \text{Rs. } 810$

28. A sum of money amounts to Rs. 2880 in 2 years and 3456 in 3 years at compound interest. Find the sum.

(a) Rs. 1800 (b) Rs. 2000 (c) Rs. 2400 (d) Rs. 2500

Solution:

(b) Rs. 2880 amounts to Rs. 3456 in one year.

The sum amounts to $\frac{3456}{2880} = \frac{6}{5}$ times of itself

$\therefore$ Principal $= 2880 \div \left(\frac{6}{5}\right)^2 = 2880 \times \frac{5}{6} \times \frac{5}{6} =$ Rs. 2000

29. Find the rate percent of compound interest at which a sum of money will amount to $\frac{9}{4}$ times of itself in 2 years?

(a) 10% (b) 25% (c) 50% (d) 75%

Solution:

(c) Let principal = Rs. 4

Then amount $= \frac{9}{4} \times$ Rs. 4 = Rs. 9

$\therefore (1 + r)^2 = \frac{9}{4} = \left(\frac{3}{2}\right)^2$

$\therefore (1 + r) = \frac{3}{2}$

$\therefore r = \frac{3}{2} - 1 = \frac{1}{2} = 50\%$

30. Find the rate percent of compound interest at which a sum of money will become 1.44 times itself in 2 years?

(a) 10% (b) 20% (c) 25% (d) 33.33%

Solution:

(b) Let principal = Re. 1

Then amount = Rs. 1.44.

$\therefore (1 + r)^2 = 1.44$

$\therefore (1 + r) = \sqrt{1.44} = 1.2$

$\therefore r = 1.2 - 1 = 0.2$

$\therefore$ Rate of interest = 0.2 = 20% p.a.

31. A man borrows Rs. 2100 and undertakes to pay back with compound interest @ 10% p.a. in 2 equal yearly instalments at the end of first and second year. What is the amount of each instalment?

(a) Rs. 1000 (b) Rs. 1100 (c) Rs. 1200 (d) Rs. 1210

Solution:

(d) $(1 + r) = 1 + \frac{1}{10} = \frac{11}{10}$

Ratio of principals of two instalments $= 1 : \frac{10}{11} = 11 : 10$

Sum of ratios = 11 + 10 = 21

$\therefore$ Principal of first instalment $= 2100 \times \frac{11}{21} =$ Rs. 1100

$\therefore$ Instalment = Principal of first instalment × (1 + r)

$= 1100 \times \frac{11}{10} =$ Rs. 1210

32. A man borrows Rs. 820 and undertakes to pay back with compound interest @ 5% p.a. in 2 equal yearly instalments at the end of first and second year. What is the amount of each instalment?

(a) Rs. 400 (b) Rs. 420 (c) Rs. 441 (d) Rs. 450

Solution:

(c) $(1 + r) = 1 + \frac{1}{20} = \frac{21}{20}$

Ratio of principals of two instalments = $1 : \frac{20}{21} = 21 : 20$

Sum of ratios = 21 + 20 = 41

∴ Principal of first instalment = $\frac{21}{41} \times 820$ = Rs. 420

∴ Instalment = Principal of first instalment × (1 + r)

$= 420 \times \frac{21}{20}$ = Rs. 441

33. A man borrows Rs. 1820 and undertakes to pay back with compound interest @ 20% p.a. in 3 equal yearly instalments at the end of first, second and third years. What is the amount of each instalment?

(a) Rs. 500 (b) Rs. 560 (c) Rs. 750 (d) Rs. 864

Solution:

(d) $(1 + r) = 1 + \frac{1}{5} = \frac{6}{5}$

Ratio of principals for three years = $1 : \frac{5}{6} : \left(\frac{5}{6}\right)^2$

$= 6^2 : 6 \times 5 : 5^2$ (On multiplying each ratio by 6^2)

= 36 : 30 : 25

Sum of the ratios = 36 + 30 + 25 = 91

∴ Principal of first instalment = $\frac{36}{91} \times 1820$ = Rs. 720

∴ Instalment = Principal of first instalment × (1 + r)

$= 720 \times \frac{6}{5}$ = Rs. 864

34. A man borrows a certain sum and pays it back in two equal annual instalments. If he pays back Rs. 676 annually and rate of interest is 4%, what is the sum borrowed ?

(a) Rs. 1352 (b) Rs. 1300 (c) Rs. 1275 (d) Rs. 1250

Solution :

(c) $(1 + r) = 1 + \frac{1}{25} = \frac{26}{25}$

∴ Principal of first instalment $= \frac{25}{26} \times 676$ = Rs.650

And Principal of second instalment $= 676 \times \left(\frac{25}{26}\right)^2$ = Rs.625

∴ Sum borrowed = Rs. 650 + Rs. 625 = Rs. 1275

35. A certain sum is to be divided between A and B so that after 5 years the amount received by A is equal to the amount received by B after 7 years. The rate of interest is 10%, interest compounded annually. Find the ratio of amounts invested by them.

(a) 10 : 11 (b) 100 : 121 (c) 11 : 10 (d) 121 : 100

Solution : (d) $(1+r) = 1 + \frac{1}{10} = \frac{11}{10}$

Let the sum (principal) received by A and B are x and y.

Then $\frac{x}{y} = \left(\frac{11}{10}\right)^{7-5} = \left(\frac{11}{10}\right)^2 = \frac{121}{100}$

Hence the ratio in which the sum is divided = 121 : 100.

36. A father divides Rs. 5100 between his two sons, Mohan and Sohan who are 23 and 24 at present in such a way that if their shares are invested at compound interest @ 4% p.a., they will receive equal amount on attaining the age of 26 years. Find Mohan's share.

(a) Rs. 2400 (b) Rs. 2500 (c) Rs. 2550 (d) Rs. 2600

Solution : (b) $(1+r) = 1 + \frac{1}{25} = \frac{26}{25}$

Let Mohan and Sohan receives Rs. x and Rs. y respectively at present.

Then $\frac{x}{y} = \left(\frac{26}{25}\right)^{2-3} = \left(\frac{26}{25}\right)^{-1} = \frac{25}{26}$

∴ Mohan's share = $\frac{25}{51}$ × Rs. 5100 = Rs. 2500

37. A certain sum, invested at compound interest becomes Rs. 3000 in 3 years and Rs. 6000 in 6 years. Find the amount after 9 years.

(a) Rs. 7500 (b) Rs. 9000 (c) Rs. 10000 (d) Rs. 12000

Solution:

(d) In 3 years (i.e. 6 years – 3 years), principal doubles itself i.e. from Rs. 3000 to Rs. 6000.

In the next 3 years, amount will become double of Rs. 6000.

∴ Amount after 9 years will become 2 × Rs. 6000 = Rs. 12000.

38. Population of a town increases at a certain rate per cent per annum. Present population of the town is 3600 and in 5 years' time it becomes 4800. How much will it be in 10 years time?

(a) 5000 (b) 6000 (c) 6400 (d) 7000

Solution:

(c) Let rate of increase in population = r % p.a.

Then $4800 = 3600\left(1+\frac{r}{100}\right)^5$

$\therefore \left(1+\frac{r}{100}\right)^5 = \frac{4800}{3600} = \frac{4}{3}$

Population in the next 5 years will become $4800 \times \frac{4}{3} = 6400$.

39. Present population of a town is 12000. Increasing at a certain rate, the population becomes 27000 after six years. What will the size of population after 3 years from now?

(a) 15000 (b) 18000 (c) 19500 (d) 20000

Solution:

(b) Population becomes $\frac{27000}{12000} = \frac{9}{4}$ times of itself in 6 years.

$\therefore$ In 3 years it will become $\sqrt{\frac{9}{4}} = \frac{3}{2}$ times

$\therefore$ Population after 3 years = $12000 \times \frac{3}{2} = 18000$

40. Find the difference between compound interest on Rs. 1000 for $1\frac{1}{2}$ years at 20% p.a. when interest is compounded annually and half-yearly respectively.

(a) Rs. 11 (b) Rs. 10 (c) Rs. 21 (d) Rs. 20

Solution:

(a) Difference between compound interests for $1\frac{1}{2}$ years

$$= P\left(1 + \frac{r}{2}\right)\left(\frac{r}{2}\right)^2$$

$$= 1000 \times \frac{11}{10} \times \frac{1}{10} \times \frac{1}{10} = \text{Rs. } 11$$

Chapter 26

RELATION BETWEEN C.I. AND S.I.

In this chapter, we will discuss relation among Compound Interest, Simple Interest and the difference between Simple Interest and Compound Interest.

IMPORTANT RULES

1. How to **find Compound Interest** for two years, when **Simple interest is given**:

Method:

Compound Interest

= Total Simple interest + (Simple interest for one year × Rate of Interest)

Note: Unless mentioned otherwise, it is presumed that interest is compounded annually.

2. **Difference between Simple interest and Compound Interest for two years, when Principal is given**:

$$\text{Difference} = \text{Principal} \times \left(\frac{\text{Rate}}{100}\right)^2$$

$$\therefore \text{Principal} = \text{Difference} \times \left(\frac{1}{r}\right)^2$$

Logic: Compound interest for first year = Simple Interest for first year

Compound Interest for second year

= Simple Interest for first year + Interest on Simple Interest for the first year

∴ Difference = Interest on Simple interest of first year

Proof: Compound interest for two years = $P(1 + r)^2 - P$

Simple interest for two years = $P \times r \times 2 = 2Pr$

Difference between compound interest and simple interest for two years

$= [P(1 + r)^2 - P] - (2Pr)$

$= P(1 + 2r + r^2) - P - (2Pr)$

$= P + 2Pr + Pr^2 - P - 2Pr = Pr^2$

Hence difference between compound interest and simple interest @ r % p.a. for two years = $P(r^2)$

$$\therefore \text{Principal} = \text{Difference} \times \left(\frac{1}{r}\right)^2$$

Similarly we can find **difference between compound interest and simple interest for 3 and 4 years**.

Difference for 3 years = $Pr^2(3 + r)$

Difference for 4 years = $Pr^2(6 + 4r + r^2)$

Here 'r' means $\frac{\text{Rate of interest}}{100}$

3. Difference between Compound Interest and Simple interest, when Simple Interest is given:

(a) For 2 years: $\frac{1}{2}$ r of simple interest

(b) For 3 years: $\frac{1}{3}$ r (3 + r) of simple interest

Note: 'r' is $\frac{\text{Rate of interest}}{100}$

Proof:

(a) Difference between compound interest and simple interest for 2 years

$= P(r^2) = Pr(r)$

But $Pr = \text{Principal} \times \frac{\text{Rate}}{100} = \text{Simple Interest for one year}$

$= \frac{1}{2} \times \text{Total Simple Interest}$

∴ Difference between compound interest and simple interest for 2 years

$= Pr(r) = \frac{1}{2} \times \text{Simple Interest} \times (r)$

$= \frac{1}{2}$ r of Simple Interest

(b) Difference between compound interest and simple interest for 3 years

$= Pr^2(3 + r) = Pr(r)(3 + r)$

Now $Pr = \text{Principal} \times \frac{\text{Rate}}{100} = \text{Simple Interest for one year}$

$= \frac{1}{3} \times \text{Total Simple Interest}$

∴ Difference between compound interest and simple interest for 3 years

$= Pr(r)(3 + r) = \frac{1}{3} \times \text{Simple Interest} \times (r)(3 + r)$

$= \frac{1}{3}$ r (3 + r) of Simple Interest

4. How **to find simple interest** when **compound interest is given**?

Method:

If difference between compound interest and simple interest is $\frac{1}{x}$ of Simple interest, then

Difference between compound interest and simple interest is $\frac{1}{x-1}$ of Compound Interest.

Hint : Compound Interest = Simple Interest + Difference

∴ If we shift base from Simple Interest to Compound Interest, smaller fraction is required and vice-versa.

SOLVED EXERCISE

1. Difference between Compound Interest and Simple Interest on Rs. 8000 for 2 years at 10% p.a. is:

(a) Nil (b) Rs. 80 (c) Rs. 160 (d) Rs. 800

Solution:

(b) Difference between Compound Interest and Simple Interest for 2 years

$$= Pr^2 = \text{Rs. } 8000 \times \frac{1}{10} \times \frac{1}{10} = \text{Rs. } 80$$

2. Find the difference between Compound Interest and Simple Interest on Rs. 4000 for 1 year at 10% p.a., if the interest is compounded half-yearly.

(a) Nil (b) Rs. 5 (c) Rs. 10 (d) Rs. 40

Solution:

(c) Since interest is compounded half-yearly

$\therefore$ Rate of interest is halved and time is doubled.

$$\therefore \text{Rate} = \frac{10}{2}\% = 5\% = \frac{1}{20}$$

Time = 2 × 1 = 2 half-years.

$\therefore$ Difference between Compound Interest and Simple Interest

$$= \text{Rs. } 4000 \times \frac{1}{20} \times \frac{1}{20} = \text{Rs. } 10$$

3. If difference between Compound Interest and Simple Interest on a certain amount at 5% p.a. for two years is Rs.30. Find the Principal.

(a) Rs. 10000 (b) Rs. 12000 (c) Rs. 12500 (d) Rs. 15000

Solution:

(b) $r = 5\% = \frac{1}{20}$

$$\therefore \frac{1}{r} = 1 \div \frac{1}{20} = 20$$

$$\therefore \text{Principal} = \text{Difference} \times \left(\frac{1}{r}\right)^2 = \text{Rs. } 30 \times 20 \times 20 = \text{Rs. } 12000$$

4. Two persons lent equal amount of money at 10% p.a., first at Compound Interest and second at Simple Interest. If first person gets Rs. 20 more than the second after a period of 2 years. Find sum lent by each of them.

(a) Rs. 1000 (b) Rs. 2000 (c) Rs. 3000 (d) Rs. 4000

Solution:

(b) $r = 10\% = \frac{1}{10}$

$$\therefore \frac{1}{r} = 1 \div \frac{1}{10} = 10$$

$$\therefore \text{Sum lent} = \text{Difference} \times \left(\frac{1}{r}\right)^2 = \text{Rs. } 20 \times 10 \times 10 = \text{Rs. } 2000$$

5. Find the difference between Compound Interest and Simple Interest on Rs. 1000 for 3 years at 10% p.a., if interest is compounded annually.

(a) Rs. 30 (b) Rs. 31 (c) Rs. 50 (d) Rs. 331

Solution:

(b) Difference between Compound Interest and Simple Interest for 3 years

$$= Pr^2(3 + r) = \text{Rs. } 1000 \times \frac{1}{10} \times \frac{1}{10} \times \left(3 + \frac{1}{10}\right) = \text{Rs. } 31$$

6. Find the difference between Compound Interest and Simple Interest on Rs. 10000 for 4 years at 10% p.a., if interest is compounded annually.

(a) Rs. 4641 (b) Rs. 4000 (c) Rs. 641 (d) Rs. 600

Solution:

(c) Difference between Compound Interest and Simple Interest for 4 years

$= Pr^2 (6 + 4r + r^2) = 10000 \times \frac{1}{10} \times \frac{1}{10} \times \left(6 + \frac{4}{10} + \frac{1}{100}\right)$

$= 10000 \times \frac{1}{100} \times \frac{641}{100} =$ Rs. 641

7. If Simple Interest on a certain sum at 10% p.a. for 2 years is Rs. 80. The Compound Interest on the same sum for the same period is:

(a) Rs. 80 (b) Rs. 82 (c) Rs. 84 (d) Rs. 88

Solution:

(c) Compound interest = Simple Interest + $\frac{\text{Simple Interest}}{2}$ × Rate

$= 80 + \frac{80}{2} \times 10\% =$ Rs. 84

8. If Simple interest on a certain sum for 2 years @ 10% p.a. is Rs. 200, the Compound Interest will be?

(a) Rs. 200 (b) Rs. 210 (c) Rs. 220 (d) Rs. 225

Solution:

(b) Compound interest = Simple Interest + $\frac{\text{Simple Interest}}{2}$ × Rate

= Rs. 200 + $\frac{200}{2} \times 10\%$ = Rs. 210

9. If simple interest and compound interest on a certain sum for 2 years are Rs. 400 and Rs. 440 respectively. What is the rate of interest?

(a) 5% (b) 10% (c) 20% (d) Data Insufficient

Solution:

(c) Simple interest for first year and second year = Rs. $\frac{400}{2}$ = Rs. 200 each

Difference between Compound Interest and Simple Interest

= Rs. 440 – Rs. 400 = Rs. 40

∴ Rs. 40 is interest on simple interest for one year i.e. interest on Rs. 200.

∴ Rate of interest = $\frac{40 \times 100}{200 \times 1}$ = 20% p.a.

10. If simple interest and compound interest on a certain sum for 2 years is Rs. 160 and Rs. 168 respectively. What is the Principal?

(a) Rs. 800 (b) Rs. 1600 (c) Rs. 2000 (d) Data Insufficient

Solution:

(a) Simple interest for first year and second year = Rs. $\frac{160}{2}$ = Rs. 80 each

∴ Rs. 8 (i.e. Rs. 168 – Rs. 160) is interest on simple interest for one year i.e. interest on Rs. 80

Alternative Method:

Compound Interest for 3 years

$$= \text{Simple Interest} + \text{Simple Interest (r)} + \frac{SI}{3}(r)^2$$

$$= 300 + 300 \times \frac{1}{10} + 100 \times \frac{1}{10} \times \frac{1}{10}$$

$$= 300 + 30 + 1 = \text{Rs. } 331$$

14. If simple interest for 3 years @ 5% is Rs. 1200, Compound Interest is:

(a) Rs. 1206 (b) Rs. 1241 (c) Rs. 1251 (d) Rs. 1261

Solution:

(d) Difference between Compound Interest and Simple Interest for 3 years

$$= \frac{1}{3}r\,(3 + r) = \frac{1}{3} \times \frac{1}{20} \times \frac{61}{20} = \frac{61}{1200} \text{ of Simple Interest}$$

$$= \text{Rs. } 1200 \times \frac{61}{1200} = \text{Rs. } 61$$

$\therefore$ Compound Interest = Rs. 1200 + Rs. 61 = Rs. 1261

Hint: $r = 5\% = \frac{1}{20}$

$\therefore (3 + r) = 3 + \frac{1}{20} = \frac{61}{20}$

Alternative Method:

Compound Interest = Simple Interest + Simple Interest (r) + $\frac{SI}{3}(r)^2$

$$= 1200 + 1200 \times \frac{1}{20} + 400 \times \frac{1}{20} \times \frac{1}{20}$$

$$= 1200 + 60 + 1 = \text{Rs. } 1261$$

15. If Compound Interest on a certain sum for 2 years @ 5% p.a. is Rs. 328, the Simple interest will be?

(a) Rs. 300 (b) Rs. 310 (c) Rs. 320 (d) Rs. 325

Solution:

(c) Suppose Compound Interest for first year = Rs. 100

Then Compound Interest for second year = Rs. 105

Total Compound Interest for two years = (Rs.100 + Rs.105) = Rs.205

And Simple Interest for two years = 2 × Rs. 100 = Rs. 200

If Compound Interest is Rs. 205, Simple Interest = Rs. 200

If Compound Interest is Rs. 328, Simple Interest = Rs. 328 × $\frac{200}{205}$

= Rs. 320

Alternative Method:

Rate = 5% = $\frac{1}{20}$

Difference between Compound interest and Simple interest

$= \frac{1}{2} \times \frac{1}{20} = \frac{1}{40}$ of simple interest

$= \frac{1}{41}$ of the compound interest $= \frac{1}{41} \times$ Rs. 328 = Rs. 8

∴ Simple interest = Compound interest – Difference

= Rs. 328 – Rs. 8 = Rs. 320

16. If compound interest on a certain sum at 10% p.a. for 2 years is Rs. 210, find the simple interest for the same period on same amount.

(a) Rs. 190 (b) Rs. 195 (c) Rs. 200 (d) Rs. 205

Solution:

(c) Difference between Compound Interest and Simple Interest for 2 years

$= \frac{1}{2} \times r$ of Simple Interest

$= \frac{1}{2} \times \frac{1}{10} = \frac{1}{20}$ of Simple interest

$= \frac{1}{20+1}$ of Compound Interest $= \frac{1}{21} \times$ Rs. 210 = Rs. 10

∴ Simple interest = Rs. 210 – Rs. 10 = Rs. 200

17. If compound interest for a certain sum at 5% p.a. for 2 years is Rs. 492, find the simple interest for the same period.

(a) Rs. 460 (b) Rs. 470 (c) Rs. 472 (d) Rs. 480

Solution:

(d) Difference between Compound Interest and Simple Interest for 2 years

$= \frac{1}{2} \times r$ of Simple Interest

$= \frac{1}{2} \times \frac{1}{20}$ of simple interest $= \frac{1}{40}$ of simple interest

$= \frac{1}{40+1}$ of Compound Interest $= \frac{1}{41} \times$ Rs. 492 = Rs. 12

∴ Simple interest = Rs. 492 – Rs. 12 = Rs. 480

18. If compound interest on a certain sum for 3 years at 10% is Rs. 662, find simple interest on that sum for 3 years.

(a) Rs. 570 (b) Rs. 600 (c) Rs. 625 (d) Rs. 630

Solution:

(b) Re. 1 will become Rs. $\left(\frac{11}{10}\right)^3$ or $\frac{1331}{1000}$ in 3 years

∴ Rs. 1000 will become Rs. 1331

∴ Compound interest = 1331 – 1000 = 331

And simple interest on Rs. 1000 for 3 years at 10% = Rs. 300

Chapter 27

TIME AND WORK

Time taken to do a work depends on many factors such as 'number of persons' doing the work, their 'efficiency' in doing the work, 'amount of work', 'number of days' and time spent per day'. In absence of any specific information to the contrary in the question, we will assume that the same conditions apply.

FORMULAE

1. If a man can do a piece of work in 'x' days.

   Then his work for one day $= \frac{1}{x}$.
2. If the ratio of time taken by A and B in doing a work = x : y.

   Then his ratio of work done by A and B = $\frac{1}{x} : \frac{1}{y} = y : x$.

   And ratio in which the wages is to be distributed = y : x.
3. If three men can do a work in x, y and z days respectively.

   Then ratio in which the wages is to be distributed = $\frac{1}{x} : \frac{1}{y} : \frac{1}{z}$
4. If A is $\frac{x}{y}$ times as good a workman as B.

   Then he will take $\frac{y}{x}$ of the time that B takes in doing the work.
5. If number of persons engaged in doing a certain job is changed in the ratio of x : y, the days required in completion of work will change in the ratio of y : x.
6. If 'm' men can do a piece of work in 'd' days.
   Then 1 man can do the same work in (d × m) days.

   And 'n' men can do the work in $\frac{d \times m}{n}$ days
7. If A can do a piece of work in 'x' days and B in 'y' days. In how many days, both working together will finish the work?

Solution:

A's one day's work = $\frac{1}{x}$

B's one day's work = $\frac{1}{y}$

A's and B's one day's work = $\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy}$

$\therefore$ A and B will do the work in $\frac{xy}{x + y}$ days

Rule:

If A can do a piece of work in 'x' days and B in 'y' days.

Then A and B together will do the work in $\frac{xy}{x + y}$ days

One day work of A and B = $\frac{x + y}{xy}$

Example:

If A can do a piece of work in 20 days and B in 30 days. In how many days, both working together will finish the job?

Solution:

Time taken by A and B = $\frac{20 \times 30}{20 + 30}$ = 12 days

8. A and B working together can do a piece of work in 'x' days whereas B working alone can do the same work in 'y' days. How many days will A alone take to do the work?

Solution:

A's and B's one day's work = $\frac{1}{x}$

B's one day's work = $\frac{1}{y}$

∴ A's one day's work = $\frac{1}{x} - \frac{1}{y} = \frac{y - x}{xy}$

Hence, A alone will complete the work in $\frac{xy}{y - x}$ days.

Rule:

If A and B working together can do a piece of work in 'x' days whereas B alone can do the same work in 'y' days.

Then A alone will complete the work in $\frac{xy}{y - x}$ days.

Example:

A and B working together can do a piece of work in 12 days, whereas B working alone can do the same work in 18 days. How many days will A alone take to do the work?

Solution:

Time taken by A = $\frac{18 \times 12}{18 - 12}$ = 36 days

9. Pipe A can fill a tank in 'x' hours and pipe B can empty it in 'y' hours.

If both the pipes opened together, the tank will be filled in $\frac{xy}{y - x}$ hours.

And 1 hour's work of both the pipes = $\frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$

Hint: Pipe B is doing negative work here.

10. A pipe can fill a cistern in 'x' hours but due to leakage in the bottom, it is filled in 'y' hours.

Then time taken by the leak to empty the cistern = $\frac{xy}{y - x}$

11. A and B can do a work in a and b days respectively. They started a work together but A left x days before completion of the work. Then time taken to finish the work is $\frac{b \times (a + x)}{a + b}$

Proof :

Let the work is finished in y days

∴ A worked at it for (y – x) days and B for y days.

$$\therefore \frac{y-x}{a}+\frac{y}{b}=1$$

$$\therefore y=\frac{b\times(a+x)}{a+b}$$

12. If 'a' men or 'b' women can do a piece of work in 'x' days.

Then 'm' men and 'n' women together can finish the work in $\frac{abx}{an + bm}$ days

Proof :

b women's work = a men's work

n women's work = $n\left(\frac{a}{b}\right)$ men's work

∴ (m men + n women)'s work $= m + n\left(\frac{a}{b}\right)$ men's work $= \frac{mb+an}{b}$ men's work

a men can do a work in x days.
1 men can do a work in ax days.

∴ m men and n women can do a work in $\dfrac{ax}{\frac{mb+an}{b}} = \dfrac{abx}{an + bm}$ days

OR $$\frac{x}{\frac{m}{a}+\frac{n}{b}}$$

13. If A and B can finish a work in 'x' and 'ax' days respectively. In other words, 'A' is 'a' times efficient than 'B' or 'A' can do 'a' times the work as much as B can do.

Then working together they can finish the work in $\frac{ax}{a+1}$ days.

14. If A is 'x' times efficient than B, and working together they finish a work in 'y' days. Then,

Time taken by A = $\frac{y(x + 1)}{x}$

Time taken by B = y (x + 1)

15. 'x' workers can do a work in 'y' days. If there were 'a' men more, the work would have finished in 'b' less days.

Then workers originally engaged = $\frac{a(y-b)}{b}$

∴ Workers originally engaged

$$= \frac{\text{Extra men engaged} \times \text{Days taken in second situation}}{\text{Less days taken}}$$

16. A group consisting of 'a' men and 'b' women can do a piece of work in 'm' days and other group of 'x' men and 'y' women in 'n' days, then

$|(am - xn)|$ men's work = $|(bm - yn)|$ women's work

SOLVED EXERCISE

1. If 20 men can do a piece of work in 40 days, in how many days will 40 men do the same work?

(a) 20 days (b) 40 days (c) 60 days (d) 80 days

Solution:

(a) Time taken by 40 men = $\frac{20 \times 40}{40}$ = 20 days

2. If 30 men can do a piece of work in 30 days, in how many days will 15 men do the same work?

(a) 15 days (b) 30 days (c) 45 days (d) 60 days

Solution:

(d) Time taken by 15 men = $\frac{30 \times 30}{15}$ = 60 days

3. 12 men can do a work in 25 days. How long will 10 men take to complete the work?

(a) 27 days (b) 28 days (c) 30 days (d) 50 days

Solution:

(c) Time taken by 10 men = $\frac{12 \times 25}{10}$ = 30 days

4. 12 men can do a piece of work in 15 days. How many men are required to complete the work in 10 days?

(a) 10 men (b) 12 men (c) 15 men (d) 18 men

Solution:

(d) Men required to complete the work in 10 days = $\frac{12 \times 15}{10}$ = 18 men

5. A work can be done by a certain number of workers in 8 days. How many days will they take to finish a work four times the original work?

(a) 2 days (b) 12 days (c) 24 days (d) 32 days

Solution:

(d) More work, more days i.e. Direct Proportion

$\therefore$ Days required = 8 × 4 = 32 days

6. A and B together can type 35 pages in 5 minutes whereas A alone can type 20 pages in 4 minutes. How many pages can B type in 1 minute?

(a) 2 pages (b) 4 pages (c) 5 pages (d) 6 pages

Solution:

(a) Together they can type $\frac{35}{5}$ = 7 pages in 1 minute

A alone can type $\frac{20}{4}$ = 5 pages in 1 minute

12. A, B and C together can do a piece of work in 12 days. A and B together can do the same work in 20 days. In how many days C working alone can finish the work?

(a) 8 days (b) 16 days (c) 25 days (d) 30 days

Solution:

(d) C alone can do the work in $\frac{20 \times 12}{20-12} = \frac{20 \times 12}{8} = 30$ days

13. A alone can do a piece of work in 21 days. B who is 40% more efficient than A, will finish the work in:

(a) 10 days (b) 12 days (c) 15 days (d) 18 days

Solution:

(c) Ratio of work done by A and B = 100 : 140 = 5 : 7

$\therefore$ Ratio of time taken = 7 : 5

$\therefore$ Time taken by B = $\frac{5}{7} \times 21$ days = 15 days.

14. A, B and C did a work together and earned Rs. 729 for the work. If the ratio of work done by A, B and C is 2 : 3 : 4, find C's share.

(a) Rs. 81 (b) Rs. 162 (c) Rs. 243 (d) Rs. 324

Solution:

(d) Wages is distributed in ratio of work done i.e. in the ratio of 2 : 3 : 4.

Sum of ratios = 2 + 3 + 4 = 9

$\therefore$ C's share = $\frac{4}{9} \times 729$ = Rs. 324

15. A alone can do a piece of work in 12 days and B alone in 10 days. They undertook to do the work for Rs. 330. How much will A get?

(a) Rs. 120 (b) Rs. 150 (c) Rs. 160 (d) Rs.180

Solution:

(b) Ratio of wages = $\frac{1}{12} : \frac{1}{10} = 10 : 12 = 5 : 6$

$\therefore$ A's wages = $\frac{5}{11} \times 330$ = Rs. 150

16. A, B and C together undertook a work for Rs. 550. A and B together done $\frac{7}{11}$ of the work. Find C's share.

(a) Rs. 50 (b) Rs. 350 (c) Rs. 200 (d) Data Insufficient

Solution:

(c) Work done by $C = 1 - \frac{7}{11} = \frac{4}{11}$

$\therefore$ His share in wages = $\frac{4}{11} \times$ Rs. 550 = Rs. 200

17. A can do a piece of work in 10 days. With the help of B, he finished the work in 6 days. If they earn Rs. 200 for doing the job, find A's share.

(a) Rs. 60 (b) Rs. 80 (c) Rs. 100 (d) Rs. 120

Solution:

(d) Work done by A in 6 days $= \frac{1}{10} \times 6 = \frac{6}{10}$

$\therefore$ A's share in wages $= \frac{6}{10} \times 200 =$ Rs. 120

18. A, B and C can do a piece of work in 12, 18, and 30 days respectively. If they do a work together, find the ratio in which the amount is to be distributed.

(a) 2 : 3 : 5 (b) 5 : 3 : 2 (c) 6 : 10 : 15 (d) 15 : 10 : 6

Solution:

(d) Ratio of wages $= \frac{1}{12} : \frac{1}{18} : \frac{1}{30}$

LCM of 12, 18 and 30 is $3 \times 4 \times 3 \times 5 = 180$.

On multiplying the ratios by 180, we get 15 : 10 : 6.

19. A, B and C can do a piece of work in 12, 15 and 20 days respectively. In how many days working together will they finish the work.

(a) 5 days (b) 10 days (c) 15 days (d) 47 days

Solution:

(a) 1 day's work of A, B and C $= \frac{1}{12} + \frac{1}{15} + \frac{1}{20} = \frac{5+4+3}{60} = \frac{12}{60} = \frac{1}{5}$

$\therefore$ Days taken to finish the work = 5 days

20. A and B can do a piece of work in 12 days, B and C in 15 days, A and C in 20 days. In how many days they working together will finish the work.

(a) 5 days (b) 10 days (c) 12 days (d) 18 days

Solution:

(b) $(A + B) + (B + C) + (A + C) = \frac{1}{12} + \frac{1}{15} + \frac{1}{20}$

$\therefore 2(A + B + C) = \frac{1}{5}$

$\therefore$ Work of A, B and C for 2 days $= \frac{1}{5}$

$\therefore$ Work of A, B and C for one day $= \frac{1}{5} \times \frac{1}{2} = \frac{1}{10}$

$\therefore$ Working together they will finish the work in 10 days.

21. A and B can do a piece of work in 20 days and 15 days respectively. With the help of C, they finished the work in 5 days. How many days will C alone take to finish the work?

(a) 8 days (b) 10 days (c) 12 days (d) 18 days

Solution:

(c) Time taken by C alone $= \frac{1}{5} - \frac{1}{20} - \frac{1}{15} = \frac{12-3-4}{60} = \frac{5}{60} = \frac{1}{12}$

$\therefore$ C alone will take 12 days.

22. A and B together can do $\frac{11}{19}$ of a work. In the same time, B and C together can do $\frac{14}{19}$ of the work. Find the ratio of work done by each of them.

(a) 3 : 4 : 5 (b) 4 : 5 : 7 (c) 5 : 6 : 8 (d) 5 : 7 : 8

Solution:

(c) Work done by A $= 1 - (B + C) = 1 - \frac{14}{19} = \frac{5}{19}$

Work done by B $= (A + B) + (B + C) - 1 = \frac{11}{19} + \frac{14}{19} - 1 = \frac{6}{19}$

Work done by C $= 1 - (A + B) = 1 - \frac{11}{19} = \frac{8}{19}$

∴ Ratio of work done by them $= \frac{5}{19} : \frac{6}{19} : \frac{8}{19} = 5 : 6 : 8$

23. A and B can do a piece of work in 20 days. With the help of C, they finished the work in 15 days. Find the time C alone will take to finish the work.

(a) 60 days (b) 50 days (c) 35 days (d) Data Insufficient

Solution:

(a) (A and B)'s work in 15 days $= \frac{1}{20} \times 15 = \frac{3}{4}$

∴ C's work for 15 days $= 1 - \frac{3}{4} = \frac{1}{4}$

∴ C will finish the work in 4 × 15 = 60 days.

Alternative Method:

One day's work of C $= \frac{1}{15} - \frac{1}{20} = \frac{1}{60}$

∴ C alone can do the work in 60 days.

24. A alone can do a work in 4 days, B and C in 3 days, A and C in 2 days. How many days will B alone take to finish the work ?

(a) 6 days (b) 9 days (c) 12 days (d) 24 days

Solution :

(c) C's work for 1 day $= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

∴ B's work for 1 day $\frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

∴ B will finish the work in 12 days.

25. A and B can do a piece of work in 12 days, B and C can do the same work in 15 days. If A is twice as good a worker as C, find the time that C alone will take to do the work.

(a) 27 days (b) 33 days (c) 45 days (d) 60 days

Solution:

(d) (A + B)'s one day's work = $\frac{1}{12}$

$\therefore$ (2C + B)'s one day's work = $\frac{1}{12}$

But (B + C)'s one day's work = $\frac{1}{15}$

$\therefore$ C's one day's work = (2C + B) – (B + C) = $\frac{1}{12} - \frac{1}{15} = \frac{1}{60}$

$\therefore$ C alone can do the work in 60 days.

26. A tap can fill a tank in 15 minutes while another tap can empty it in 20 minutes. If both the taps are opened simultaneously, when the tank be full?

(a) 5 minutes (b) 35 minutes (c) 60 minutes (d) 70 minutes

Solution:

(c) Time taken = $\frac{15 \times 20}{20 - 15} = \frac{15 \times 20}{5}$ = 60 minutes

27. A pipe can fill a tank in 20 minutes. Due to leakage in the bottom of the tank, it takes 25 minutes to fill the tank. If the tank is full, how long will the leak take to empty the tank?

(a) 50 minutes (b) 60 minutes (c) 80 minutes (d) 100 minutes

Solution:

(d) Time taken = $\frac{20 \times 25}{25 - 20} = \frac{20 \times 25}{5}$ = 100 minutes

28. A cistern is normally filled in 9 hours but due to leak in the bottom, it takes one hour more to be filled. If the cistern is full, in what time will the leak empty it?

(a) 1 hour (b) 19 hours (c) 90 hours (d) 100 hours

Solution:

(c) When both pipes are opened, tank is filled in 9 + 1 = 10 hours.

$\therefore$ Time taken = $\frac{9 \times 10}{10 - 9}$ = 90 hours

29. A cistern is normally filled in 8 hours but it takes 2 hours longer to fill because of a leak in the bottom. If the cistern is full, in what time will the leak empty it?

(a) 10 hours (b) 16 hours (c) 40 hours (d) 80 hours

Solution:

(c) When both pipes are opened, tank is filled in 8 + 2 = 10 hours.

$\therefore$ Time taken = $\frac{8 \times 10}{10 - 8}$ = 40 hours

30. A tap can fill a tank in 6 hours. After half the tank is filled, three more similar taps are opened. What is the total time taken to fill the tank completely ?

(a) 3 hours 15 minutes (b) 3 hours 30 minutes

(c) 3 hours 45 minutes (d) 4 hours 30 minutes

Solution :

(c) Half of the tank is filled in $\frac{1}{2} \times 6 = 3$ hours

$\therefore$ Time taken to fill the remaining half $= \frac{1}{2} \times 6 \times \frac{1}{4}$ hour $= 45$ minutes

$\therefore$ Total time taken = 3 hours and 45 minutes.

31. A tap can empty a tank in 30 minutes while second tap empty it in 20 minutes. If both the taps are kept open, how much time is required to empty the cistern?

(a) 12 minutes (b) 25 minutes (c) 50 minutes (d) 1 hour

Solution:

(a) Time taken $= \frac{30 \times 20}{50} = 12$ minute

Hint: Both taps are doing negative work.

$\therefore$ Work done by them is added.

32. Two pipes A and B can fill a tank in 12 hours and 16 hours respectively, while a third pipe C can empty the tank in 8 hours. If all the three pipes are opened simultaneously, the tank will be filled in:

(a) 16 hours (b) 24 hours (c) 42 hours (d) 48 hours

Solution:

(d) 1 hour's work of each of the three pipes

$$= \frac{1}{12} + \frac{1}{16} - \frac{1}{8} = \frac{4 + 3 - 6}{48} = \frac{1}{48}$$

$\therefore$ The tank will be filled in 48 hours.

33. Pipe A can fill a tank in 15 hours, B in 20 hours and C can empty the tank in 30 hours. If all the three taps are opened together, how much time is required to fill the tank?

(a) 5 hours (b) 10 hours (c) 12 hours (d) 25 hours

Solution:

(c) One hour's work of each of the three pipes $= \frac{1}{15} + \frac{1}{20} - \frac{1}{30} = \frac{4 + 3 - 2}{60} = \frac{1}{12}$

$\therefore$ Time taken to fill the tank = 12 hours

34. Pipe A can fill a tank in 15 minutes and pipe B can drain 40 litre per minute. If both the pipes are opened together, the cistern is full in 45 minutes, find the capacity of the cistern.

(a) 600 litres (b) 750 litres (c) 900 litres (d) 1800 litres

Solution:

(c) 1 minute's work of pipe A $= \frac{1}{15}$

$\therefore$ Water filled in 45 minutes $= \frac{1}{15} \times 45 = 3$ cisterns

Total work of two pipes in 45 minutes = 1 cistern

∴ Work of waste-pipe in 45 minutes = 3 – 1 = 2 cisterns

But waste-pipe drains (45 × 40) litres in 45 minutes

∴ 2 cisterns = 45 × 40 litres

$$\therefore \text{ 1 cistern} = \frac{45 \times 40}{2} = 900 \text{ litres}$$

Alternative Method:

$$\text{Leak in one minute} = \frac{1}{15} - \frac{1}{45} = \frac{2}{45} \text{ of cistern}$$

But waste-pipe drains 40 litre per minute.

$$\therefore \text{ Capacity of cistern} = 40 \times \frac{45}{2} = 900 \text{ litres.}$$

35. A, B and C together can do a work in 4 days. If A and B can do the work in 5 days, B and C in 10 days, find the days taken by A and C to do the work.

(a) 4 days (b) 5 days (c) 6 days (d) 8 days

Solution:

(b) (A + C) = 2 (A + B + C) – (A + 2B + C)

= 2 (A + B + C) – (A + B) – (B + C)

$$= 2 \times \frac{1}{4} - \frac{1}{5} - \frac{1}{10} = \frac{1}{2} - \frac{1}{5} - \frac{1}{10} = \frac{5-2-1}{10} = \frac{2}{10} = \frac{1}{5}$$

∴ A and C together will finish the work in 5 days.

36. A man and a woman working together can do a piece of work in 30 days. If they can do the work in the ratio of 6 : 7, how many days will a man working alone take to do the work.

(a) 18 days (b) 21 days (c) 42 days (d) 65 days

Solution:

(d) One day's work of a man and a woman = $\frac{1}{30}$

Ratio of work done by a man and a woman = 6 : 7.

∴ The man does $\frac{6}{13}$th of the work in the given period.

$$\therefore \text{ 1 day's work of a man} = \frac{1}{30} \times \frac{6}{13} = \frac{1}{65}$$

∴ 1 man can do the work in 65 days.

37. 2 men or 4 women can do a work in 28 days. How long will 4 men and 8 women take to complete the work?

(a) 7 days (b) 14 days (c) 28 days (d) 56 days

Solution:

$$\text{(a) Time taken} = \frac{2 \times 4 \times 28}{2 \times 8 + 4 \times 4} = \frac{2 \times 4 \times 28}{32} = 7 \text{ days}$$

Alternative Method :

$$\text{Time taken} = \frac{28}{\frac{4}{2}+\frac{8}{4}} = \frac{28}{4} = 7 \text{ days}$$

38. 2 men or 4 women can do a work in 22 days. How long will 4 men and 3 women take to complete the work?

(a) 4 days (b) 8 days (c) 16 days (d) 26 days

Solution:

(b) $\text{Time taken} = \frac{2 \times 4 \times 22}{2 \times 3 + 4 \times 4} = \frac{2 \times 4 \times 22}{22} = 8 \text{ days}$

39. If 2 men or 3 women can do a piece of work in 30 days, in how many days can the work be done by 2 men and 6 women?

(a) 4 days (b) 5 days (c) 10 days (d) 15 days

Solution:

(c) $\text{Time taken} = \frac{2 \times 3 \times 30}{2 \times 6 + 3 \times 2} = \frac{2 \times 3 \times 30}{18} = 10 \text{ days}$

Alternative Method :

$$\text{Time taken} = \frac{30}{\frac{2}{2}+\frac{6}{3}} = \frac{30}{3} = 10 \text{ days}$$

40. 4 men and 7 women can do a piece of work in 8 days. In how many days will 5 men and 2 women finish the work if each woman takes twice the time taken by one man?

(a) 5 days (b) 7 days (c) 9 days (d) 10 days

Solution:

(d) 1 man's work ≡ 2 women's work

∴ 4 men's work ≡ 8 women's work,

5 men's work ≡ 10 women's work

∴ 4 men + 7 women ≡ 15 women can do the work in 8 days.

∴ 5 men + 2 women ≡ 12 women can do the work in $\frac{8 \times 15}{12} = 10$ days

41. 8 men can do a piece of work in 5 days whereas 6 women can do the same work in 10 days. In how many days can 4 men and 9 women finish the work?

(a) 2 days (b) 3 days (c) 4 days (d) 6 days

Solution:

(c) (8 × 5) men's work ≡ (6 × 10) women's work.

∴ 40 men's work ≡ 60 women's work,

∴ 1 women's work ≡ $\frac{40}{60} = \frac{2}{3}$ men's work

9 women ≡ $\frac{2}{3} \times 9 = 6$ men's work

∴ 4 men + 9 women ≡ 4 men + 6 men ≡ 10 men

8 men can do the work in 5 days.

∴ 10 men can do the work in = $\frac{8 \times 5}{10} = 4$ days

42. If 2 men and 3 women can do a piece of work in 8 days and 3 men and 2 women in 7 days. In how many days can the work be done by 5 men and 4 women working together?

(a) 4 days (b) 5 days (c) 10 days (d) 15 days

Solution:

(a) $|2 \times 8 - 3 \times 7|$ men's work ≡ $|3 \times 8 - 2 \times 7|$ women's work

∴ 5 men's work ≡ 10 women's work

∴ 1 man's work ≡ 2 women's work

2 men and 3 women can do the work in 8 days.

2 men's work ≡ 4 women's work

∴ 2 men's work + 3 women's work ≡ (4 + 3) women's work

≡ 7 women's work

∴ 7 women can do the work in 8 days.

∴ Days taken by 5 men + 4 women ≡ 14 women are $\frac{7 \times 8}{14} = 4$ days

43. A team consisting of 10 men and 9 women can do a piece of work in 9 days and another team consisting of 8 men and 15 women in 8 days. In how many days can the work be done by 16 men and 12 women?

(a) 4 days (b) 5 days (c) 6 days (d) 8 days

Solution:

(c) $|10 \times 9 - 8 \times 8|$ men's work ≡ $|9 \times 9 - 15 \times 8|$ women's work

∴ 26 men's work ≡ 39 women's work

∴ 1 women's work ≡ $\frac{26}{39} = \frac{2}{3}$ men's work

10 men and 9 women can do the work in 9 days.

∴ 10 men + $\frac{2}{3} \times 9$ men ≡ 16 men can do the work in 9 days.

∴ 16 men + 12 women ≡ 16 men + $\frac{2}{3} \times 12$ men ≡ 24 men will take = $\frac{16 \times 9}{24} = 6$ days

44. A team consisting of 2 men and 5 women can do a piece of work in 12 days and another team of 3 men and 7 women in 8 days. In how many days can the work be done by 5 men and 12 women working together?

(a) 4 days (b) 4.8 days (c) 6.4 days (d) 10 days

A and B will finish the remaining work in $\frac{3}{5} \times 25 = 15$ days

But A and B has already worked for 6 days with C.

∴ A and B alone will work at it for 15 – 6 = 9 days.

55. A and B can do a piece of work in 12 days and 20 days respectively. They started work together and then A left after working for some days. If the remaining work is finished by B in 12 days, how many days has A worked on it before leaving?

(a) 3 days (b) 4 days (c) 6 days (d) 8 days

Solution:

(a) B alone has done the work for 12 days in the end.

B's 12 days work = $\frac{1}{20} \times 12 = \frac{3}{5}$

Remaining work = $1 - \frac{3}{5} = \frac{2}{5}$

This work is done by A and B together in $\frac{2}{5} \times \frac{12 \times 20}{32} = 3$ days

∴ A has worked on it for 3 days.

56. A alone can do a piece of work in 6 days and B alone in 12 days. They started work together but 3 days before the completion of the work, A leaves. The work will be completed in:

(a) 3 days (b) 4 days (c) 5 days (d) 6 days

Solution:

(d) For last 3 days, B alone has done the work.

∴ B's work for last 3 days = $\frac{1}{12} \times 3 = \frac{1}{4}$

Remaining work = $1 - \frac{1}{4} = \frac{3}{4}$

Time taken by A and B to do the remaining work = $\frac{3}{4} \times \frac{6 \times 12}{18}$

= 3 days

∴ Total time taken = 3 + 3 = 6 days

Alternative Method :

$$\text{Time taken} = \frac{(6+3)\times 12}{6+12} = \frac{9\times 12}{18} = 6 \text{ days}$$

57. A, B and C can do a piece of work in 10, 12 and 15 days respectively. They started the work together, but B leaves after 3 days. Find the days in which A and C will finish the remaining work.

(a) 1 day (b) 1.5 days (c) 3 days (d) 4.5 days

∴ Days taken by B and C to do $\frac{5}{8}$ of the work = $\frac{5}{8} \times \frac{16 \times 24}{40}$ = 6 days

∴ Total time taken = 6 + 3 = 9 days.

Hint: For the first 3 days A has worked with B and C.

∴ Those 3 days are already included in 6 days and hence not added again.

60. A alone can do a piece of work in 8 days and B alone can do the same work in 24 days. Working together they will finish the work in:

(a) 4 days (b) 6 days (c) 12 days (d) 16 days

Solution:

(b) Time taken by B is three times of that taken by A.

∴ Together they will finish the work in $\frac{24}{3+1}$ = 6 days

61. A and B can do a piece of work in 15 days and 30 days respectively. Working together, they will finish the work in:

(a) 10 days (b) 15 days (c) 22.5 days (d) 45 days

Solution:

(a) $30 = 15 \times 2$

∴ A is two times efficient than B.

∴ Time taken by A and B = $\frac{30}{1+2}$ = 10 days

62. A is 4 times as good a workman as B. If A could finish a work in 15 days less than B, find the time taken by them to finish the work while working together.

(a) 4 days (b) 5 days (c) 6 days (d) $7\frac{1}{2}$ days

Solution:

(a) Ratio of time taken by A and B = 1 : 4

Difference between ratio of time taken = 4 – 1 = 3

But actual difference is 15 days (i.e. 5 times of 3)

∴ Days taken by A and B are 5 × 1 and 5 × 4 or 5 and 20 respectively.

∴ Working together they will finish the work in $\frac{20}{4+1}$ = 4 days

63. A is twice as good a workman as B. Working together they can finish a work in 16 days. How many days will A take to finish the work?

(a) 4 days (b) 8 days (c) 24 days (d) 48 days

Solution:

(c) Time taken by A = $\frac{16 \times (2+1)}{2}$ = 24 days

64. A is thrice as good a workman as B and together they finish a work in 24 days. In how many days it can be done by B?

(a) 18 days (b) 24 days (c) 72 days (d) 96 days

Solution:

(d) Time taken by B = 24 × (3 + 1) = 24 × 4 = 96 days

65. A does half as much work as B in one sixth of the time. If together they take 12 days to complete the work, how much time shall B take to do it alone ?

(a) 6 days (b) 24 days (c) 36 days (d) 48 days

Solution :

(d) Ratio of work done by A and B $=\left(\frac{1}{2}\times 6\right):1 = 3:1$

Time taken by B = 12 × (3 + 1) = 48 days

66. A and B can do a work in 10 days and 15 days respectively. If they work at it on an alternate day, A starting first, in how many days the work will be completed?

(a) 6 days (b) 10 days (c) 12 days (d) 15 days

Solution:

(c) Work done by A on the first day $= \frac{1}{10}$

Work done by B on the second day $= \frac{1}{15}$

∴ Two days' work (one day each of A and B) $= \frac{1}{10} + \frac{1}{15} = \frac{25}{10\times 15} = \frac{1}{6}$

∴ Time taken by A and B to do the work = 2 × 6 = 12 days.

67. A, B and C can do a piece of work in 10, 30 and 60 days respectively. If A is assisted by B and C on an alternate day, find number of days taken to finish the work.

(a) 4 days (b) 6 days (c) 8 days (d) 12 days

Solution:

(c) Work done by A and B on the first day $= \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$

Work done by A and C on the second day $= \frac{1}{10} + \frac{1}{60} = \frac{7}{60}$

∴ Work done in first two days $= \frac{4}{30} + \frac{7}{60} = \frac{15}{60} = \frac{1}{4}$

∴ Total time taken = 4 × 2 = 8 days

68. Two men A and B can finish a work in 8 and 16 hours respectively. If they work on it alternatively for one hour each, A beginning first, in how many hours will the work be finished?

(a) 5 hours (b) $5\frac{1}{2}$ hours (c) 10 hours (d) $10\frac{1}{2}$ hours

Solution:

(d) Work of A and B for 1 hour each $= \frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

$\therefore$ Total time taken $= 33 + 1 + \frac{1}{3} = 34\frac{1}{3}$ minutes

73. A certain number of men can complete a job in 60 days. If there are 8 more men, the work can be completed in 10 days less. How many men were there originally?

(a) 40 men (b) 45 men (c) 36 men (d) 50 men

Solution:

(a) Extra workers = 8 men

Days taken in second situation = 60 – 10 = 50 days

Less days taken = 10 days

$\therefore$ Men (originally) $= \frac{8 \times 50}{10} = 40$ men

74. A certain number of men can complete a job in 10 days. If 10 men are absent, the remaining man can do the work in 12 days. How many men were there originally?

(a) 10 men (b) 20 men (c) 50 men (d) 60 men

Solution:

(d) Extra workers = – 10 men

Days taken in second situation = 12 days

Less days taken = 10 – 12 = – 2 days

$\therefore$ Men (originally) $= \frac{(-10) \times 12}{-2} = 60$ men

75. If 18 men working 7 hours a day can finish a work in 24 days. In how many days 21 men working 8 hours per day will finish the same work?

(a) 18 days (b) 21 days (c) 24 days (d) 27 days

Solution:

(a)

Men	Hours	Days
18	7	24
21	8	x

More men, less days, i.e. inverse proportion

More hours, less days, i.e. inverse proportion

$\therefore$ Time taken $= 24 \times \frac{18}{21} \times \frac{7}{8} = 18$ days

76. If 60 men working 7 hours a day can do a piece of work in 18 days, in how many days will 21 men working 8 hours a day can do the same work ?

(a) 15 days (b) 25 days (c) 30 days (d) 45 days

Solution:

(d)

Men	Hours	Days
60	7	18
21	8	x

Less men, more days, i.e. inverse proportion

More hours, less days, i.e. inverse proportion

$\therefore$ Days taken $= 18 \times \frac{60}{21} \times \frac{7}{8} = 45$ days

Less men, more days, i.e. inverse proportion

More hours, less days, i.e. inverse proportion

Less length, less days, i.e. direct proportion

$\therefore$ Days taken $= 32 \times \frac{30}{20} \times \frac{5}{8} \times \frac{100}{300} = 10$ days.

81. A contractor agreed to complete a work in 120 days. He engaged 150 men for the work and after 70 days, only half of the work was completed. How many additional men must he employ to complete the work in agreed time?

(a) 60 men (b) 90 men (c) 150 men (d) 210 men

Solution:

(a) Days remaining after 70 days' work = 120 – 70 = 50 days

150 men has completed half the work in 70 days.

Remaining work $= 1 - \frac{1}{2} = \frac{1}{2}$ which is equal to work already done.

Men	Days	Work
150	70	1
x	50	1

Less days, more men, i.e. inverse proportion

$\therefore$ Men required $= 150 \times \frac{70}{50} = 210$ men

$\therefore$ Additional men required = 210 – 150 = 60 men

82. A contractor agreed to complete a work in 40 days. He engaged 25 men for the work and after 24 days, only one-third of the work was completed. How many more men must he employ to complete the work 4 days earlier?

(a) 50 men (b) 60 men (c) 75 men (d) 100 men

Solution:

(c) Days remaining after 24 days' work = 40 – 24 = 16 days

25 men has completed $\frac{1}{3}$ work in 24 days.

Remaining work $= 1 - \frac{1}{3} = \frac{2}{3}$ which is equal to double of work already done.

Remaining work is to be completed in 16 – 4 = 12 days.

Men	Days	Work
25	24	1
x	12	2

Less days, more men, i.e. inverse proportion

More work, more men, i.e. direct proportion

$\therefore$ Men required $= 25 \times \frac{24}{12} \times \frac{2}{1} = 100$ men

$\therefore$ Additional men required = 100 – 25 = 75 men

Less men, more days, i.e. inverse proportion

More work, more days, i.e. direct proportion

$\therefore$ Time taken = $15 \times \frac{12}{10} \times \frac{2}{1} = 36$ days

87. A alone can do a piece of work in 90 days. He works at it for 15 days and then B alone finishes the remaining work in 50 days. Working together they will finish the work in:

(a) 36 days (b) 40 days (c) 60 days (d) 90 days

Solution:

(a) A's 15 days work = $\frac{15}{90} = \frac{1}{6}$

Remaining work = $1 - \frac{1}{6} = \frac{5}{6}$ which is completed by B in 50 days.

$\therefore$ B alone can do the work in $50 \times \frac{6}{5} = 60$ days

$\therefore$ A and B together can do the work in $\frac{90 \times 60}{150} = 36$ days

88. A, B and C together can do a work in 8 days. If B takes double time of A, C takes double time of B, find the time taken by A alone to do the work.

(a) 14 days (b) 20 days (c) 28 days (d) 35 days

Solution :

(a) Let A alone takes x days to do the work.

$\therefore$ B and C takes 2x and 4x days

$\therefore$ (A + B + C)'s 1 day's work $= \frac{1}{x} + \frac{1}{2x} + \frac{1}{4x} = \frac{1}{8}$

$\Rightarrow \frac{7}{4x} = \frac{1}{8}$

$\Rightarrow x = \frac{8 \times 7}{4} = 14$ days.

89. A can do a piece of work in 80 days. He alone works at it for 20 days and then B alone finished the remaining work in 36 days. In how many days A and B together can complete the work?

(a) 25 days (b) 30 days (c) 35 days (d) 40 days

Solution:

(b) A's 20 days work = $\frac{1}{80} \times 20 = \frac{1}{4}$

Remaining work = $1 - \frac{1}{4} = \frac{3}{4}$ which is completed by B in 36 days

$\therefore$ B alone can do the work in $36 \times \frac{4}{3} = 48$ days

$\therefore$ A and B together can do the work in $\frac{80 \times 48}{128} = 30$ days

89. A and B can do a piece of work in 30 days. They worked together for 6 days and then A leaves off. If B finishes the remaining work in 36 days, how many days will A alone take to do the work?

(a) 45 days (b) 60 days (c) 80 days (d) 90 days

Solution:

(d) 6 days' work of A and B = $\frac{6}{30} = \frac{1}{5}$

Remaining work = $1 - \frac{1}{5} = \frac{4}{5}$ which is completed by B in 36 days

∴ B alone can do the work in $36 \times \frac{5}{4} = 45$ days

∴ A alone can do the work in $\frac{30 \times 45}{15} = 90$ days.

91. 25 men can do a piece of work in 17 days. They worked at it for 5 days and then some more men joined the group and all of them together finished the remaining work in 10 days. How many more men joined the group?

(a) 5 men (b) 6 men (c) 7 men (d) 8 men

Solution:

(a) Days remaining after 5 days' work = 17 – 5 = 12 days

Work remaining = 25 men's work for 12 days

∴ In 10 days, remaining work will be finished by $\frac{25 \times 12}{10} = 30$ men

∴ Additional men joined = 30 – 25 = 5 men

Alternative Method:

Let additional men employed = x

Days taken in second case = 10 days

Less days taken = 12 – 10 = 2 days

$$\text{Men (originally)} = \frac{\text{Extra men engaged} \times \text{Days taken in second situation}}{\text{Less days taken}}$$

$\therefore 25 = \frac{x \times 10}{2}$

∴ x = 5 men

92. A can do a certain work in the same time in which B and C together can do it. If A and B together could do it in 10 days and C alone in 50 days, then B alone could do the work in :

(a) 15 days (b) 20 days (c) 25 days (d) 30 days

Solution :

(c) Work of A, B and C for 1 day $= \frac{1}{10} + \frac{1}{50} = \frac{6}{50}$

Let A can do the work in x days

Then work of A for one day $= \frac{1}{x}$

$\therefore \quad 5 \times \frac{1}{12} + 2 \times \frac{1}{16} + 11z = 1$

$\therefore \quad 11z = 1 - \frac{5}{12} - \frac{1}{8} = \frac{11}{24}$

$\therefore \quad z = \frac{11}{24} \times \frac{1}{11} = \frac{1}{24}$

$\therefore$ C alone will take $\frac{1}{z} = 24$ days

Now $y + z = \frac{1}{16}$

$\therefore \quad y = \frac{1}{16} - \frac{1}{24} = \frac{1}{48}$

$\therefore$ B can finish the work in $\frac{1}{y} = 48$ days

Chapter 28

TIME AND DISTANCE

Distance covered, average speed and time are the three basic terms used in this chapter.

Distance covered = Time taken × Average speed

Note: A train crosses a pole or a standing man when it covers distance equal to its own length. A train crosses another standing train or a platform when it covers distance equal to the total of its own length and that of other train, platform etc.

Time taken = $\frac{\text{Distance covered}}{\text{Average speed}}$

> If the speed of two trains are in the ratio of x : y, then ratio of time taken to cover a certain distance by them = $\frac{1}{x} : \frac{1}{y} = y : x$.

Average Speed = $\frac{\text{Distance covered}}{\text{Time taken}}$

Note: Speed of the object is deemed to be constant during the whole journey, unless stated otherwise.

> 1 km per hour = $\frac{5}{18}$ metre per second
>
> ∴ To convert speed of an object from km/h to m/s, multiply the speed by $\frac{5}{18}$.

Example:

Convert speed of 36 km/h into speed m/s.

Solution:

36 km/h = 36 × $\frac{5}{18}$ m/s = 10 m/s

1 metre per second = $\frac{18}{5}$ km per hour

∴ To convert speed of an object from m/s to km/h, multiply the speed by $\frac{18}{5}$.

Example:

Convert speed of 20 m/s into speed km/h.

Solution:

20 m/s = 20 × $\frac{18}{5}$ km/h = 72 m/s

Relative Speed:

> If two trains are moving in **opposite directions**, their relative speed is equal to **'sum of their speeds'**.

Example:

Two trains are running in opposite directions at 80 km/h and 35 km/h, find their relative speed.

Solution:

Relative speed = 80 + 35 = 115 km/h

If two trains are moving in the **same direction**, their relative speed is equal to '**difference of their speeds**'.

Example:

Two trains are running in the same direction at 90 km/h and 40 km/h, find their relative speed.

Solution:

Relative speed = 90 – 40 = 50 km/h

MORE FORMULAE

1. Two persons left from place A for B at the speed of 'x' km/h and 'y' km/h respectively. If the first person (faster one) reaches at place B and immediately returns for place A. How far from B will both the persons meet?

Solution:

Let total distance = 'd' km.

And distance between B and the point of meeting = 'm' km

∴ Distance covered by the first person = d + m

And distance covered by the second person = d – m

Since time taken by both are equal.

$$\therefore \quad \frac{d+m}{x} = \frac{d-m}{y}$$

$$\therefore \quad dy + my = dx - mx$$

$$\therefore \quad m \times (x+y) = d \times (x-y)$$

$$\therefore \quad m = d \times \frac{x-y}{x+y}$$

∴ Distance between B and meeting point

$$= \text{Distance (i.e. AB)} \times \frac{\text{Difference between speeds}}{\text{Sum of speeds}}$$

Alternative Method:

Ratio of speeds of two persons = x : y.

∴ Ratio of distance travelled by them = x : y

$$\therefore \text{ Distance covered by first person} = \frac{x}{x+y} \times \text{Total distance covered}$$

$$\text{Distance covered by second person} = \frac{y}{x+y} \times \text{Total distance covered}$$

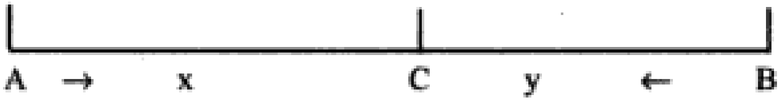
Note: Total distance covered = AB + BA.

2. A train leaves a station at x km/h and another train leaves the same station after T hours at y km/h. They will meet after $\frac{xT}{y-x}$ hours.

   And distance $= \frac{x\,yT}{y-x}$

3. A train leaves station A for B and another train leaves station B for station A at the same time. The trains cross each other en route and reach their destinations in 'm' and 'n' hours respectively after crossing to each other. Find the speed of the two trains.

Solution:



Let speeds of two trains are 'a' and 'b' km/h respectively.

And the two trains meet at point C, after covering a distance of 'x' and 'y' km respectively from the starting points *i.e.* from A and B respectively.

Then $\frac{x}{a} = \frac{y}{b} \Rightarrow b = \frac{ay}{x}$ (1)

After crossing each other, they reach to the other side in 'm' and 'n' hours after covering the distance of 'y' and 'x' km respectively.

$\therefore \frac{y}{a} = m \Rightarrow y = am$ (2)

And $\frac{x}{b} = n \Rightarrow x = nb$ (3)

Now $b = \frac{ay}{x} = \frac{a(am)}{nb}$

$$\therefore b^2 = \frac{a^2m}{n}$$

$$\therefore \frac{b^2}{a^2} = \frac{m}{n}$$

$$\therefore \frac{b}{a} = \sqrt{\frac{m}{n}}$$

$$\therefore \frac{a}{b} = \sqrt{\frac{n}{m}}$$

Rule:

> Speeds of first and second trains are in the ratio of square roots of time taken by the second and the first train in reaching their destination after crossing each other.

4. If two trains travels a certain distance at the speed of 'a' km/h, and 'b' km/h respectively and difference between time taken by two trains is 'T' hours.

   Then distance between the starting point and the destination $= \frac{a \times b}{a - b} \times T$

$$\therefore \text{Distance} = \frac{\text{Multiplication of Speeds} \times \text{Difference of Time taken}}{\text{Difference of speeds}}$$

Proof:

Let the distance covered = 'x' km.

Then $\frac{x}{a} - \frac{x}{b} = T$

$$\therefore \quad \frac{(a-b)\,x}{a \times b} = T$$

$$\therefore \quad x = \frac{a \times b}{a - b} \times T$$

5. If a person goes from A to B @ x km/h and returns @ y km/h, then his average speed during the whole journey is: $\frac{2xy}{x+y}$

Proof:

Let distance between two places is 1 km.

Then time taken to travel from A to B = $\frac{1}{x}$ hours

And time taken to travel from B to A = $\frac{1}{y}$ hours

Total distance covered = 1 + 1 = 2 km.

Total time taken = $\frac{1}{x} + \frac{1}{y}$

Average speed during the whole journey = $\frac{\text{Distance covered}}{\text{Time taken}} = \frac{2}{\frac{1}{x}+\frac{1}{y}} = \frac{2}{\frac{x+y}{xy}} = \frac{2xy}{x+y}$

$\therefore$ Distance covered = $\frac{2xy}{x+y} \times$ Time taken

Distance from A to B = $\frac{2xy}{x+y} \times$ Time Taken $\times \frac{1}{2}$

6. A man covers $\frac{1}{3}$rd of his journey at 'x' km/h, next $\frac{1}{3}$rd of his journey at 'y' km/h and remaining $\frac{1}{3}$rd journey at 'z' km/h, then his average speed during the journey = $\frac{3xyz}{xy + yz + xz}$

7. **Two trains of equal lengths** running at **different speeds** take 'a' and 'b' seconds respectively to cross a pole. Then **time taken by the trains to cross each other** when they are:

Solution:

(b) Let speed of train A = 100 km/h

Then speed of train B = 125 km/h

Ratio of speeds of trains A and B = 100 : 125 = 4 : 5

∴ Ratio of time taken by them to cover equal distance = 5 : 4

Time taken by train A = 35 minutes

Time take by train B = $\frac{4}{5} \times 35$ minutes = 28 minutes.

Alternative Method:

Speed of train B is faster by 25% = $\frac{1}{4}$

∴ Decrease in time taken by train B = $\frac{1}{5} \times 35$ minutes = 7 minutes.

∴ Time taken by train B = 35 – 7 = 28 minutes.

5. Two persons start walking in opposite directions at 5 km/h and 4 km/h respectively. After how many hours will they be 45 km apart?

(a) 5 hours (b) 7 hours (c) 9 hours (d) 11 hours

Solution:

(a) Relative speed of two persons = 5 + 4 = 9 km/h

∴ Time taken = $\frac{45}{9}$ = 5 hours

6. Two persons start walking in the same direction at 5 km/h and 4 km/h respectively. In how many hours will they be 45 km apart?

(a) 5 hours (b) 7 hours (c) 9 hours (d) 45 hours

Solution:

(d) Relative speed of two persons = 5 – 4 = 1 km/h

∴ Time taken = $\frac{45}{1}$ = 45 hours

7. A man takes 8 hours in walking to a certain place and riding back. He would have gained 2 hours by riding both ways. What time would he take to walk both ways?

(a) 3 hours (b) 5 hours (c) 6 hours (d) 10 hours

Solution:

(d) One way walking time + One way riding time = 8 hours

Two ways walking time = (8 + 2) = 10 hours

8. A 100 metre long train takes 5 seconds to pass a standing man. Find the time taken by the train in crossing a railway platform of 260 metre in length.

(a) 18 seconds (b) 13 seconds (c) 26 seconds (d) 36 seconds

Solution:

(a) Speed of the train = $\frac{100}{5}$ = 20 m/s

$\therefore$ Time taken by the train to cross the platform = $\dfrac{260 + 100}{20}$ = 18 seconds

9. A train running at the average speed of 45 km/h passes a standing man in 10 seconds. Find the length of the train.

(a) 75 metre (b) 100 metre (c) 125 metre (d) 150 metre

Solution:

(c) Speed of train = 45 km/h = $45 \times \dfrac{5}{18}$ m/s

$\therefore$ Distance covered in 10 seconds = $45 \times \dfrac{5}{18} \times 10$ = 125 metre

$\therefore$ Length of train = 125 metre

10. Find the time taken by a 200 metre long train, running @ 72 km/h in crossing a standing man.

(a) 5 seconds (b) 10 seconds (c) 15 seconds (d) 20 seconds

Solution:

(b) Speed of the train = $\dfrac{5}{18} \times 72$ m/s = 20 m/s

Distance covered to cross the man = 200 metre

$\therefore$ Time taken = $\dfrac{200}{20}$ = 10 seconds

11. A 250 metre train is running at the average speed of 90 km/h. How long will it take to pass a standing man.

(a) 8 seconds (b) 10 seconds (c) 12.5 seconds (d) 20 seconds

Solution:

(b) Speed of the train = $\dfrac{5}{18} \times 90$ = 25 m/s

$\therefore$ Time taken to cover 250 metre = $\dfrac{250}{25}$ = 10 seconds

12. Find the time taken by a 100 metre long train to cross a platform of 400 metre in length, if the train is running @ 90 km/h.

(a) 12.5 seconds (b) 15 seconds (c) 20 seconds (d) 25 seconds

Solution:

(c) Total distance covered = 100 + 400 = 500 metre

Speed of the train = $\dfrac{5}{18} \times 90$ m/s = 25 m/s

$\therefore$ Time taken = $\dfrac{500}{25}$ = 20 seconds

13. A train running at the speed of 15 m/s takes 20 seconds to pass a man running at the speed of 5 m/s in the same direction. Find the length of the train.

(a) 50 metre (b) 100 metre (c) 200 metre (d) 250 metre

But the man is going at the speed of 5 km/h in the opposite direction.

∴ Speed of the train = 40 – 5 = 35 km/h.

26. A and B walk around a circular path of 900 metre in circumference, starting together from the same point in the same direction. If their speeds are 150 metre per minute and 200 metre per minute respectively, after how many minutes will they be again at the starting point?

(a) 4.5 minutes (b) 6 minutes (c) 15 minutes (d) 18 minutes

Solution:

(d) They will be together at the starting point when B (the faster one) has gained 900 metre (i.e. one round) over A.

In 1 minute B gains (200 – 150) metre = 50 metre over A.

∴ B will gain 900 metres in $\frac{900}{50} = 18$ minutes

27. A and B walk around a circular path of 5 km in circumference, starting together from the same point in the same direction. If their speeds are 4 km/h and 3 km/h respectively, after how many hours will they be again at the starting point?

(a) 3 hours (b) 4 hours (c) 5 hours (d) 6 hours

Solution:

(c) They will be together at the starting point when A (the faster one) has gained 5 km (i.e. one round) over B.

In 1 hour A gains (4 km – 3 km) i.e. 1 km over B.

∴ A will gain 5 km in 5 hours.

Alternative Method:

Time taken by A to cover 5 km = $\frac{5}{4}$ hours

Time taken by B to cover 5 km = $\frac{5}{3}$ hours

LCM of $\frac{5}{4}$ and $\frac{5}{3}$ is 5 hours.

∴ They will be together again at the starting point after 5 hours.

27. A and B can walk around a circular path in 4 hours and 6 hours respectively. If they start from the same point in the same direction, when will they be together at the starting point again?

(a) 2 hours (b) 6 hours (c) 8 hours (d) 12 hours

Solution:

(d) LCM of time taken by A and B = LCM of 4 and 6 = 12 hour.

∴ They will be together again at the starting point after 12 hours.

28. A man is walking at a speed of 10 km/h. After every km, he takes rest for 4 minutes. How much time will he take to cover a distance of 10 km?

(a) 1 hour (b) 90 minutes (c) 96 minutes (d) 100 minutes

Solution:

(c) He covers 10 km in 1 hour (i.e. in 60 minutes)

∴ He will take 6 minutes in covering 1 km.

He rests for 4 minutes after every km.

Time taken = (6 + 4) minutes = 10 minutes for every km.

∴ Time taken (for first 9 km) = 9 × 10 = 90 minutes.

Time taken to cover 10th km = 6 minutes

∴ Total time taken = 90 + 6 = 96 minutes.

Hint: Rest time after 10th km is not added as he has reached his destination.

30. Two men A and B walk from P to Q, a distance of 21 km at 3 km/h and 4 km/h respectively. B reaches at point Q and returns immediately and he meets A at point R. Find the distance from P to R.

(a) 9 km (b) 18 km (c) 24 km (d) 27 km

Solution:

(b) Both of them together have walked twice the distance from P to Q i.e. 42 km.

Ratio of speeds of A and B = 3 : 4.

∴ Distance travelled by A = PR = $\frac{3}{7} \times 42 = 18$ km.

31. Two men A and B walk from P to Q, a distance of 36 km at 7 km/h and 5 km/h respectively. A reaches at Q and immediately returns and meets B at a place R. Find the distance between P and R.

(a) 15 km (b) 21 km (c) 30 km (d) 42 km

Solution:

(c) Total distance travelled = 2 × 36 = 72 km

Ratio of speeds of A and B = 7 : 5

∴ Distance travelled by B = PR = $\frac{5}{12} \times 72 = 30$ km.

32. Train A takes 6 hours more than train B to cover a distance of 800 km. If the speed of train A is doubled it takes 2 hours less than train B. What is the speed of train B?

(a) 50 km/h (b) 60 km/h (c) 75 km/h (d) 80 km/h

Solution:

(d) Let time taken by train B = x hours.

Then time taken by train A = (x + 6) hours.

Time taken by train A on doubling the speed = (x – 2) hours.

We know that if speed is doubled, time taken is halved.

∴ $2(x - 2) = x + 6$

∴ $2x - 4 = x + 6$

∴ $x = 10$ hours

∴ Speed of train B = $\frac{800}{10}$ = 80 km/h

33. A man standing on a railway platform notices that a train going in one direction takes 10 seconds to pass him and other train of the same length takes 15 seconds to pass him. Find the time taken by the two trains to cross each other when they are running in the opposite directions.

(a) 6 seconds (b) 10 seconds (c) 12 seconds (d) 15 seconds

Solution:

(c) Time taken = $\frac{2ab}{a+b} = \frac{2 \times 10 \times 15}{25} = 12$ seconds

34. A man standing on a railway platform notices that a train going in one direction takes 9 seconds to pass him and other train of the same length takes 6 seconds to pass him. Find the time taken by the two trains to cross each other when they are running in the same direction.

(a) 3 seconds (b) 27 seconds (c) 30 seconds (d) 36 seconds

Solution:

(d) Time taken = $\frac{2ab}{a-b} = \frac{2 \times 9 \times 6}{3} = 36$ seconds

35. Two trains running in the opposite directions at the speeds of 44 km/h and 55 km/h take 12 seconds to cross each other. If the length of one train is 150 metre, what is the length of the other train?

(a) 120 metre (b) 180 metre (c) 220 metre (d) 250 metre

Solution:

(b) Their relative speed = 44 + 55 = 99 km/h

$= \frac{5}{18} \times 99$ m/s $= \frac{55}{2}$ m/s

Distance covered in 12 seconds = $12 \times \frac{55}{2} = 330$ metre.

∴ Length of the second train = 330 – 150 = 180 metre.

36. Two trains of 175 metre and 250 metre cross a pole in 7 seconds and 10 seconds respectively. In what time will they cross each other, if they are running in the opposite directions ?

(a) 3 seconds (b) 5 seconds (c) 8.5 seconds (d) 17 seconds

Solution :

(c) Time taken to cross each other $= \frac{7+10}{2} = 8.5$ seconds

37. Two trains running in the same direction at the speeds of 68 km/h and 48 km/h take 36 seconds to cross each other. If the length of one train is 85 metre, what is the length of the other train?

(a) 115 metre (b) 125 metre (c) 150 metre (d) 160 metre

Solution:

(a) Their relative speed = 68 – 48 = 20 km/h

Distance covered in 36 seconds = $\frac{5}{18} \times 20 \times 36 = 200$ metre.

∴ Length of the second train = 200 – 85 = 115 metre.

38. A 220 metre long train running at 90 km/h crosses a 180 metre train coming from the opposite direction in 9 seconds. The speed of the second train is:

(a) 50 km/h (b) 70 km/h (c) 100 km/h (d) 160 km/h

Solution:

(b) Distance covered = (220 + 180) metres = 400 metres

Relative speed of two trains = $\frac{400}{9}$ m/s = $\frac{400}{9} \times \frac{18}{5}$ km/h

= 160 km/h

∴ Speed of the second train = 160 – 90 = 70 km/h

39. Two trains of length of 250 metre and 200 metre are running in the same direction. If the trains cross each other in 45 seconds and the speed of the first train is 27 km/h, find speed of the second train.

(a) 36 km/h (b) 54 km/h (c) 63 km/h (d) 72 km/h

Solution :

(c) Total distance covered = (250 + 200) metres = 450 metres

Relative speed of two trains $= \frac{450}{45}$ m/s $= \frac{450}{45} \times \frac{18}{5}$ km/h = 36 km/h

∴ Speed of the second train = 27 + 36 = 63 km/h

40. Two trains of the length of 200 metre and 250 metre are running in the opposite directions at the speed of 35 km/h and 55 km/h respectively. Find the time in which the faster train will pass a man sitting in the slower train.

(a) 2 seconds (b) 8 seconds (c) 10 seconds (d) 18 seconds

Solution:

(c) Relative speed = 35 + 55 = 90 km/h = $\frac{5}{18} \times 90$ m/s = 25 m/s

The faster train will completely cross a man sitting in the slower train when a distance equal to length of faster train is covered.

Distance to be covered = 250 metre.

∴ Time taken = $\frac{250}{25}$ = 10 seconds.

41. Two trains start at the same time from station A and B towards station B and A at the speed of 50 km/h and 55 km/h respectively. When the two trains meet it is found that the faster train has travelled 30 km more than the slower one. What is the distance between the two stations.

(a) 300 km (b) 330 km (c) 550 km (d) 630 km

Solution:

(d) Faster train covers 5 km/h (i.e. 55 – 50) extra than the slower train.

∴ To cover 30 km extra it will take $\frac{30}{5}$ = 6 hours

Distance covered by the two trains in 1 hour = 55 + 50 = 105 km

∴ Distance covered in 6 hours = 6 × 105 km = 630 km.

∴ Distance between the two stations = 630 km.

42. Two trains start at the same time from two stations and proceed towards each other at the speed of 15 km and 20 km per hour respectively. When the trains meet, it is found that one train has travelled 50 km more than the other. Find the distance between the two stations.

(a) 10 km (b) 35 km (c) 350 km (d) 500 km

Solution :

(b) Sum of ratio of speeds of two trains = 4 + 5 = 9

∴ Distance covered by the first train till meeting point $=\frac{4}{9}\times 117=52$ km

Time taken by the first train $=\frac{52}{60}\times 60=52$ minutes

47. A train is running at 70 km/h but due to stoppages it could cover a certain distance at 63 km/h. How much time the train stops on an average (per hour)?

(a) 6 minutes (b) 10 minutes (c) 12 minutes (d) 15 minutes

Solution:

(a) Time spent on stoppages (per hour)

= Time in which the train could have travelled (70 – 63) km = 7 km

$=\frac{7}{70}\times 60=6$ minutes/hour

48. Two trains measuring 120 metre and 180 metre cross each other completely in 10 seconds when running in opposite directions and the faster train takes 30 seconds to cross the slower train when the two trains are running in the same direction. Find the speed of the faster train.

(a) 30 km/h (b) 36 km/h (c) 54 km/h (d) 72 km/h

Solution:

(d) Total distance to be covered = 120 + 180 = 300 metre

Time taken to cross each other, when trains are running in opposite directions = 10 seconds

∴ Relative speed (i.e. sum of speeds) of the trains $=\frac{300}{10}=30$ m/s

Time taken to cross each other, when trains are running in the same direction = 30 seconds

∴ Relative speed (i.e. difference of the speeds) of the trains $=\frac{300}{30}=10$ m/s

∴ Speed of the faster train $=\frac{1}{2}(30+10)=20$ m/s $=20\times\frac{18}{5}$ km/h = 72 km/h

49. The distance between two stations A and B is 300 km. One train leaves station A towards station B at the average speed of 40 km/h. At the same time, another train leaves station B towards A at the average speed of 80 km/h. The distance from station A where the two trains meet is:

(a) 80 km (b) 100 km (c) 200 km (d) 160 km

Solution:

(b) Ratio of distance covered = Ratio of speeds = 40 : 80 = 1 : 2

∴ Distance covered are 100 km and 200 km respectively.

∴ Distance from station A = 100 km.

50. The distance between two stations A and B is 550 km. One train leaves station A towards station B at the average speed of 100 km/h. After half an hour, another train leaves station B towards A at the average speed of 150 km/h. The distance from station A where the two trains meet is:

(a) 100 km (b) 200 km (c) 250 km (d) 300 km

$\therefore$ Second train will meet the first train after $\frac{240}{30} = 8$ hours.

Distance covered by the faster train in 8 hours = 8 × 110 = 880 km

$\therefore$ The faster train will meet after covering 880 km.

Alternative Method:

Distance covered till meeting place

$$= \frac{\text{First speed} \times \text{Difference in Time} \times \text{Second speed}}{\text{Difference in speeds}}$$

$$= \frac{80 \times 3 \times 110}{30} = 880 \text{ km}$$

58. A man walking at $\frac{3}{4}$th of the speed, reaches his office late by 2 hours. What is the usual time?

(a) 3 hours (b) 5 hours (c) 6 hours (d) 12 hours

Solution:

(c) Walking at $\frac{3}{4}$th of his usual speed, he will take $\frac{4}{3}$ of usual time.

$\therefore$ $\frac{4}{3}$ of the usual time = Usual time + 2 hours

$\therefore$ $\frac{1}{3}$ of the usual time = 2 hours

$\therefore$ Usual time = 3 × 2 hours = 6 hours.

59. Walking at $\frac{5}{4}$th of the usual speed, I reach my office 12 minutes too early. what is the usual time?

(a) 48 minutes (b) 60 minutes (c) 70 minutes (d) 80 minutes

Solution:

(b) If I walk at $\frac{5}{4}$th his usual speed, I will take $\frac{4}{5}$ of usual time.

$\therefore$ Usual time = $\frac{4}{5}$ of usual time + 12 minutes

$\therefore$ $\frac{1}{5}$ of usual time = 12 minute

$\therefore$ Usual time = 5 × 12 minutes = 60 minutes

60. If a man goes to a place at an average speed of 10 km/h and then returns at the average speed of 15 km/h. Find his average speed during the whole journey?

(a) 12 km/h (b) 12.5 km/h (c) 13 km/h (d) 15 km/h

Solution:

(a) Average speed $= \frac{2 \times 10 \times 15}{10 + 15} = \frac{2 \times 10 \times 15}{25} = 12$ km/h

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Chapter 29

BOATS AND STREAMS

A boat is said to be moving downstream when it is going with the stream and it is said to be going upstream when it is going against the stream.

If speed of a boat in still water is 'x' km/h and speed of the stream is 'y' km/h, Then speed of the boat downstream = (x + y) km/h

And Speed of the boat upstream = (x – y) km/h

If we are given speed of boat upstream and downstream then we can find speed of boat in still water and speed of stream as follows:

Speed of boat in still water = $\frac{1}{2}$(Downstream + Upstream)

Speed of stream = $\frac{1}{2}$(Downstream – Upstream)

FORMULAE

1. A person rows a boat to a certain place and then comes back to the starting point. If he rows the boat at the speed of 'x' km/h in still water and the rate of the current is 'y' km/h, then

$$\text{Average speed during the journey} = \frac{2(x+y)(x-y)}{(x+y)+(x-y)} = \frac{x^2-y^2}{x}$$

Hint: (x + y) and (x – y) are speeds of boat downstream and upstream respectively.

2. If ratio of downstream and upstream speeds of a boat is a : b.
   Then ratio of time taken = b : a.

   $$\text{Speed of stream} = \frac{a-b}{a+b} \times \text{Speed in still water.}$$

   $$\text{Speed in still water} = \text{Speed of stream} \times \frac{a+b}{a-b}.$$

Proof:

Let speed of the boat in still water = x km/h

And speed of the stream = y km/h

Speed of boat downstream = (x + y) km/h

Speed of boat upstream = (x – y) km/h

$(x + y) : (x - y) = a : b$

$\therefore b(x + y) = a(x - y)$

Solving, we get:

$$y = \frac{a-b}{a+b} \times x$$

$= \frac{a-b}{a+b} \times$ Speed of boat in still water

SOLVED EXERCISE

1. A man can row a boat 12 km/h with the stream and 8 km/h against the stream. Find his speed in still water.

(a) 2 km/h (b) 4 km/h (c) 8 km/h (d) 10 km/h

Solution:

(d) Speed of boat in still water = $\frac{12+8}{2}$ = 10 km/h

2. A man can row a boat 27 km/h with the stream and 11 km/h against the stream. Find speed of stream.

(a) 2 km/h (b) 4 km/h (c) 8 km/h (d) 10 km/h

Solution:

(c) Speed of stream = $\frac{27-11}{2}$ = 8 km/h

3. A man can row a boat 15 km/h with the current and speed of current is 3 km/h. Find his speed against the current.

(a) 9 km/h (b) 12 km/h (c) 18 km/h (d) 21 km/h

Solution:

(a) Speed of boat in still water = 15 – 3 = 12 km/h

∴ Speed of boat against current = 12 – 3 = 9 km/h

Direct Method:

Speed of boat against current = 15 – 2 × 3 = 9 km/h

4. The speed of a boat in still water is 12 km/h and its speed against the current is 8.5 km/h. Find the speed of boat with the current.

(a) 3.5 km/h (b) 7 km/h (c) 15.5 km/h (d) 20.5 km/h

Solution :

(c) Speed of current = 12 – 8.5 = 3.5 km/h

Downstream speed = 12 + 3.5 = 15.5 km/h

Alternative Method :

$2 \times 12 - 8.5 = 15.5$ km/h

Logic : $x + y = 2x - (x - y)$

5. A man can row a boat 40 km downstream in 5 hours and 18 km upstream in 3 hours. Find the speed of the current.

(a) 1 km/h (b) 2 km/h (c) 11 km/h (d) 22 km/h

Solution:

(a) Speed of the boat downstream = $\frac{40}{5}$ = 8 km/h

Speed of the boat upstream = $\frac{18}{3}$ = 6 km/h

Speed of current only moves the boat.

Boat is moved 1 km by the current in (10 + 10) minutes = 20 minutes = $\frac{1}{3}$ hour.

∴ Speed of current = 1 × 3 = 3 km/h.

10. A man rows a boat 3 hours downstream and 3 hours upstream. Find speed of boat in still water, if he covers a total distance of 12 km.

(a) 2 km/h (b) 3 km/h (c) 4 km/h (d) Data insufficient

Solution :

(*a*) Let speed of boat and current are x km/h and y km/h respectively.

Then $3 \times (x + y) + 3 \times (x - y) = 12$

Solving x = 2 km/h

11. Speed of a boat in still water is 16 km/h. If it can travel 20 km downstream in the same time as it can travel 12 km upstream, the rate of stream is.

(a) 1 km/h (b) 2 km/h (c) 4 km/h (d) 5 km/h

Solution:

(c) Speed downstream : Speed upstream = 20 : 12 = 5 : 3.

∴ Speed of current = $\frac{5-3}{5+3} \times 16 = 4$ km/h

12. A man can row a boat at 12 km/h in still water. If he takes twice the time to go upstream as to row downstream, find the speed of the boat upstream.

(a) 4 km/h (b) 5 km/h (c) 6 km/h (d) 8 km/h

Solution:

(d) Ratio of time taken (downstream and upstream) = 1 : 2

∴ Ratio of speeds (downstream and upstream) = 2 : 1

∴ Speed of current = $\frac{2-1}{2+1} \times 12 = \frac{1}{3} \times 12 = 4$ km/h.

Speed of boat upstream = 12 – 4 = 8 km/h

Alternative Method:

Speed in still water = $\frac{1}{2}$(speed upstream + speed downstream)

∴ Speed upstream + speed downstream = 2 × 12 = 24 km

But speed upstream : speed downstream = 1 : 2

∴ Speed upstream = $\frac{1}{3} \times 24 = 8$ km/h

13. A man can row a boat to a certain distance upstream in 4 hours and takes 3 hours to row downstream the same distance. What is the speed of boat in still water, if the speed of the stream is 2 km/h ?

(a) 9 km/h (b) 10 km/h (c) 12 km/h (d) 14 km/h

Solution :

(d) Ratio of speed downstream and upstream = 3 : 4

$\therefore$ Speed in still water $= \dfrac{4+3}{4-3} \times 2 = 14$ km/h

14. A man can row a boat to a certain distance upstream in 20 hours and the same distance downstream in 12 hours. What is the speed of boat in still water, if the speed of the stream is 6 km/h ?

(a) 15 km/h (b) 18 km/h (c) 24 km/h (d) 30 km/h

Solution :

(c) Ratio of time taken upstream and downstream = 20 : 12 = 5 : 3

Ratio of speeds upstream and downstream = 3 : 5

$\therefore$ Speed in still water $= \dfrac{5+3}{5-3} \times 6 = 24$ km/h.

15. A man can row a boat at 6 km/h in still water and speed of the current is 2 km/h. If he takes 45 minutes to row the boat to a place and back. Find the distance between the two places.

(a) 2 km (b) 4 km (c) 5 km (d) 6 km

Solution:

(a) Speed of the boat downstream = 6 + 2 = 8 km/h

Speed of the boat upstream = 6 – 2 = 4 km/h

$\therefore$ Average speed $= \dfrac{2 \times 8 \times 4}{8+4} = \dfrac{2 \times 8 \times 4}{12}$ km/h

Time taken = 45 minute $= \dfrac{45}{60}$ hour $= \dfrac{3}{4}$ hour

Distance travelled $= \dfrac{2 \times 8 \times 4}{12} \times \dfrac{3}{4} = 4$ km

$\therefore$ Distance between the two places $= \dfrac{1}{2} \times 4 = 2$ km.

16. A man can row a boat at 5 km/h in still water and speed of the current is 1 km/h. He takes 5 hours to row boat to a place and come back to the starting point. Find the distance between the two places.

(a) 4 km (b) 6 km (c) 12 km (d) 24 km

Solution:

(c) Speed of boat downstream = 5 + 1 = 6 km/h

Speed of boat upstream = 5 – 1 = 4 km/h

$\therefore$ Distance between two places = Average Speed × Time $\times \dfrac{1}{2}$

$= \dfrac{2 \times 6 \times 4}{10} \times \dfrac{5}{2} = 12$ km.

17.. A man rows boat to a place covering 72 km distance and back in 15 hours. He finds that he can row 3 km with the stream in the same time as 2 km against the stream. Find the speed of the stream.

(a) 1 km/h (b) 2 km/h (c) 3 km/h (d) 4 km/h

Solution:

(b) Speed downstream : Speed upstream = 3 : 2

∴ Time taken to row downstream : Time taken to row upstream = 2 : 3

But total time taken = 15 hours

∴ Time taken to row downstream $= \frac{2}{5} \times 15$ hours = 6 hours

Time taken to row upstream = 15 – 6 = 9 hours

∴ Speed downstream $= \frac{72}{6} = 12$ km/h

Speed upstream $= \frac{72}{9} = 8$ km/h

∴ Speed of the stream $= \frac{12-8}{2} = 2$ km/h

Alternative Method:

Let downstream and upstream speeds are 3 km/h and 2 km/h respectively.

Then distance covered in 15 hours $= \frac{2 \times 3 \times 2}{5} \times 15 = 36$ km

But actual distance covered = 2 × 72 km = 144 km (i.e. 4 times 36 km)

∴ Speeds are 4 × 3 and 4 × 2, i.e. 12 and 8 km/h

∴ Speed of the stream $= \frac{12-8}{2} = 2$ km/h

18. The speed of a boat in still water is 12 km/h and the speed of the stream is 3 km/h. If total time taken to travel from A and B and back is 8 hours, find the distance between A and B.

(a) 25 km (b) 30 km (c) 40 km (d) 45 km

Solution :

(d) Average speed $= \frac{(12+3)\times(12-3)}{12} = \frac{15\times9}{12}$

Distance between A and B $= \frac{1}{2} \times \frac{15\times9}{12} \times 8 = 45$ km.

19. A boat whose speed in the still water is 16 km/h, covers a distance of 30 km upstream and then returns taking 4 hours in total. What is the speed of the current?

(a) 2 km/h (b) 3 km/h (c) 4 km/h (d) 5 km/h

Solution:

(c) Total distance covered = 2 × 30 km = 60 km

Time taken = 4 hours,

∴ Average speed $= \frac{60}{4} = 15$ km/h.

Let the speed of current = 'x' km/h

Then $\frac{16^2 - x^2}{16} = 15$

$\therefore \quad 16^2 - x^2 = 16 \times 15$

$\therefore \quad x^2 = 16^2 - 16 \times 15 = 16 \times (16 - 15) = 16$

$\therefore \quad x = \sqrt{16} = 4$

$\therefore$ Speed of current = 4 km/h

20. **The speed of current is 5 km/h. A boat goes 10 km upstream and then returns to the starting point in 50 minutes. What is the speed of boat in still water ?**

(a) 20 km/h (b) 25 km/h (c) 28 km/h (d) 30 km/h

Solution :

(b) Total distance covered = 2 × 10 km = 20 km

$\therefore$ Average speed $= 20 \times \frac{60}{50} = 24$ km/h.

Let speed of boat in still water = *x* km/h

Then $\frac{x^2 - 5^2}{x} = 24$

$\therefore x^2 - 24x - 25 = 0$

$x^2 + x - 25x - 25 = 0$

$x(x + 1) - 25(x + 1) = 0$

$\therefore x = 25$ or -1

$= 15 \times \left(1+\frac{1}{11}\right) = 15 + \frac{15}{11} = 15 + 1\frac{4}{11} = 16\frac{4}{11}$ minutes.

Hence the Hands are opposite to each other at $16\frac{4}{11}$ minutes past 9.

3. At what time between 4'o clock and 5'o clock will the hands of a clock be at right angle?

Solution:

At 4'o clock, the Minute Hand is 20 minutes space behind the Hour Hand.

The hands of the clock will be at right angle twice between 4 and 5'o clock.

(a) When the Minute Hand is 15 minute space behind the Hour Hand, i.e., when Minute Hand gains (20 – 15) = 5 minutes' space over Hour Hand.

Time taken by Minute Hand to gain 5 minutes

$= 5 \times \left(1+\frac{1}{11}\right) = 5 + \frac{5}{11} = 5\,\frac{5}{11}$ minutes.

Hence the Hands are at right angle at $5\,\frac{5}{11}$ minutes past 4.

(b) The hands will be at right angle again when Minute Hand is 15 minute space ahead of Hour Hand *i.e.* when Minute Hand has gained (20 + 15) = 35 minute's space over Hour Hand.

Time taken by Minute Hand to gain 35 minutes

$= 35 \times \left(1+\frac{1}{11}\right) = 35 + \frac{35}{11} = 35+3\frac{2}{11} = 38\frac{2}{11}$ minutes

∴ They will be at right angle at $38\frac{2}{11}$ minutes past 4.

4. What will be the acute angle between hands of a clock at 2 : 30?

Solution.

At 2'O Clock, Minute Hand will be 10 × 6 = 60° behind the Hour hand.

In 30 minutes, Minute hand will gain $\left(5\frac{1}{2}\right)^{\circ} \times 30 = 150 + 15 = 165°$

∴ Angle between Hour Hand and Minute Hand = 165 – 6 0 = 105°.

Chapter 31

LOG

Logarithms (Log): The Log of a number to a given base is the power to which the base must be raised in order to get the given number.

If $a^m = b$, then $\text{Log}_a b = m$

Note: When no base is mentioned with the Log, it is assumed that the base is 10.

FORMULAE

1. $\text{Log}(m \times n) = \text{Log } m + \text{Log } n$
2. $\text{Log}\left(\frac{m}{n}\right) = \text{Log } m - \text{Log } n$
3. $\text{Log } m^n = n \text{ Log } m$
4. $\text{Log } m^{-n} = -n \text{ Log } m$
5. $\log \sqrt[n]{m} = \log m^{1/n} = \frac{1}{n} \log m$
6. $\text{Log } x + \text{Log}\left(\frac{1}{x}\right) = \text{Log } x \times \frac{1}{x} = \text{Log } 1 = 0$
7. $\text{Log } \frac{x}{y} = -\text{Log } \frac{y}{x}$
8. $\text{Log}_a a = 1$, since $a^1 = a$
9. $\text{Log}_a 1 = 0$, since $a^0 = 1$
10. $\text{Log } x^a \div \text{Log } x = a \log x \div \log x = a$
11. $\text{Log}_a b = \frac{1}{\text{Log}_b a}$

    $\therefore \text{Log}_a b \times \text{Log}_b a = 1$
12. $\text{Log}_a x = \text{Log}_a b \times \text{Log}_b x$
13. $\text{Log}_a x = \frac{\text{Log}_b x}{\text{Log}_b a}$

SOLVED EXERCISE

1. $\text{Log } 5 + \text{Log } 6 - \text{Log } 3 = ?$

Solution:

$\text{Log } 5 + \text{Log } 6 - \text{Log } 3$

9. Log 72 = ?

Solution:

$$\text{Log } 72 = \text{Log } (2 \times 2 \times 2 \times 3 \times 3) = \text{Log } (2^3 \times 3^2)$$

$$= \text{Log } (2^3) + \text{Log } (3^2) = 3 \text{ Log } 2 + 2 \text{ Log } 3$$

10. Log 20 = ?

Solution:

$$\text{Log } 20 = \text{Log } \left(\frac{100}{5}\right) = \text{Log } 100 - \text{Log } 5$$

$$= \text{Log } (10^2) - \text{Log } 5 = 2 \text{ Log } 10 - \text{Log } 5 = 2 - \text{Log } 5$$

Alternative Solution:

$$\text{Log } 20 = \text{Log } (10 \times 2) = \text{Log } 10 + \text{Log } 2 = 1 + \text{Log } 2$$

11. If Log 2 = 0.3010, then Log 5 = ?

Solution:

$$\text{Log } 5 = \text{Log} \frac{10}{2} = \text{Log } 10 - \text{Log } 2 = 1 - 0.3010 = 0.6990$$

12. If Log 2 = 0.301, then Log 50 = ?

Solution:

$$\text{Log } 50 = \text{Log } \frac{100}{2} = \text{Log } 100 - \text{Log } 2$$

$$= \text{Log } (10^2) - \text{Log } 2 = 2 \text{ Log } 10 - \text{Log } 2$$

$$= 2 - 0.301 = 1.699$$

13. If Log 2 = 0.3010, Log 3 = 0.4771, then Log 15 = ?

Solution:

$$\text{Log } 15 = \text{Log } \left(\frac{10 \times 3}{2}\right) = \text{Log } 10 + \text{Log } 3 - \text{Log } 2$$

$$= 1 + 0.4771 - 0.3010 = 1.1761$$

14. If Log 3 = 0.4771, then Log $(30)^4$

Solution:

$$\text{Log } (30)^4 = 4 \text{ Log } 30 = 4 \text{ Log } (10 \times 3) = 4 \text{ (Log } 10 + \text{Log } 3)$$

$$= 4 \text{ (}1 + \text{Log } 3) = 4 + 4 \text{ Log } 3 = 4 + 4 \text{ (}0.4771) = 4 + 1.9084 = 5.9084$$

15. If Log 27 = 1.431, Log 9 = ?

Solution:

$$\text{Log } 27 = \text{Log } 3^3 = 3 \text{ Log } 3$$

$$\therefore \text{ Log } 3 = \frac{\text{Log } 27}{3} = \frac{1.431}{3} = 0.477$$

$$\text{Now Log } 9 = \text{Log } 3^2 = 2 \text{ Log } 3 = 2 \times 0.477 = 0.954$$

Solution:

$$\text{Log } x = \text{Log } 5 + 2 \text{ Log } 3 - \frac{1}{2}\text{Log } 25$$

$\therefore$ Log x = Log 5 + Log 3^2 − Log $25^{\frac{1}{2}}$

$\therefore$ Log x = Log 5 + Log 9 – Log 5

$\therefore$ Log x = Log 9

$\therefore$ x = 9

22. If Log 9 = 0.9542, how many digits are there in 9^{20}?

Solution:

9^{20} = Antilog (Log 9^{20}) = Antilog (20 Log 9) = Antilog (20 × 0.9542) = Antilog 19.084

In Antilog of 19.084, decimal will be after 19 + 1 = 20 digits from the left side.

$\therefore$ 9^{20} has 20 digits.

23. If Log 2 = 0.3010, how many digits are there in 2^{15}?

Solution:

2^{15} = Antilog (Log 2^{15}) = Antilog (15 Log 2) = Antilog (15 × 0.3010) = Antilog 4.515

In Antilog of 4.515, decimal will be after 4 + 1 = 5 digits from the left side.

$\therefore$ 2^{15} has 5 digits.

13. If **length** of rectangle **is increased by x%** and **breadth by y%,**

Then percent **increase in its area** $= x + y + \frac{xy}{100}$

Alternative Method :

$$\frac{(100+x)\times(100+y)}{100}-100$$

14. If **all the sides** of a two-dimensional figure are **increased by x%,**

Then percent **increase in its area** $= x + x + \frac{x^2}{100} = 2x + \frac{x^2}{100}$

Alternative Method :

$$\frac{(100+x)^2}{100}-100$$

15. If all sides of a quadrilateral are increased by x%,

Then its corresponding diagonals will also increase by x%.

16. Ratio between area and circumference of a circle is r : 2, where 'r' is radius of the circle.

Proof:

Area of the circle $= \pi r^2$

Circumference of the circle $= 2\pi r$

Ratio between area and circumference of a circle $= \pi r^2 : 2\pi r = r : 2$

17. Ratio between area of a square and area of the square drawn on its diagonal is 1 : 2.

Proof:

Let side of the small square = 1 cm

Then side of the bigger square = Diagonal of the smaller square = $\sqrt{2}$ cm

$\therefore$ Ratio between the areas $= (1)^2 : (\sqrt{2})^2 = 1 : 2$

18. Area of the largest circle inscribed in a square of side 'a' is $\pi\left(\frac{a}{2}\right)^2$.

Proof:

Diameter of circle = Side of square = a

$\therefore$ Radius of circle $= \frac{a}{2}$

$\therefore$ Area of circle $= \pi\left(\frac{a}{2}\right)^2$

19. Area of the largest triangle that can be inscribed in a semi-circle of radius of 'r' cm is r^2 cm^2.

Proof:

Area of triangle = $\frac{1}{2}$ × Base × Height

= $\frac{1}{2} \times 2r \times r = r^2$ cm^2

20. Area of a square inscribed in a circle of radius 'r' is $2r^2$.

Proof:

Diagonal of square = Diameter of circle = 2r

Area of square = $\frac{1}{2}(\text{Diagonal})^2 = \frac{1}{2}(2r)^2 = 2r^2$

21. Area of a square circumscribing a circle is double to that of the one inscribed in the circle.

Proof:

Let side of square circumscribing the circle = 'a'

Then diameter of the circle = 'a'

Diagonal of inner square = Diameter of circle = 'a'

∴ Area of inner square = $\frac{1}{2}(a^2)$

∴ Ratio between areas of two squares = $a^2 : \frac{1}{2}a^2 = 2 : 1$

22. Area of a circle circumscribing a square is double to that of one inscribed in the square.

Proof:

Let radius of inner circle = r

Then side of square = 2r

Diagonal of square = $2\sqrt{2}\,r$

∴ Radius of outer circle = $\frac{1}{2} \times 2\sqrt{2}\,r = \sqrt{2}\,r$

Area of inner circle = πr^2

Area of outer circle = $\pi\left(\sqrt{2}\,r\right)^2 = 2\pi r^2$

∴ Ratio between areas of two circles = $2\pi r^2 : \pi r^2 = 2 : 1$

23. If a circle is drawn in a square, then the area of square not covered by the circle is $\frac{6}{7}x^2$, where 'x' is radius of the circle.

Solution:

Let radius of circle = x

Then side of square = 2x

Area of square = $(2x^2) = 4x^2$

Area of circle = πx^2

∴ Area of square which is not covered by the circle

$$= 4x^2 - \pi x^2 = x^2\,(4-\pi) = x^2\left(4-\frac{22}{7}\right) = \frac{6}{7}x^2$$

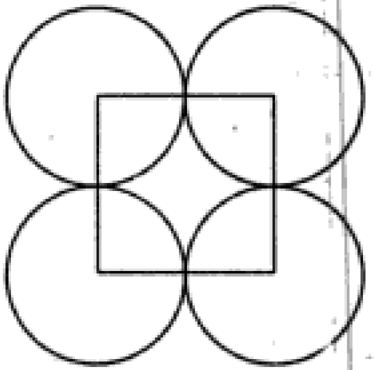
Note: 'x' = $\frac{1}{2}$ × Side of the square = Radius of circle

24. If four circles of equal radius are drawn at the each corner of a square, then the area of the square which is not covered by any of the circles is $\frac{6}{7}x^2$, where 'x' is radius of the circle.

Solution:

Let side of square = 2x

Then radius of one circle = x



Area of square = ($2x^2$) = $4x^2$

Area of one circle = πx^2

Area of one circle which is covered by square = $\frac{1}{4}\pi x^2$

∴ Area covered by 4 circles = $4 \times \frac{1}{4}\pi x^2 = \pi x^2$

∴ Area of square which is not covered by any of the circle

$$= 4x^2 - \pi x^2 = x^2\,(4-\pi) = x^2\left(4-\frac{22}{7}\right) = \frac{6}{7}x^2$$

25. If a **square field** with 'x' cm a side is **surrounded by a path of uniform width** of 'y' cm. Then area of path = 4y (x + y)

Proof:

Area of inner square = x^2

Side of outer square = (x + 2y)

Area of outer square = $(x + 2y)^2 = x^2 + 4xy + 4y^2$

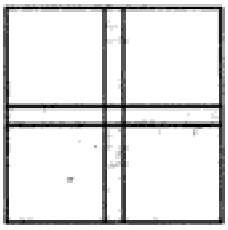
∴ Area of path = $x^2 + 4xy + 4y^2 - x^2$

= $4xy + 4y^2$

= 4y (x + y)

26. Area of **uniform path outside the rectangular field** surrounding it:

= 2 × Width of path × [Length of field + Breadth of field + 2 (Width of path)]



Area of **uniform path inside a rectangular field,** surrounded by the field:

= 2 × Width of path × [Length of field + Breadth of field – 2 (Width of path)]

Area of **uniform path** of 'x' metre width **running from the centre of each side of a rectangular field to the centre of opposite side:**

= Width of path × (Length of field + Breadth of field) – (Width of path)^2

7. If the length of a rectangle is 8 cm and its diagonal is 10 cm, find its area.

(a) 64 cm^2 (b) 40 cm^2 (c) 48 cm^2 (d) 80 cm^2

Solution :

(c) Breadth of the rectangle $= \sqrt{10^2 - 8^2} = 6$ cm

$\therefore$ Area = 6 × 8 = 48 cm^2

8. A man takes 15 minutes to walk along the diagonal of a square field at the rate of 2 km/h. The area of the field is :

(a) 6250 m^2 (b) 12500 m^2 (c) 25000 m^2 (d) 50000 m^2

Solution :

(b) $\text{Diagonal} = 2 \times 1000 \times \frac{15}{60} = 500\text{m}$

$\therefore$ $\text{Area} = \frac{1}{2} \times (500)^2 = 12500 \text{ m}^2$

9. Area of a square plot is 200 m^2. Find the length of its diagonal.

(a) 20 metre (b) $2\sqrt{10}$ metre

(c) 10 metre (d) 15 metre

Solution:

(a) Diagonal = $\sqrt{2 \times \text{Area of square}} = \sqrt{2 \times 200} = 20$ metre

10. Find the area of a triangle whose base is 8 cm and height is 5 cm.

(a) 13 cm^2 (b) 20 cm^2 (c) 40 cm^2 (d) 80 cm^2

Solution:

(b) Area of triangle = $\frac{1}{2} \times 8 \times 5 = 20$ cm^2

11. The base of a triangle is $\frac{4}{3}$ times its height. If area of the triangle is 600 cm^2, find its base.

(a) 30 cm (b) 40 cm (c) 50 cm (d) 60 cm

Solution :

(b) Let height of the traingle = 3x cm

Then base of the traingle = 4x cm

$\therefore$ $\text{Area} = \frac{1}{2} \times 3x \times 4x = 6x^2 = 600 \text{ cm}^2$

$\Rightarrow$ $x = \sqrt{\frac{600}{6}} = 10 \text{ cm}$

$\therefore$ Base of the triangle = 4 × 10 = 40 cm

12. The angles of a triangle are in the ratio of 3 : 4 : 5. What is degree of the smallest angle?

(a) 15° (b) 30° (c) 45° (d) 60°

Solution:

(c) Sum of ratios = 3 + 4 + 5 = 12

$\therefore$ Smallest angle = $\frac{3}{12} \times 180° = 45°$

13. The base of a right angle triangle is 8 cm and hypotenuse is 10 cm. Find its area.

(a) 12 cm^2 (b) 24 cm^2 (c) 40 cm^2 (d) 80 cm^2

Solution:

(b) Height of the right-angle triangle = $\sqrt{10^2 - 8^2} = \sqrt{36} = 6$

$\therefore$ Area of triangle = $\frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

14. Two equal sides of a right isosceles triangle are 8 cm each. What is its area ?

(a) 8 cm^2 (b) 16 cm^2 (c) 32 cm^2 (d) 64 cm^2

Solution :

(c) Two equal sides are base and height of the triangle.

$\therefore$ Area of the triangle $= \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2} \times 8 \times 8 = 32 \text{ cm}^2$

15. If sides of a triangle are 8 cm, 15 cm and 17 cm respectively. Find its area.

(a) 60 cm^2 (b) 120 cm^2 (c) 240 cm^2 (d) 360 cm^2

Solution:

(a) $s = \frac{8 + 15 + 17}{2} = 20 \text{ cm}$

$\therefore$ Area = $\sqrt{20 \times (20 - 8) \times (20 - 15) \times (20 - 17)}$

$= \sqrt{20 \times 12 \times 5 \times 3}$

$= \sqrt{4 \times 5 \times 3 \times 4 \times 5 \times 3} = 4 \times 5 \times 3 = 60 \text{ cm}^2$

Trick:

The triangle is right angle triangle as $17^2 = 15^2 + 8^2$

$\therefore$ Area of right angle triangle = $\frac{1}{2} \times 8 \times 15 = 60 \text{ cm}^2$

16. If diameter of a circle is 28 cm, then area of the circle is:

(a) 77 cm^2 (b) 154 cm^2 (c) 308 cm^2 (d) 616 cm^2

Solution:

(d) Radius of the circle = $\frac{1}{2} \times 28 \text{ cm} = 14 \text{ cm}$

$\therefore$ Area of the circle = $\frac{22}{7} \times 14 \times 14 = 616 \text{ cm}^2$

17. Two parallel sides of a trapezium are 4 cm and 5 cm respectively. The perpendicular distance between the parallel sides is 6 cm. Find the area of the trapezium.

(a) 27 cm^2 (b) 30 cm^2 (c) 40 cm^2 (d) 54 cm^2

Solution:

(a) Area of trapezium = $\frac{1}{2} \times 6 \times (4 + 5) = 27$ cm^2

18. In a quadrilateral, length of one diagonal is 15 cm and the lengths of the perpendiculars drawn from the opposite vertices are 7 cm and 9 cm. Find the area of the quadrilateral.

(a) 30 cm^2 (b) 60 cm^2 (c) 90 cm^2 (d) 120 cm^2

Solution:

(d) Area of quadrilateral = $\frac{1}{2} \times 15 \times (7 + 9) = 120$ cm^2

19. If circumference of a circle is 88 cm. Find radius of the circle.

(a) 7 cm (b) 14 cm (c) 22 cm (d) 44 cm

Solution:

(b) Radius of circle = $\frac{\text{Circumference}}{2\pi} = 88 \times \frac{1}{2} \times \frac{7}{22} = 14$ cm

20. If area of a circle is 154 cm^2, find its radius.

(a) 7 cm (b) 14 cm (c) 21 cm (d) 28 cm

Solution:

(a) Area of circle = πr^2

$\therefore r^2 = \frac{\text{Area}}{\pi}$

$\therefore r = \sqrt{154 \times \frac{7}{22}} = 7$ cm

21. The length and breadth of a rectangle are in the ratio of 5 : 3. If area of the rectangle is 60 cm^2. Find its length.

(a) 2 cm (b) 5 cm (c) 6 cm (d) 10 cm

Solution:

(d) Let length and breadth are 5x and 3x respectively.

Then Area = (5x) (3x) = 60

$\therefore x^2 = \frac{60}{5 \times 3} = 4$

$\therefore x = \sqrt{4} = 2$

$\therefore$ Length of rectangle = $2 \times 5 = 10$ cm.

22. The length and breadth of a rectangle are in the ratio of 5 : 3. Find its area, if the perimeter is 64 cm.

(a) 15 cm^2 (b) 60 cm^2 (c) 150 cm^2 (d) 240 cm^2

Solution:

(d) Let length and breadth are 5x and 3x respectively.

Then Perimeter = 2 (5x + 3x) = 64

$\therefore x = \frac{64}{16} = 4$ cm

$\therefore$ Length and Breadth are 5×4 and $3 \times 4 = 20$ cm and 12 cm respectively.

$\therefore$ Area of rectangle $= 20 \times 12 = 240$ cm^2

23. Area of a rectangle plot is 1200 m^2. Find the perimeter of the plot if its length and breadth are in the ratio of 4 : 3.

(a) 120 metre (b) 140 metre (c) 700 metre (d) 1400 metre

Solution:

(b) Let length and breadth of the plot are 4x and 3x respectively.

Then $(4x) \times (3x) = 1200$

$\therefore 12x^2 = 1200$

$$\therefore x^2 = \frac{1200}{12} = 100$$

$$\therefore x = \sqrt{100} = 10 \text{ metre}$$

$\therefore$ Length and breadth of the plot are 40 metre and 30 metre respectively.

$\therefore$ Perimeter of the plot $= 2\,(40 + 30) = 140$ metre

24. The cost of flooring a plot at the rate of Rs. 50 per metre is Rs. 30000. If ratio of length and breadth of the plot is 3 : 2, find the cost of fencing at the rate of Rs. 25 per metre.

(a) Rs. 2500 (b) Rs. 5000 (c) Rs. 10000 (d) Rs. 15000

Solution :

(a) Area of plot $= \dfrac{30000}{50} = 600 \text{ m}^2$

Area of plot $= (3x) \times (2x) = 600$ m^2

$\therefore x = 10$ metre

Perimeter of plot $= 2 \times (30 + 20) = 100$ m

$\therefore$ Cost of fencing $= 25 \times 100 =$ Rs. 2500

25. The cost of carpeting a hall at Rs. 30 per square metre is Rs. 5400. Had the length been 4 metre less, the cost would have been Rs. 3960. Find the breadth of the hall.

(a) 12 m (b) 14 m (c) 15 m (d) 16 m

Solution :

(a) Difference in costs = Rs. 5400 – Rs. 3960 = Rs. 1440

Difference in area $= 1440 \div 30 = 48$ m^2

$\therefore$ Breadth of hall $= 48 \div 4 = 12$ m

26. Perimeter of a rectangle is 28 cm and its area is 48 sq. cm. Find length of its diagonal.

(a) 10 cm (b) 5 cm (c) $5\sqrt{2}$ cm (d) $10\sqrt{2}$ cm

Solution:

(a) Perimeter $= 2\,(l + b) = 28$ cm.

$\therefore (l + b) = 14$ cm

Area $= (l \times b) = 48$

$l^2 + b^2 = (l + b)^2 - 2lb = (14)^2 - 2 \times 48 = 196 - 96 = 100$

$$\therefore \text{Diagonal} = \sqrt{l^2 + b^2} = \sqrt{100} = 10 \text{ cm.}$$

27. If perimeter and area of a square are equal. Side of the square (in cm) is:

(a) 1 cm (b) 2 cm (c) 4 cm (d) 8 cm

Solution:

(c) Given $(\text{Side})^2 = 4 \times (\text{Side})$

$\therefore$ Side = 4 cm

28. The diameter of a wheel is 14 cm. How many revolutions the wheel is required to make, to cover a distance of 880 cm?

(a) 10 (b) 20 (c) 25 (d) 40

Solution:

(b) Radius of the wheel $= \frac{1}{2} \times 14 = 7$ cm

Distance covered in 1 revolution = Circumference of the wheel

$= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm.

$\therefore$ Revolutions required to make to cover 880 cm $= \frac{880}{44} = 20$

29. The diameter of the wheel of a vehicle is 5 metre. It makes 7 revolutions per 9 seconds. What is speed of the vehicle in km/h?

(a) 30 km/h (b) 36 km/h (c) 40 km/h (d) 44 km/h

Solution:

(d) Radius of the wheel $= \frac{5}{2}$ metre

Distance covered in 1 revolution = Circumference of the wheel = $2\pi r$

$= 2 \times \frac{22}{7} \times \frac{5}{2}$ metre

$\therefore$ Distance covered in one second $= 2 \times \frac{22}{7} \times \frac{5}{2} \times \frac{7}{9}$ metre

$\therefore$ Speed per hour $= 2 \times \frac{22}{7} \times \frac{5}{2} \times \frac{7}{9} \times \frac{18}{5} = 44$ km/h

30. How many revolutions are required to be made by a wheel of 42 cm in diameter, to cover the same distance as covered by a wheel of 49 cm in radius in 18 revolutions ?

(a) 21 (b) 42 (c) 49 (d) 84

Solution :

(b) Ratio of two radii $= \frac{1}{2} \times 42 : 49 = 3 : 7$

$\therefore$ Revolutions made by smaller wheel $= \frac{7}{3} \times 18 = 42$

31. The perimeter of a rhombus is 60 cm and one of its diagonal is 24 cm. Find the other diagonal of the rhombus.

(a) 12 cm (b) 18 cm (c) 24 cm (d) 36 cm

$=2 \times 2.5 \times (40+35+2 \times 2.5)$

$=5 \times (75+5) = 400\,\text{m}^2$

Alternative Method :

$45 \times 40 - 40 \times 35 = 1800 - 1400 = 400$

or $45 \times 40 - 40 \times 35 = 40 \times (45 - 35) = 400$

36. A field is 100 metre long and 60 metre wide. A path of uniform width of 5 metre runs round it on the inside. Find the area of the path.

(a) 200 m^2 (b) 800 m^2 (c) 1500 m^2 (d) 1600 m^2

Solution:

(c) Area of path

$= 2 \times \text{Width} \times [\text{Length} + \text{Breadth} -- (2 \times \text{Width})]$

$= 2 \times 5 \times (100 + 60 - 10)$

$= 10 \times 150 = 1500\,\text{m}^2$

Alternative Method :

$100 \times 60 - 90 \times 50 = 6000 - 4500 = 1500$

37. Find area of uniform path of width 2 metre running from centre of each side to the opposite side of a rectangle field measuring 17 metre by 12 metre.

(a) 24 m^2 (b) 34 m^2 (c) 48 m^2 (d) 54 m^2

Solution:

(d) Area of path

$= \text{Width of path} \times (\text{Length of field} + \text{Breadth of field}) - (\text{Width of path})^2$

$= 2 \times (17 + 12) - (2)^2$

$= 58 - 4 = 54\,\text{m}^2$

38. If length of a rectangle is reduced by 20% and width remaining the same. Area of the resulting rectangle will be decreased by:

(a) 10% (b) 20% (c) 25% (d) 40%

Solution:

(b) Decrease in area = 20%

39. If length of a rectangle is increased by 50%, by what percent should its width be decreased to get the same area as before?

(a) 25% (b) 33.33% (c) 50% (d) 100%

Solution:

(b) Increase in length = $50\% = \frac{1}{2}$

$\therefore$ Decrease in width $= \frac{1}{2+1} = \frac{1}{3} = 33.33\%$

40. If length of a rectangle is decreased by 20%, by what percent should its width be increased to get the same area as before?

(a) 16.67% (b) 20% (c) 25% (d) 30%

Solution:

(c) Decrease in length = $20\% = \frac{1}{5}$

∴ Increase in width = $\frac{1}{5-1} = \frac{1}{4} = 25\%$

41. If the length of a rectangle is increased by 30% and width by 20%, its area will increase by:

(a) 50% (b) 25% (c) 56% (d) 60%

Solution:

(c) Increase in Area = $30 + 20 + \frac{30 \times 20}{100} = 56\%$

Alternative Method :

13 × 12 = 156

156 – 100 = 56 %

42. If all the sides of a square are increased by 20%, its area is increased by:

(a) 4% (b) 20% (c) 40% (d) 44%

Solution:

(d) Increase in Area = $2 \times 20 + \frac{(20)^2}{100} = 40 + 4 = 44\%$

Alternative Method :

12 × 12 = 144

144 – 100 = 44 %

43. If all the sides of a square are increased by 200%, its area is increased by :

(a) 200% (b) 400% (c) 600% (d) 800%

(d) Increase in Area = $2 \times 200 + \frac{(200)^2}{100} = 400 + 400 = 800\%$

44. If radius of a circle is increased by 10%. Area of the resulting circle will be increased by:

(a) 11% (b) 20% (c) 21% (d) 25%

Solution:

(c) Increase in Area = $2 \times 10 + \frac{(10)^2}{100} = 20 + 1 = 21\%$

Alternative Method :

11 × 11 = 121

121 – 100% = 21 %

45. If all the sides of a square are decreased by 10%, its area will be decreased by:

(a) 19% (b) 20% (c) 21% (d) 22%

Solution:

(a) Increase in Area = $2 \times (-10) + \frac{(-10)^2}{100} = -20 + 1 = (-)\ 19\%$

∴ Decrease in Area = 19%

Alternative Method :

9 × 9 = 81

81 – 100 = – 19%

Solution:

(b) Let radii of the circles are 'R' and 'r' respectively.

Then $2\pi R = 176$ and $2\pi r = 132$

$\therefore 2\pi R - 2\pi r = 176 - 132$

$\therefore 2\pi(R - r) = 44$

$\therefore (R - r) = \frac{44}{2\pi} = 44 \times \frac{1}{2} \times \frac{7}{22} = 7$ metre

66. A wire is bent into the shape of a square, encloses an area of $81 cm^2$. If the wire is bent into a semicircular shape, find its area.

(a) $44 cm^2$ (b) $77 cm^2$ (c) $154 cm^2$ (d) $231 cm^2$

Solution:

(b) Perimeter of square $= 4 \times \sqrt{81} = 36$ cm

Boundary of semi-circle $= \pi r + 2r = 26$ cm

$\therefore (\pi + 2)r = 36$

$$\therefore \left(\frac{22}{7} + 2\right)r = 36$$

$$\therefore r = 36 \times \frac{7}{36} = 7 \text{ cm}$$

$$\text{Area of semi-circle} = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 = 77 \text{ cm}^2$$

67. A wire, bent in the form of a square, encloses an area of $484 cm^2$. If the same wire is bent so as to form a circle, then area of the circle is :

(a) $440 cm^2$ (b) $475 cm^2$ (c) $525 cm^2$ (d) $616 cm^2$

Solution:

(d) Ratio of area of square and circle with same perimeter = 11 : 14

$$\therefore \text{Area of circle} = \frac{14}{11} \times 484 = 616 \text{ cm}^2$$

68. The parallel sides of a trapezium are 15 cm and 29 cm. If the non-parallel sides are 13 cm and 15 cm, what is the area of trapezium ?

(a) $225 cm^2$ (b) $264 cm^2$ (c) $308 cm^2$ (d) $616 cm^2$

Solution:

(b) Let ABCD is a trapezium and CE || AD.

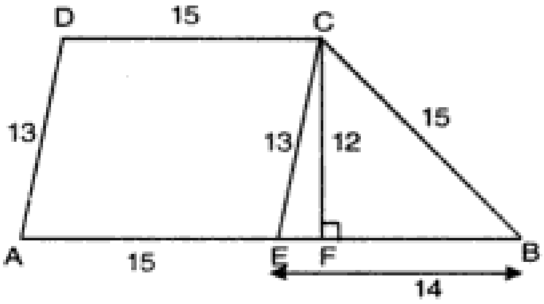
AECD is parallelogram.

CE = AD = 13 cm

EB = AB – AE = 29 – 15 = 14 cm,

CB = 15 cm

In triangle CEB, $s = \frac{1}{2} \times (13 + 14 + 15) = 21$ cm



$$\text{Area} = \sqrt{21\times8\times7\times6} = 84\ \text{cm}^2$$

$$\therefore\ CF = \frac{84\times2}{14} = 12\ \text{cm}$$

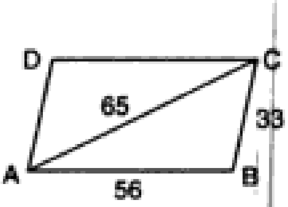
$$\text{Area of trapezium ABCD} = \frac{1}{2}\times(15+29)\times12 = 264\ \text{cm}^2$$

69. In a parallelogram two adjacent sides are 33 cm and 56 cm and one diagonal is 65 cm. What is its area?

(a) $824\ \text{cm}^2$ (b) $924\ \text{cm}^2$ (c) $1648\ \text{cm}^2$ (d) $1848\ \text{cm}^2$

Solution :

(d) In traingle ABC, $s = \frac{1}{2}\times(56+33+65) = 77$ cm



$$\text{Area of } \Delta ABC = \sqrt{77\times21\times44\times12} = 924\ \text{cm}^2$$

$$\text{Area of ABCD} = 2\times\Delta ABC = 2\times924 = 1848\ \text{cm}^2$$

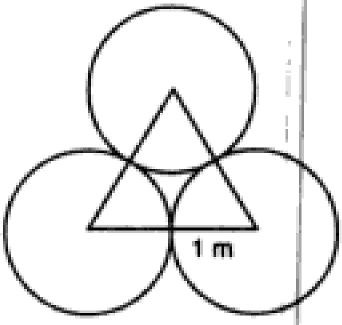
70. Three coins of the same size are placed on a table such that each of them touched the other two. Find the area enclosed by the coins, if radius of each coin is 1 cm.

(a) $\left(\frac{\pi}{2}-\sqrt{3}\right)\text{cm}^2$ (b) $\left(\sqrt{3}-\frac{\pi}{2}\right)\text{cm}^2$

(c) $\left(\pi-\sqrt{3}\right)\text{cm}^2$ (d) $\left(\sqrt{3}-\pi\right)\text{cm}^2$

Solution :

(b) AB = BC = AC = 2 cm



$$\Delta ABC = \frac{\sqrt{3}}{4}\times(2)^2 = \sqrt{3}\ \text{cm}^2$$

$$\angle A = \angle B = \angle C = 60°$$

$$\text{Area of three sectors} = 3\times\pi\times(1)^2\times\frac{60}{360} = \frac{\pi}{2}\ \text{cm}^2$$

$$\therefore\ \text{The area enclosed by the coins} = \left(\sqrt{3}-\frac{\pi}{2}\right)\text{cm}^2$$

Chapter 33

MENSURATION – THREE DIMENSIONAL

Three-dimensional figures: Three-dimensional figures have three dimensions viz. length, breadth and height.

Surface Area: Total of surface areas of each side of the figure. It is always measured in square units.

Volume: Space occupied by the object is called its volume. It is always measured in cube units.

THREE - DIMENSIONAL FIGURES

1. **Cube**: A figure with six square sides.

   Surface Area = $6 \times (\text{Side})^2$

   Volume = $(\text{Side})^3$

   Diagonal = $\sqrt{3} \times$ Side

2. **Cuboid**: A figure with six rectangular sides.

   Surface Area = $2\,(lb + bh + lh)$

   Volume = Area of Base × Height = $l \times b \times h$

   Diagonal = $\sqrt{l^2 + b^2 + h^2}$

   Where l, b and h are length, breadth and height of the cuboid.

3. **Sphere**:

   Surface Area = $4\pi r^2$

   Volume = $\frac{4}{3}\pi r^3$

4. **Hemisphere**:

   Curved Surface Area = $2\pi r^2$

   Total Surface Area = $3\pi r^2$

   Volume = $\frac{2}{3}\pi r^3$

5. **Cylinder:**

   Curved Surface Area = $2\pi rh$ = Circumference of Base × Height

   Total Surface Area = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$

   Volume = $\pi r^2 h$ = Area of Base × Height

6. **Cone:**

   Curved Surface Area = πrl

   Where, 'l' is slant height of the Cone and 'l' = $\sqrt{h^2 + r^2}$

   Total Surface Area = $\pi rl + \pi r^2 = \pi r\,(l + r)$

Volume = $\frac{1}{3}\pi r^2h$

7. **Room:**

Area of four walls = 2 × Height × (Length + Breadth)

Floor Area = Length × Breadth

Volume of Room = Length × Breadth × Height = Floor Area × Height

Note: Diagonal of the room i.e. $\sqrt{l^2 + b^2 + h^2}$.

Where l, b and h respectively are length, breadth and height of the room.

FORMULAE

8. If **ratio of sides** of two three-dimensional figures as spheres, cubes etc. = x : y.

Then **ratio of their surface areas** = $x^2 : y^2$

And **ratio of volumes** = $x^3 : y^3$.

9. If length, breadth and height of a cuboid are increased by x%, y% and z% respectively.

Then increase in its volume = $(x + y + z) + \frac{xy + yz + xz}{100} + \frac{xyz}{100^2}$

10. If all the sides of a three-dimensional figure are increased by x%.

Then increase in surface area = $x + x + \frac{x^2}{100} = 2x + \frac{x^2}{100}$

And increase in volume = $(x + x + x) + \frac{x^2 + x^2 + x^2}{100} + \frac{x^3}{100^2}$

$= 3x + \frac{3x^2}{100} + \frac{x^3}{100^2}$

11. If outer and inner radii of a cylinder are 'R' and 'r' respectively.

Then metal required to make the cylinder = $\pi (R^2 - r^2) h$

Proof:

Volume (outer) of the cylinder = πR^2h

Volume (inner) of the cylinder = πr^2h

∴ Volume of Metal = $\pi R^2h - \pi r^2h = \pi (R^2 - r^2) h$

12. A spherical ball is fitted into a cubical box. Then ratio of volume of the cube and that of the sphere is 6 : π or 21 : 11.

Proof:

Let radius of sphere = x

Then side of cube = 2 × x = 2x

∴ Volume of sphere = $\frac{4}{3}\pi x^3$

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(a) 154 cm^2 (b) 308 cm^2 (c) 462 cm^2 (d) 616 cm^2

Solution:

(b) Radius of hemisphere $= \frac{1}{2} \times 14$ cm = 7 cm.

$\therefore$ Curved surface Area $= 2\pi r^2 = 2 \times \frac{22}{7} \times 7 \times 7 = 308$ cm^2

3. Find the volume of air in a room whose floor area is 40 m^2 and the height is 5 metre.

(a) 8 m^3 (b) 100 m^3 (c) 200 m^3 (d) 400 m^3

Solution:

(c) Volume of air in the room = Floor area × Height $= 40 \times 5 = 200$ m^3

4. Find the height of a cuboid whose volume and base area are 144 m^3 and 18 m^2 respectively.

(a) 6 metre (b) 7 metre (c) $7\frac{1}{2}$ metre (d) 8 metre

Solution:

(d) Height of the cuboid $= \frac{\text{Volume}}{\text{Base Area}} = \frac{144}{18} = 8$ metre

5. Find the length of the longest pole that can be placed in a room 12 metre long, 8 metre broad and 9 metre high.

(a) 15 metre (b) 17 metre (c) 20 metre (d) 29 metre

Solution:

(b) Diagonal of room $= \sqrt{\text{Length}^2 + \text{Breadth}^2 + \text{Height}^2}$

$= \sqrt{12^2 + 8^2 + 9^2} = \sqrt{289} = 17$ metre.

$\therefore$ Length of longest pole = 17 metre

6. The length, breadth and height of a room are 7 metre, 6 metre and 5 metre respectively. Find the area of four walls.

(a) 130 m^2 (b) 144 m^2 (c) 154 m^2 (d) 210 m^2

Solution:

(a) Area of the walls = 2 × Height (Length + Breadth)

$= 2 \times 5 \times (7 + 6) = 130$ m^2

7. The surface area of a cube is 486 cm^2. Find its volume.

(a) 81 cm^3 (b) 243 cm^3 (c) 529 cm^3 (d) 729 cm^3

Solution:

(d) Surface Area of cube $= 6\,(\text{Side})^2 = 486$

$(\text{Side})^2 = \frac{486}{6} = 81$ cm^3

Side of the cube $= \sqrt{81} = 9$ cm.

$\therefore$ Volume of the cube $= (9\text{ cm})^3 = 729$ cm^3.

8. The areas of three adjacent sides of a cuboid are 15 m^2, 12m^2 and 5m^2. What is the volume of the cuboid ?

(a) 30 m^3 (b) 60 m^3 (c) 180 m^3 (d) 75 m^3

MISCELLANEOUS

1. A certain number when successively divided by two different numbers leaves some remainders. Same number when divided by the product of the two divisors will leave remainder equal to:
   First Divisor × Second Remainder + First Remainder

Proof:

Let the number is 'P' and it is divided successively by 'x' and 'y' leaving remainders 'a' and 'b' respectively.

We know that, Dividend = (Divisor × Quotient) + Remainder

Let $P = mx + a$ (where 'm' is assumed quotient)

And $m = ny + b$ (where 'n' is assumed quotient)

$\therefore P = mx + a = (ny + b)x + a = nxy + bx + a$

Clearly, if we divide this number by 'xy', the remainder $= bx + a$.

i.e. First Divisor × Second Remainder + First Remainder

2. Difference between square of a two-digit number (xy) and square of a number formed by interchanging the digits (now yx), is always divisible by 99.

Proof:

$(10x + y)^2 - (10y + x)^2$

$= (100x^2 + y^2 + 20xy) - (100y^2 + x^2 + 20xy)$

$= 99x^2 - 99y^2 = 99(x^2 - y^2) = 99(x - y)(x + y)$

3. Two numbers 'A' and 'B' when separately divided by 'x', leaves remainders 'a' and 'b' respectively.
   (A × B) when divided by 'x', will leave remainder (a × b).

Logic: $(x + a)(x + b) = x^2 + (a + b)x + ab$

Example 1:

Find the remainder on dividing 25 × 26 by 24.

Solution:

25 and 26 when divided by 24, leave remainders 1 and 2 respectively.

$\therefore$ 25 × 26 when divided by 24, leaves remainder $1 \times 2 = 2$

Example 2:

Find the remainder on dividing 153 × 799 by 5.

Solution:

153 and 799 when divided by 5, leave remainders 3 and 4 respectively.

$\therefore$ 153 × 799 when divided by 5, leaves remainder $3 \times 4 = 12$

We know, remainder cannot exceed the divisor.

$$= \frac{4}{3} \times \frac{6 \times 6 \times 6}{3 \times 3} = 32 \text{ cm}$$

∴ Length of the wire is 32 cm.

32. A solid cylinder of metal, whose radius is 6 cm and height is 125 cm is melted into a sphere. Find the raidus of the sphere.

(a) 3 cm (b) 5 cm (c) 12 cm (d) 15 cm

Solution :

(d) Volume of cylinder $= \pi r^2 h = \pi(6)^2 125$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$\therefore\ r^3 = \pi \times 6 \times 6 \times 125 \times \frac{3}{4} \times \frac{1}{\pi}$$

$r = 3 \times 5 = 15$ cm

33. A well 2 metre in radius has been dug 5 metre deep and the earth taken out was spread to make an embankment 1 metre wide round the well. What is the height of the embankment?

(a) 2 metre (b) 4 metre (c) 5 metre (d) 8 metre

Solution:

(b) Volume of earth dug out $= \pi r^2 h = \pi (2)^2 \times 5 = 20\pi$

Area of embankment $= \pi R^2 - \pi r^2 = \pi (3^2 - 2^2) = 5\pi$

Volume of the embankment = Volume of earth dug out

∴ Height of embankment $= \frac{20\pi}{5\pi} = 4$ metre

Short-cut Method:

Height of embankment $= \frac{r^2 h}{R^2 - r^2} = \frac{(2)^2 5}{3^2 - 2^2} = \frac{20}{5} = 4$ metre

34. A field is 90 metre long and 50 metre broad. A 25 metre long, 20 metre broad and 4 metre deep tank dug in the field and the earth taken out is spread evenly over the remaining field. How much the level of field will rise?

(a) 0.5 metre (b) 1 metre (c) 1.5 metre (d) 2 metre

Solution:

(a) Area of field = 90 × 50 = 4500 m^2

Area of field dug out = 25 × 20 = 500 m^2

∴ Area of remaining field = 4500 m^2 – 500 m^2 = 4000 m^2

Volume of the earth dug out = 25 × 20 × 4 = 2000 m^3

∴ Field will rise by $\frac{2000}{4000} = 0.5$ metre

Internal volume of box = $(11 - 1) \times (9 - 1) \times (7 - 0.5) = 10 \times 8 \times 6.5 = 520 \text{ cm}^3$

$\therefore$ Volume of wood = $693 - 520 = 173 \text{ cm}^3$.

28. How much metal is required to make a 20 metre long pipe, if its inner and outer diameter are 6 metre and 8 metre respectively.

(a) 400 m^3 (b) 440 m^3 (c) 800 m^3 (d) 880 m^3

Solution:

(b) Outer and inner radius of the pipe are 4 metre and 3 metre respectively.

$\therefore$ Quantity of metal required = Outer volume of the pipe – Inner volume of the pipe

$$= \pi (R^2 - r^2) h = \frac{22}{7} \times (4^2 - 3^2) \times 20 = \frac{22}{7} \times 7 \times 1 \times 20 = 440 \text{ m}^3$$

Hint: Pipe is a form of cylinder.

29. A sphere of radius 3 cm is put into water contained in a cylinder of 6 cm radius. If the sphere is completely immersed in the water, the water level in the cylinder rises by:

(a) 1 cm (b) $\frac{1}{3}$ cm (c) $\frac{1}{2}$ cm (d) $\frac{2}{3}$ cm

Solution:

(a) Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi (3)^3 = 36\pi$

On immersing the ball in the cylinder, volume of water will rise by volume of ball.

$\therefore$ π (Radius of cylinder)^2 (Rise in water) = 36π

$\therefore$ Rise in water $= \frac{36}{(6)^2} = 1$

$\therefore$ Level of water will rise by 1 cm.

30. How many solid spherical balls of radius 3 cm each are required to immerse in a cylindrical jar of radius 4 cm to raise the level of water in the jar by 9 cm?

(a) 4 (b) 6 (c) 9 (d) 16

Solution :

(a) Volume of one ball $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = 36\pi$

Volume of cylindrical jar (for 4 cm of water level rise) $= \pi r^2 h = \pi(4)^2 9 = 144\pi$

$\therefore$ Number of balls required $= \frac{144}{36\pi} = 4$

3.. A wire is drawn from a solid iron sphere of radius 6 cm. Find the length of the wire, if its diameter is 6 cm.

(a) 16 cm (b) 32 cm (c) 50 cm (d) 64 cm

Solution:

(b) An iron wire is a form of cylinder.

$\therefore$ Volume of cylinder = Volume of Sphere

$\therefore$ π (Radius of cylinder)^2 (Height of cylinder) $= \frac{4}{3}\pi$ (Radius of sphere)^3

$$\therefore \text{ Height of the cylinder} = \frac{4}{3} \times \frac{\pi \text{ (Radius of sphere)}^3}{\pi \text{ (Radius of cylinder)}^2}$$

23. How many spheres of diameter 2 cm can be made by melting one sphere of radius 4 cm?

(a) 16 (b) 32 (c) 64 (d) 128

Solution:

(c) Radius of big sphere = 4 cm

Radius of smaller sphere = $\frac{2}{2}$ = 1 cm

Ratio between radii of two spheres = 4 : 1

Ratio between volumes of two spheres = $4^3 : 1^3 = 64 : 1$

∴ 64 spheres can be made from the bigger sphere.

24. How many spheres of radius 2 cm can be made by melting two spheres of radius 12 cm each ?

(a) 18 (b) 108 (c) 216 (d) 432

Solution:

(d) Radius of big sphere = 12 cm

Radius of smaller sphere = 2 cm

Ratio between radii of two spheres = 12 : 2 = 6 : 1

Ratio between volumes of two spheres = $6^3 : 1^3 = 216 : 1$

∴ 216 spheres can be made from one big sphere.

∴ From two big spheres, smaller spheres made = 216 × 2 = 432

25. A cube of 9 cm a side is melted and 27 small cubes are made from that. Find side of the smaller cubes.

(a) 1.5 cm (b) 2 cm (c) 2.5 cm (d) 3 cm

Solution:

(d) We know, Volume of cube = Side^3

∴ Volume of the small cube = $\frac{9^3}{27} = 27$

∴ Side of the small cube = $\sqrt[3]{27}$ = 3 cm.

26. A box is made of 1 cm thick wood. If outer dimensions of box are 12 cm by 10 cm by 7 cm find volume of wood used.

(a) 246 cm^3 (b) 440 cm^3 (c) 616 cm^3 (d) 729 cm^3

Solution:

(b) Outer dimensions of box = 12 cm, 10 cm, 7 cm

Inner dimensions of box = 10 cm, 8 cm, 5 cm

∴ Volume of wood = $(12 \times 10 \times 7)\ \text{cm}^3 - (10 \times 8 \times 5)\ \text{cm}^3$

$= 840\ \text{cm}^3 - 400\ \text{cm}^3 = 440\ \text{cm}^3$.

27. A wooden box without lid has external length, breadth and height 11 cm, 9 cm and 7 c respectively. If wood is 5 mm thick, find the volume of wood used in making the box.

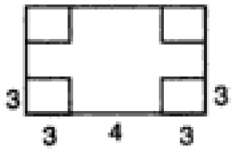
(a) 173 cm^3 (b) 213 cm^3 (c) 273 cm^3 (d) 693 cm^3

Solution :

(a) External volume of box = 11 × 9 × 7 = 693 cm

Solution :

(b) Volume of Box = l × b × h = 4 × 4 × 3 = 48 cm^3



18. Volume of two spheres are in the ratio of 27 : 125. Find radius of the bigger sphere, if radius of smaller sphere is 27 cm.

(a) 45 cm (b) 125 cm (c) 135 cm (d) 625 cm

Solution:

(a) Let radius of bigger and smaller sphere are 'R' and 'r' respectively.

Then $R : r = \sqrt[3]{125} : \sqrt[3]{27} = 5 : 3$

$\therefore R = \frac{5}{3} \times 27 = 45$ cm

19. Volume of two spheres are in the ratio of 343 : 8. Find the ratio between their surface areas.

(a) 7 : 2 (b) 49 : 4 (c) 81 : 45 (d) 343 : 8

Solution:

(b) Let radius of bigger and smaller sphere are 'R' and 'r' respectively.

Then $R : r = \sqrt[3]{343} : \sqrt[3]{8} = 7 : 2$

Ratio of their surface areas = $7^2 : 2^2 = 49 : 4$

20. Three solid cubes whose sides are 3 cm, 4 cm and 5 cm respectively are melted to form into a single cube. Find the side of new cube formed.

(a) 6 cm (b) 8 cm (c) 10 cm (d) 12 cm

Solution:

(a) Volume of new cube = Total volume of three cubes

$= 3^3 + 4^3 + 5^3 = 27 + 64 + 125 = 216 \text{ cm}^3$

$\therefore$ Side of new cube = $\sqrt[3]{216} = 6$ cm.

21. How many cubes of 2 cm, a side can be made by melting a solid cube of 6 cm, a side?

(a) 8 (b) 9 (c) 27 (d) 216

Solution:

(c) Ratio of sides of two cubes = 6 : 2 = 3 : 1

Ratio between volumes of two cubes = $3^3 : 1^3 = 27 : 1$

$\therefore$ 27 small cubes can be made from the bigger cube.

22. Find the cost of canvas cloth required for conical tent whose height is 3 m and radius is 4 m and cost of cloth is Rs. 140 per square metre.

(a) Rs. 7040 (b) Rs. 8800 (c) Rs. 5280 (d) Rs. 15840

Solution :

(b) Slant height $= \sqrt{3^2 + 4^2} = 5$ m

Curved Surface Area $= \pi r l = \frac{22}{7} \times 4 \times 5$

Cost of canvas cloth $= \frac{22}{7} \times 4 \times 5 \times 140 = \text{Rs. } 8800$

Solution :

(b) Dimensions of resulting cuboid

$l = 6 + 6 = 12$ cm, $b = 6$ cm, $h = 6$ cm

Surface Area $= 2 \times (12 \times 6 + 6 \times 6 + 12 \times 6)$

$= 2\,(72 + 36 + 72) = 2 \times 180 = 360 \text{ cm}^2$

13. If ratio of radii of two cylinders is 2 : 3 and ratio of their heights is 5 : 4, find the ratio of their volumes.

(a) 8 : 27 (b) 25 : 16 (c) 5 : 6 (d) 5 : 9

Solution :

(d) Ratio of volumes of two cylinders $= \pi(2)^2 5 : \pi(3)^2 4 = 5 : 9$

14. A pipe of diameter d can drain a certain water tank in 40 minutes. The time taken by a pipe of diameter 2d for doing the same job is :

(a) 10 minutes (b) 20 minutes (c) 80 minutes (d) 160 minutes

Solution :

(a) Ratio of diameter/radii of two pipes = d : 2d = 1 : 2

Ratio of time $= 2^2 : 1^2 = 4 : 1$

$\therefore$ Time taken by second pipe $= \frac{1}{4} \times 40 = 10$ minutes

15. The diameter and height of a cone are 14 cm and 24 cm respectively. Find curved surface area of the cone.

(a) 168 cm^2 (b) 528 cm^2 (c) 550 cm^2 (d) 1056 cm^2

Solution:

(c) Radius $= \frac{1}{2} \times 14 \text{ cm} = 7 \text{ cm}$

Slant height $= l = \sqrt{r^2 + h^2} = \sqrt{7^2 + 24^2} = \sqrt{625} = 25$ cm

Surface area $= \pi r l = \frac{22}{7} \times 7 \times 25 = 550 \text{ cm}^2$

16. The length, breadth and height of a room are 5 metre, 4 metre and 4 metre respectively. If all the four walls of the room are to be covered with 50 cm wide wall paper, find the length of the paper required.

(a) 36 metre (b) 72 metre (c) 144 metre (d) 288 metre

Solution:

(c) Area of four walls = 2 × Height (Length + Breadth)

$= 2 \times 4 \times (5 + 4) = 72 \text{ m}^2$

Width of the wall paper = 50 cm = 0.5 metre

$\therefore$ Length of the paper required $= 72 \div 0.5 = 72 \div \frac{1}{2} = 72 \times 2 = 144$ metre

17. A square card board has a side 10 cm. From each corner a square of 3 cm is cut down and then the corners are folded to make an open box. What is the volume of open box ?

(a) 27 cm^3 (b) 48 cm^3 (c) 64 cm^3 (d) 90 cm^3

Solution :

(a) Area of the sides are l × b, b × h and l × h

Volume of cube = $l \times b \times h = \sqrt{15 \times 12 \times 5} = 30\ m^3$

9. If each side of a cube is increased by 30%, then its surface area will be increased by:

(a) 50% (b) 51% (c) 60% (d) 69%

Solution:

(d) Percent increase in surface area = $2 \times 30 + \frac{30^2}{100} = 60 + 9 = 69\%$

Alternative Method :

$13 \times 13 = 169$

10. If radius and height of a cylinder are increased by 20%, find percentage increase in its volume.

(a) 40% (b) 44% (c) 66% (d) 72.8%

Solution:

(d) Percent increase in Volume = $(3 \times 20) + \frac{3 \times 20^2}{100} + \frac{20^3}{100^2}$

$= 60 + 12 + 0.8 = 72.8\%$

Alternative Method:

$12^3 - 10^3 = 728$

$\therefore$ Percent increase = $\frac{728}{1000} \times 100 = 72.8\%$

Hint: We assumed that each side = 10 cm

Then increased side = 10 + 20% of 10 = 12 cm

11. Dimensions of a cuboid are 29 cm, 18 cm and 15 cm. If its sides are increased by 20%, 25% and 40% respectively, find percent increase in its volume.

(a) 85% (b) 104% (c) 110% (d) 121%

Solution:

(c) Percent increase in Volume

$$= (20 + 25 + 40) + \frac{20 \times 25 + 25 \times 40 + 20 \times 40}{100} + \frac{20 \times 25 \times 40}{100 \times 100}$$

$$= (85) + (5 + 10 + 8) + (2) = 85 + 23 + 2 = 110\%$$

Hint: Dimensions of cuboid are immaterial in this method.

Alternative Method :

$12 \times 12.5 \times 14 - 1000 = 2100 - 1000 = 1100$

$\therefore$ Per cent increase $= \frac{1100}{1000} \times 100 = 110\%$

12. Two cubes each 6 m edge are joined end to end. The surface area of the resulting cuboid is :

(a) $216\ cm^2$ (b) $360\ cm^2$ (c) $512\ cm^2$ (d) $720\ cm^2$

5. If result is '0' or a multiple of 'y', then $(x^n - a)$ is divisible by 'y'.

Example 1:

Is $7^6 - 1$ divisible by 16?

Solution:

$$7^6 - 1 = 7^2 \times 7^2 \times 7^2 - 1 = 49 \times 49 \times 49 - 1$$

Remainder obtained on dividing each part by 16

$= 1 \times 1 \times 1 - 1 = 0$

$\therefore$ $7^6 - 1$ is divisible by 16.

Example 2:

Is $3^9 - 1$ divisible by 26?

Solution:

$3^9 - 1 = 3^3 \times 3^3 \times 3^3 - 1 = 27 \times 27 \times 27 - 1$

Remainder obtained on dividing each part by 26

$= 1 \times 1 \times 1 - 1 = 0$

$\therefore$ $3^9 - 1$ is divisible by 26.

Example 3:

Is $5^5 - 1$ divisible by 11?

Solution:

$5^5 - 1 = 5^2 \times 5^3 - 1 = 25 \times 125 - 1$

Remainder obtained on dividing each part by 11

$= 3 \times 4 - 1 = 12 - 1 = 11$, which is divisible by 11.

$\therefore$ $5^5 - 1$ is divisible by 11.

Example 4:

Is $2^{12} - 4$ divisible by 11?

Solution:

$2^{12} - 4 = 2^6 \times 2^6 - 4 = 64 \times 64 - 4$

Remainder obtained on dividing each part by 11

$= 9 \times 9 - 4 = 81 - 4 = 77$, which is divisible by 11.

$\therefore$ $2^{12} - 4$ is divisible by 11.

SOLVED EXERCISE

1. Students of a class stand in a queue. If Ramesh is 19th in order from both ends, how many students are there in the queue?

(a) 20 (b) 37 (c) 38 (d) 39

Solution:

(b) Ramesh is 19th from both the sides.

$\therefore$ There are 18 students on his each side.

$\therefore$ Total students in the queue $= 18 \times 2 + 1 = 37$

2. In a row, A is 5th from the left side and B is 8th from the right side. They both interchange their positions and then A becomes 12th from the left side. How many persons are there in the row?

(a) 17 (b) 19 (c) 20 (d) 21

Solution:

(b) B is 8th from the right side.

When they interchange their places, A is at B's place and his place becomes 12th from the left side.

∴ Same place is 8th from one side and 12th from the other side.

∴ Total persons in the row = 8 + 12 – 1 = 19

3. Find the amount for which a car worth Rs. 450000 should be insured at 10% so that in case of loss of car, cost of the car as well as the amount paid towards premium is recovered.

(a) Rs. 45000 (b) Rs. 495000 (c) Rs. 500000 (d) Rs. 600000

Solution:

(c) Premium is paid on the insured value of the goods.

∴ Car worth Rs. 90 should be insured for Rs. 100.

∴ Car worth Rs. 450000 should be insured for Rs. $450000 \times \frac{100}{90}$

= Rs. 500000

4. A certain number when successively divided by 4 and 7 leaves the remainders 2 and 3 respectively. What is the remainder if the same number is divided by 28?

(a) 5 (b) 14 (c) 17 (d) 26

Solution:

(b) Remainder = First Divisor × Second Remainder + First Remainder

$= 4 \times 3 + 2 = 14$

5. A and B have 3 and 5 loaves respectively. They share their food with C who pays them Rs. 16. If each of them got equal food, find A's share in Rs. 16.

(a) Rs. 2 (b) Rs. 3 (c) Rs. 5 (d) Rs. 6

Solution:

(a) Total loaves = 3 + 5 = 8

Share of each = $\frac{8}{3}$

A's sacrifice = $3 - \frac{8}{3} = \frac{1}{3}$

B's sacrifice = $5 - \frac{8}{3} = \frac{7}{3}$

∴ Ratio of their sacrifice = $\frac{1}{3} : \frac{7}{3} = 1 : 7$

∴ A's share in Rs. 16 = $\frac{1}{1+7}$ × Rs. 16 = Rs. 2

6. If 1 is added to the numerator of a fraction, the fraction becomes 1. If 1 is added to denominator, the fraction becomes $\frac{1}{2}$, find the fraction.

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Solution:

(c) Let, the fraction = $\frac{x}{y}$

Then $\frac{x+1}{y} = 1$

$\therefore y = x + 1$ (1)

And $\frac{x}{y+1} = \frac{1}{2}$

$\therefore 2x = y + 1$ (2)

Solving (1) and (2), we get:

$x = 2$ and $y = 3$

$\therefore$ The fraction is $\frac{2}{3}$.

7. If total cost of 7 mangoes and 4 apples is equal to cost of 5 mangoes and 6 apples. Find the ratio between the cost of 1 mango and the cost of 1 apple.

(a) 1 : 1 (b) 2 : 3 (c) 5 : 6 (d) 7 : 4

Solution:

(a) In the second case, 2 (i.e. 7 – 5) mangoes are replaced by 2 (i.e. 6 – 4) apples, total cost remaining the same in both the cases.

$\therefore$ Ratio of their costs = 2 : 2 = 1 : 1

8. If the cost of 9 mangoes and 5 apples is equal to cost of 7 mangoes and 8 apples. Find the ratio between the cost of 1 mango and the cost of 1 apple.

(a) 2 : 3 (b) 3 : 2 (c) 5 : 8 (d) 9 : 7

Solution:

(b) In the second case, 2 mangoes (i.e. 9 – 7) are replaced by 3 apples (i.e. 8 – 5), total cost remaining the same in both the cases.

$\therefore$ Ratio of their costs = 3 : 2

9. Total cost of 3 tables and 2 chairs is Rs. 1300 and that of 2 tables and 3 chairs is Rs. 1200. Find the cost of one chair.

(a) Rs. 100 (b) Rs. 150 (c) Rs. 200 (d) Rs. 250

Solution:

(c) In the second case, 1 table is substituted by 1 chair.

As a result of it, total cost is decreased by Rs.1300 – Rs. 1200

= Rs. 100

$\therefore$ Cost of 1 table = Cost of 1 chair + Rs. 100

Cost of 3 tables + cost of 2 chairs = Rs. 1300

$\therefore$ 3 × (Cost of 1 chair + Rs. 100) + Cost of 2 chairs = Rs. 1300

$\therefore$ Cost of 5 chairs = Rs. 1300 – Rs. 300 = Rs. 1000

$\therefore$ Cost of 1 chair = Rs. $\frac{1000}{5}$ = Rs. 200

10. Total cost of 7 pens and 4 books is Rs. 232 and that of 4 pens and 7 books is Rs. 241. Find the cost of one book.

(a) Rs. 19 (b) Rs. 20 (c) Rs. 21 (d) Rs. 23

Solution:

(d) In the second case, 3 pens are substituted by 3 books.

Increase in total cost = Rs. 241 – Rs. 232 = Rs. 9

Difference between cost of one book and one pen = $\frac{9}{3}$ = Rs. 3

∴ Cost of 1 book = Cost of 1 pen + Rs. 3

Cost of 7 pens + cost of 4 books = Rs. 232

∴ Cost of 11 pens = Rs. 232 – 4 × Rs. 3 = Rs. 220

∴ Cost of 1 pen = Rs. $\frac{220}{11}$ = Rs. 20

∴ Cost of 1 book = Rs. 20 + Rs. 3 = Rs. 23

11. Total cost of 5 pencils and 7 pens is Rs. 45. Total cost of 7 pencils and 6 pens is Rs. 44. Find the cost of one pencil.

(a) Rs. 2 (b) Rs. 3 (c) Rs. 4 (d) Rs. 5

Solution:

(a) In the second case, 1 pen is replaced by 2 pencils.

Decrease in total cost = Rs. 45 – Rs. 44 = Re. 1

∴ Cost of 1 pen = Cost of 2 pencils + Re. 1

Cost of 5 pencils + cost of 7 pens = Rs. 45

∴ Cost of 5 pencils + 7 (cost of 2 pencils + Re. 1) = Rs. 45

∴ Cost of 19 pencils = Rs. 45 – Rs. 7 = Rs. 38

∴ Cost of 1 pencil = Rs. $\frac{38}{19}$ = Rs. 2

12. A mass of gold and silver weighting 20 grams costs Rs. 780. Had the weights of gold and silver are exchanged it would have cost Rs. 920. Find the price of 1 gram of gold, if one gram of silver costs Rs. 25.

(a) Rs. 25 (b) Rs. 40 (c) Rs. 50 (d) Rs. 60

Solution:

(d) Let the mass contains 'x' and (20 – x) grams of gold and silver respectively.

Then new mass contains (20 – x) and 'x' grams of gold and silver respectively.

Total weight of gold in two masses = x + (20 – x) = 20 grams

Total weight of silver in two masses = (20 – x) + x = 20 grams

∴ Total weight of gold and silver in two masses is 20 grams each.

Total cost of two masses = Rs. 780 + Rs. 920 = Rs. 1700

Cost of 20 grams of silver = 20 × Rs. 25 = Rs. 500

∴ Cost of 20 grams of gold = Rs. 1700 – Rs. 500 = Rs. 1200

∴ Cost of 1 gram of gold = Rs. $\frac{1200}{20}$ = Rs. 60

13. A shopkeeper has sufficient money to buy 50 books. On reduction in price of one book by Rs. 4, he could buy 10 books more. How much money does he has?

Rs. 1500 is half of the amount deposited initially.

∴ Amount deposited initially = 2 × Rs. 1500 = Rs. 3000

16. A train at the first station drops one-third of the passengers and takes 50 more. At the second station it drops half of the remaining passengers and takes 25 more. If there were 150 passengers left in the train now, how many passengers were there in the train at the starting point?

(a) 200 (b) 250 (c) 300 (d) 350

Solution:

(c) Passengers in the train at the third station = 150

Passengers in the train at the second station (before taking 25 more passengers)

= 150 – 25 = 125

Passengers in the train at the first station = 2 × 125 = 250

Passengers in the train at the first station (before taking 50 more passengers)

= 250 – 50 = 200

Passengers at the first station = $\frac{3}{2} \times 200 = 300$

Hint: At the first station, trains drops $\frac{1}{3}$ passenger.

∴ Remaining passengers = $1 - \frac{1}{3} = \frac{2}{3}$

Inverse of $\frac{2}{3} = \frac{3}{2}$

17. A shopkeeper sold half of his stock of apples and 1 more to his first customer, half of the remaining stock and 2 more to the second customer and half of the remaining stock and 3 more to the third customer. If he still has 12 apples with him, how many apples had he in the beginning?

(a) 56 (b) 91 (c) 110 (d) 130

Solution:

(d) Stock left at the end = 12

Apples left before selling to the 3rd customer = 2 (12 + 3) = 30

Apples left before selling to the 2nd customer = 2 (30 + 2) = 64

Apples left before selling to the 1st customer = 2 (64 + 1) = 130

∴ He had 130 apples at the beginning.

18. A shopkeeper sold half of his stock of apples and 1 more to his first customer, half of the remaining stock and 1 more to the second customer and half of the remaining stock and 1 more to the third customer and so on to fourth customer. If he exhausted the stock, how many apples had he in the beginning?

(a) 4 (b) 12 (c) 30 (d) 40

Solution:

(c) Stock left at the end = 0

Apples left before selling to the 4th customer = 2 (0 + 1) = 2

Apples left before selling to the 3rd customer = 2 (2 + 1) = 6

Apples left before selling to the 2nd customer = 2 (6 + 1) = 14

Apples left before selling to the 1st customer = 2 (14 + 1) = 30

= 20 + 18 + 16 = Rs. 54 each

23. A man invested Rs. 50000 at simple interest in three parts – one-third at 6% p.a., one-third at 7%. At what rate must he invest the remaining amount so that he earns 8% interest on the total amount.

(a) 9% (b) 10% (c) 11% (d) 12%

Solution:

(c) Let total amount = Rs. 300

∴ Amount invested in each part = Rs. 300 ÷ 3 = Rs. 100

Total interest earned = Rs. 3 × 8 = Rs. 24

Interest earned from first and second part = Rs. 6 + Rs. 7 = Rs. 13

Interest to be earned from the third part = Rs. 24 – Rs. 13 = Rs. 11

∴ Third part was invested at 11%.

Trick:

8% + (8% – 6%) + (8% – 7%)

8% + 2% + 1% = 11%

24. A merchant bought some articles for Rs. 1000 and sold $\frac{1}{4}$ of his articles at 5% profit. At what profit should he sell the remaining articles so as to gain 20% on selling the whole goods?

(a) 10% (b) 15% (c) 20% (d) 25%

Solution:

(d) Let articles bought = 4 of Rs. 100 each.

Then total profit made = 4 × 20 = 80

Profit on first article = 5

∴ Profit on remaining 3 articles = 80 – 5 = 75

∴ Profit on each of the three articles = $\frac{75}{3} = 25$

Hence $\frac{3}{4}$ of the goods are to be sold at 25% profit.

25. A merchant sold one-third of his goods at 10% loss. At what profit should he sell the remaining goods so as to gain 20% on selling the whole goods?

(a) 5% (b) 10% (c) 30% (d) 35%

Solution:

(d) Let articles bought = 3 of Rs. 100 each.

Then total profit made = 3 × 20 = 60

Profit on first article = – 10

∴ Profit on remaining 2 articles = 60 – (–10) = 70

∴ Profit on each of the two articles = $\frac{70}{2} = 35$

Hence $\frac{2}{3}$ of the goods are to be sold at 35% profit.

26. If $a^x = b$ and $b^y = a$, find the value of xy.

(a) 0 (b) 1 (c) a (d) b

Solution:

(b) $a = b^y = (a^x)^y = a^{xy}$

$\therefore$ C's salary $= 6x \times \frac{100}{20} = 30x$

$\therefore$ Ratio of their salaries $= \frac{40}{3} : 20 : 30 = 4 : 6 : 9$

Short-cut Method:

Ratio of their actual savings = 4 : 5 : 6

Ratio of their savings in terms of theirs salaries = 30 : 25 : 20 = 6 : 5 : 4

$$\text{Ratio of their salaries} = \frac{\text{Ratio of actual savings}}{\text{Ratio of their savings in terms of salaries}}$$

$$= \frac{4}{6} : \frac{5}{5} : \frac{6}{4} = \frac{2}{3} : 1 : \frac{3}{2} = 4 : 6 : 9$$

31. A started a business with Rs. 40000 and was later joined by B with Rs. 30000. After how many months B had joined if profit at the end of the year was divided between A and B in the ratio of 2 : 1 ?

(a) 3 months (b) 4 months (c) 5 months (d) 6 months

Solution :

(b) Ratio of the capitals = 40000 : 30000 = 4 : 3

Ratio of profit = 2 : 1

$\therefore$ Ratio of periods $= \frac{2}{4} : \frac{1}{3} = 3 : 2$

$\therefore$ B has invested for $\frac{2}{3} \times 12 = 8$ months

32. Two vessels contain mixture of milk and water in the ratio of 9 : 1 and 1 : 4 respectively. A third vessel is filled in by taking mixtures from these two vessels in such a way that the mixture in the third vessel contains half milk and half water. What is the ratio of quantities drawn from the two vessels?

(a) 3 : 4 (b) 1 : 1 (c) 4 : 9 (d) 9 : 4

Solution:

(a) Let 10 litres (i.e. 9 + 1) of mixture is taken from the first vessel.

10 litres of this mixture contains 9 litres of milk and 1 litre of water i.e. 8 litres more of milk than water.

This excess quantity of milk is to equalized by drawing mixture from the second vessel. 5 litres (i.e. 1 + 4) of mixture of second vessel contains 1 litre of milk and 4 litres of water, i.e., 3 litres more of water for every 5 litres of mixture.

$\therefore$ Mixture drawn from second vessel $= 8 \times \frac{5}{3} = \frac{40}{3}$

$\therefore$ Ratio of mixtures drawn from two vessels $= 10 : \frac{40}{3} = 30 : 40 = 3 : 4$

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